William Stallings
Data and Computer Communications

Chapter 3

Data Transmission

Transmission Terminology

- Transmission over transmission medium using electromagnetic waves.
- Transmission media
  - Guided media
    - Waves guided along physical path
    - e.g. twisted pair, coaxial cable, optical fiber
  - Unguided media
    - Waves not guided
    - e.g. air, water, vacuum

Transmission Terminology

- Direct link
  - No intermediate devices other than amplifiers and repeaters
- Point-to-point
  - Direct link
  - Only 2 devices share link
- Multi-point
  - More than two devices share the link

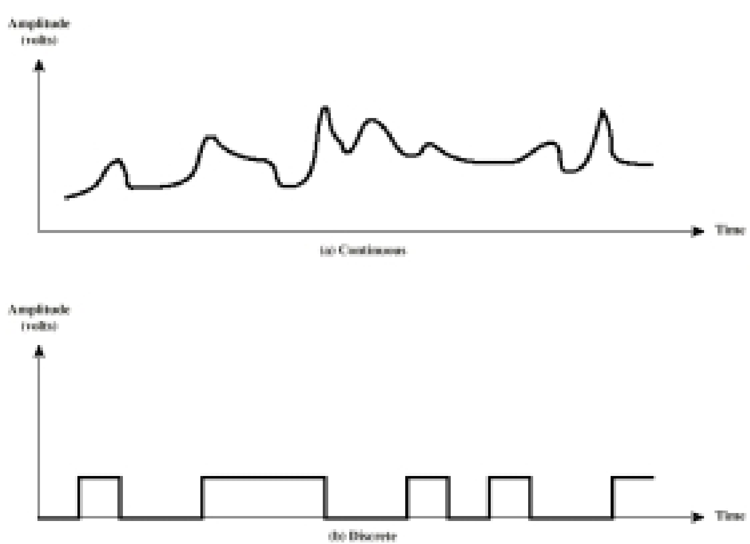
Transmission Terminology

- Simplex
  - One direction
    - e.g. Television
- Half duplex
  - Either direction, but only one way at a time
    - e.g. police radio
- Full duplex
  - Both directions at the same time
    - e.g. telephone

Frequency, Spectrum and Bandwidth

- Electromagnetic signals used to transmit data
- Signal is a function of time or frequency
- Time domain concepts
  - Continuous signal
    - Varies in a smooth way over time
  - Discrete signal
    - Maintains a constant level then changes to another constant level
  - Periodic signal
    - Same signal pattern repeated over time
  - Aperiodic signal
    - Signal pattern not repeated over time

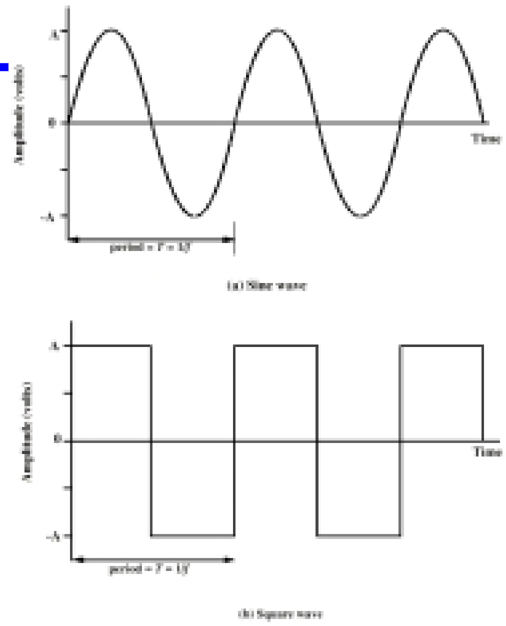
Continuous & Discrete Signals



Periodic Signals

Signal is said to be Periodic if
S(t+T) = s(t)   for all t

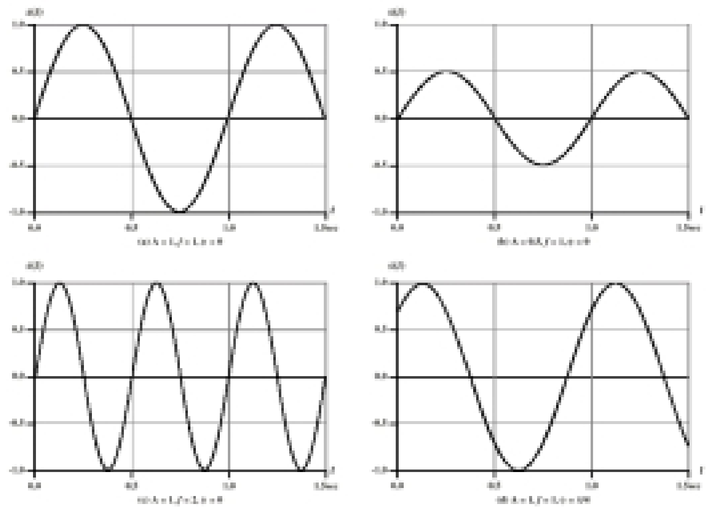
T is the period of signal



Sine Wave

- Sine wave is fundamental periodic signal
- Peak Amplitude (A)
  - maximum signal intensity over time, measured in volts
- Frequency (f)
  - Rate at which signal repeats
  - Hertz (Hz) or cycles per second
  - Period = time for one repetition (T)
  - T = 1/f
- Phase (ϕ)
  - Relative position in time within a single period
  - $s(t) = A \sin(2\pi f t + \phi)$

Varying Sine Waves



Wavelength

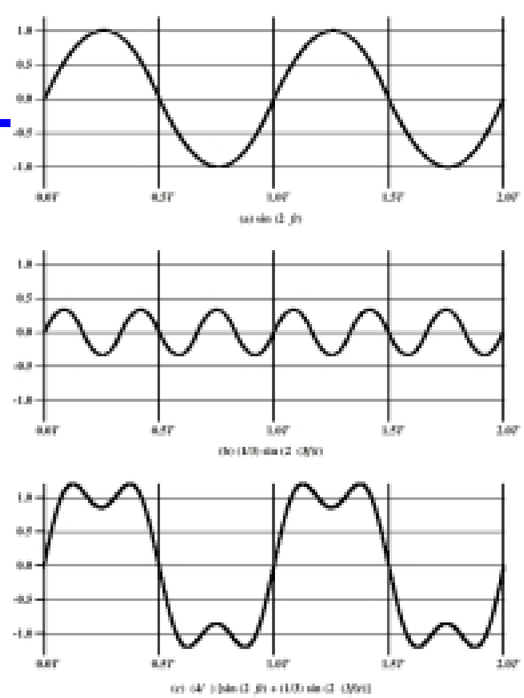
- Distance occupied by one cycle, expressed as λ
- Distance between two points of corresponding phase in two consecutive cycles
- Assuming signal velocity *v*
  - $\lambda = vT$
  - $\lambda f = v$
  - c = 3*10^8 m/s (speed of light in free space)

Frequency Domain Concepts

- Signal usually made up of many frequencies
- Components are sinusoidal waves
- Can be shown (Fourier analysis) that any signal is made up of component sinusoidal waves
- Fundamental frequency
  - Base frequency such that frequency of all components expressed as its integer multiples
  - Period of aggregate signal is same as period of fundamental frequency

Addition of Frequency Components

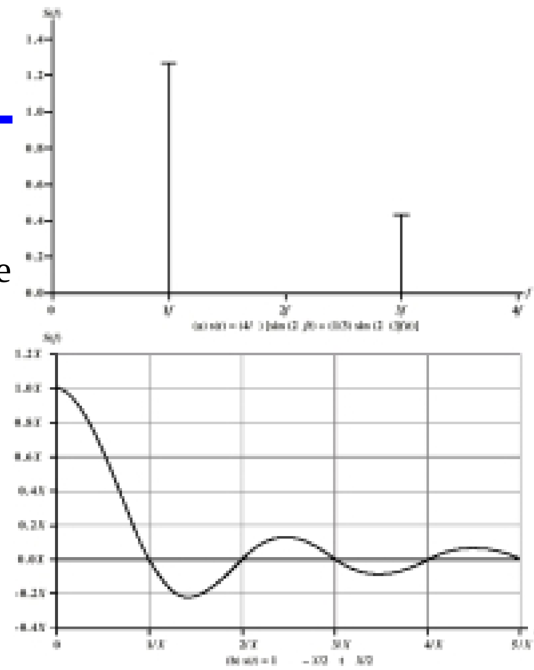
The signal
$s(t) = 4/\pi\,[\sin(2\pi f t)$
$\quad + 1/3 \sin(2\pi (3f)\, t)]$
is made up of two frequency components



Frequency Domain

Time domain function *s(t)* specifies a signal in terms of its amplitude at each instant of time.

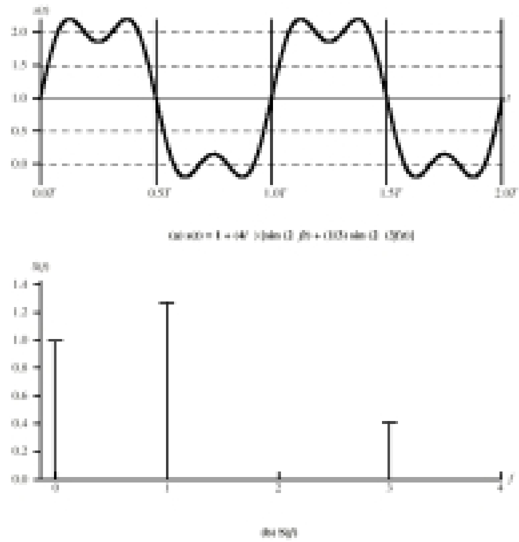
Frequency domain function *S(f)* specifies a signal in terms of its peak amplitude of constituent frequencies.



Spectrum & Bandwidth

- Spectrum
  - range of frequencies contained in signal
- Absolute bandwidth
  - width of spectrum
- Effective bandwidth
  - Often just *bandwidth*
  - Narrow band of frequencies containing most of the energy
- DC Component
  - Component of zero frequency
  - Changes average amplitude of signal to non-zero

Signal with DC Component



Data Rate and Bandwidth

- Any transmission system has a limited band of frequencies
  - Range of FM radio transmission is 88-108 MHZ
- This limits the data rate that can be carried
- Increasing bandwidth increases data rate
- A given bandwidth can support various data rates depending on receiver’s ability to distinguish 1 and 0 signals.

Data Rate and Bandwidth

- Any digital waveform has infinite bandwidth
  - Transmission system limits waveform as a signal over medium
  - Medium cost is directly proportional to transmission bandwidth
  - Signal of limited bandwidth preferable to reduce cost
  - Limiting bandwidth creates distortions making it difficult to interpret received signal

Frequency Components of a Square Wave

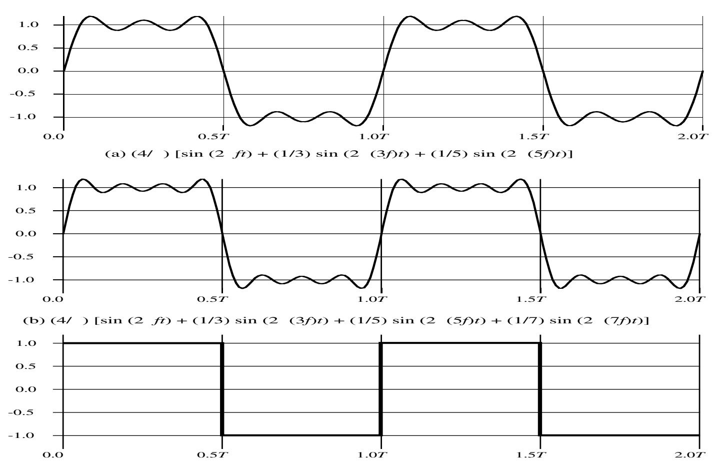


Figure 3.7 Frequency Components of Square Wave ($T = 1/f$)

Data Rate and Bandwidth

- Assume digital transmission system has bandwidth of 4 MHZ
- Transmitting sequence of alternating 1's and 0's as a square wave
- What data rate can be achieved?

Case 1

- Approximate square wave with waveform of first three sinusoidal components
  - $s(t)= 4/\pi[\sin(2\pi f\ t)+1/3\ \sin(2\pi(3f)t)+ 1/5\ \sin(2\pi(5f)t)]$
- Bandwidth = 4 MHZ = $5f - f = 4f$
  - => f = 1 MHZ $=10^6$ cycles/second
- For f = 1 MHZ, period of fundamental frequency $T=1/10^6= 10^{-6}$ = 1 us
  - One bit occurs every 0.5 us
  - Data rate is 2 x 10^6 bps or 2 Mbps

Case 2

- Assume bandwidth is 8 MHZ
- Bandwidth = 8 MHZ = $5f - f = 4f$
  - => $f = 2$ MHZ = 2×10^6 cycles/second
- For $f = 2$ MHZ, period of fundamental frequency $T = 1/(2 \times 10^6) = 0.5 \times 10^{-6} = 0.5$ us
  - One bit occurs every 0.25 us
  - Data rate is 4×10^6 bps or 4 Mbps
- Bandwidth = 4 MHZ Data Rate = 2 Mbps
- Bandwidth = 8 MHZ Data Rate = 4 Mbps

Case 3

- Approximate square wave with waveform of first two sinusoidal components
  - $s(t) = 4/\pi[\sin(2\pi f\,t) + 1/3\,\sin(2\pi(3f)t)]$
- Assume bandwidth = 4 MHZ = $3f - f = 2f$
  - => $f = 2$ MHZ $= 2 \times 10^6$ cycles/second; period $T = 0.5$ us
  - One bit occurs every 0.25 us
  - Data rate is 4×10^6 bps or 4 Mbps
- Bandwidth = 4 MHZ Data Rate = 4 Mbps
- A given bandwidth can support various data rates depending on ability of receiver to distinguish 0 & 1

Effect of Bandwidth on Digital Signal

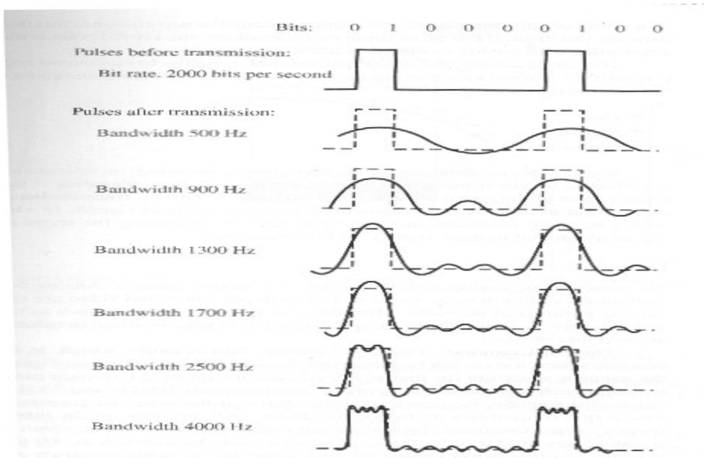


Figure 3.8 Effect of Bandwidth on a Digital Signal

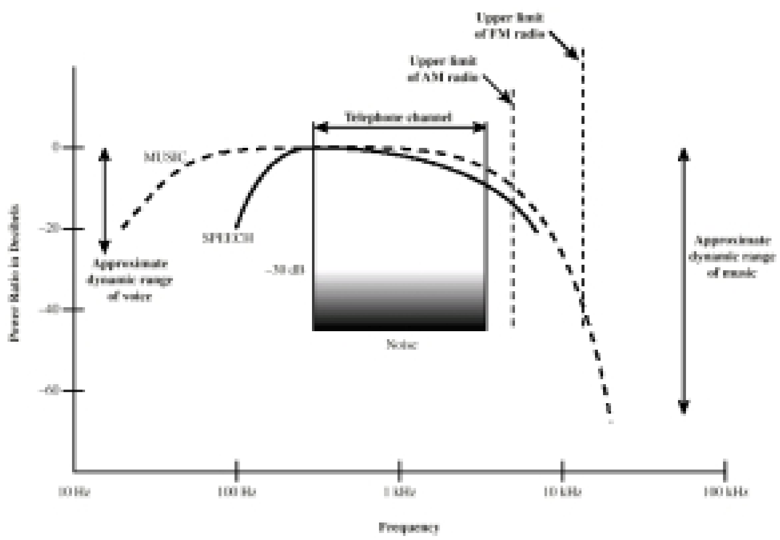
Analog and Digital Data Transmission

- Data
  - Entities that convey meaning or information
- Signals
  - Electric or electromagnetic representations of data
- Transmission
  - Communication of data by propagation and processing of signals

Data

- Analog
  - Continuous values within some interval
  - e.g. sound, video, data collected by sensors
- Digital
  - Discrete values
  - e.g. text, integers

Acoustic Spectrum (Analog)



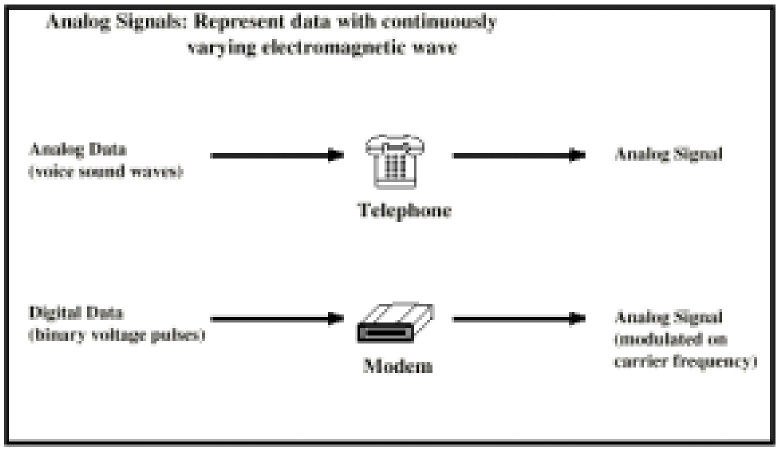
Signals

- Means by which data are propagated
- Analog
  - Continuously variable
  - Various media
    - wire, fiber optic, space
  - Speech bandwidth 100Hz to 7kHz
  - Telephone bandwidth 300Hz to 3400Hz
  - Video bandwidth 4MHz
- Digital
  - Use two DC components

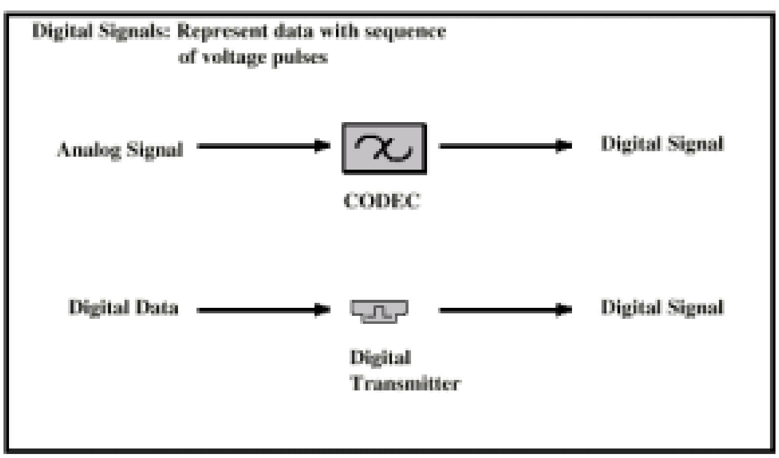
Data and Signals

- Usually use digital signals for digital data and analog signals for analog data
- Can use analog signal to carry digital data
  - Modem
- Can use digital signal to carry analog data
  - Compact Disc audio

Analog Signals Carrying Analog and Digital Data



Digital Signals Carrying Analog and Digital Data



Analog Transmission

- Analog signal transmitted without regard to content
- May be analog or digital data
- Attenuated over distance
- Use amplifiers to boost signal
- Also amplifies noise
- With amplifiers cascaded to achieve long distances, the signal becomes more distorted.

Digital Transmission

- Concerned with content
- Integrity endangered by noise, attenuation etc.
- Repeaters used
  - Repeater receives signal
  - Extracts bit pattern
  - Retransmits
  - Attenuation is overcome
- Noise is not amplified

Advantages of Digital Transmission

- Digital technology
  - Low cost LSI/VLSI technology
- Data integrity
  - Longer distances over lower quality lines
- Capacity utilization
  - High bandwidth links economical
  - High degree of multiplexing easier with digital techniques
- Security & Privacy
  - Encryption
- Integration
  - Can treat analog and digital data similarly

Decibels and Signal Strength

- Decibel is a measure of ratio between two signal levels
  - N_{dB} = number of decibels
  - P_1 = input power level
  - P_2 = output power level

$$N_{dB} = 10\log_{10}\frac{P_2}{P_1}$$

- Example:
  - A signal with power level of 10mW inserted onto a transmission line
  - Measured power some distance away is 5mW
  - Loss expressed as $N_{dB} = 10log(5/10) = 10(-0.3) = -3\ dB$

Decibels and Signal Strength

- Decibel is a measure of relative not absolute difference
  - A loss from 1000 mW to 500 mW is a loss of 3dB
  - A loss of 3 dB halves the power
  - A gain of 3 dB doubles the power
- Example:
  - Input to transmission system at power level of 4 mW
  - First element is transmission line with a 12 dB loss
  - Second element is amplifier with 35 dB gain
  - Third element is transmission line with 10 dB loss
  - Output power P2
    - (-12+35-10)=13 dB = 10 log (P_2 / 4mW)
    - P_2 = 4 x $10^{1.3}$ mW = 79.8 mW

Relationship Between Decibel Values and Powers of 10

Power Ratio	dB	Power Ratio	dB
10^1	10	10^{-1}	-10
10^2	20	10^{-2}	-20
10^3	30	10^{-3}	-30
10^4	40	10^{-4}	-40
10^5	50	10^{-5}	-50
10^6	60	10^{-6}	-60

Decibel-Watt (dBW)

- Absolute level of power in decibels
- Value of 1 W is a reference defined to be 0 dBW

$$Power_{dBW} = 10\log_{10}\frac{Power_W}{1W}$$

- Example:
  - Power of 1000 W is 30 dBW
  - Power of 1 mW is –30 dBW

Decibel & Difference in Voltage

- Decibel is used to measure difference in voltage.
- Power $P=V^2/R$

$$N_{dB} = 10\log\frac{P_2}{P_1} = 10\log\frac{V_2^{\,2}/R}{V_1^{\,2}/R} = 20\log\frac{V_2}{V_1}$$

- Decibel-millivolt (dBmV) is an absolute unit with 0 dBmV equivalent to 1mV.
  - Used in cable TV and broadband LAN

$$Voltage_{dBmV} = 20\log\frac{Voltage_{mV}}{1mV}$$

Transmission Impairments

- Signal received may differ from signal transmitted
- Analog - degradation of signal quality
- Digital - bit errors
- Caused by
  - Attenuation and attenuation distortion
  - Delay distortion
  - Noise

Attenuation

- Signal strength falls off with distance
- Depends on medium
  - Logarithmic for guided media; constant number of decibels per unit distance
  - For unguided media, complex function of distance and atmospheric conditions
- Received signal strength:
  - must be strong enough to be detected
  - must be sufficiently higher than noise to be received without error
- Attenuation is an increasing function of frequency

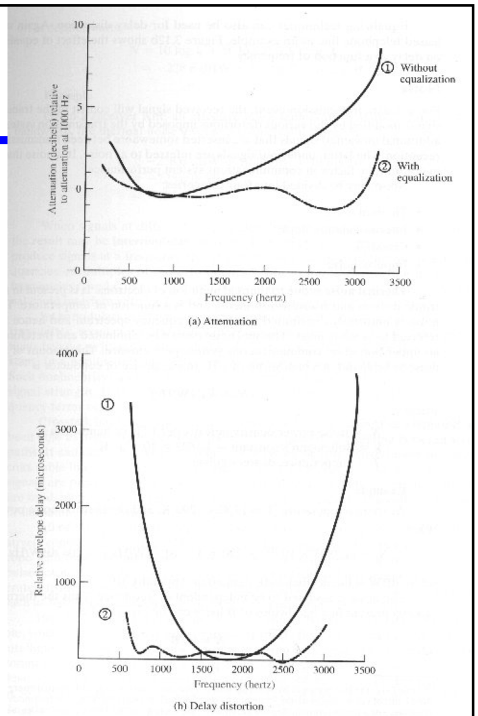
Attenuation Distortion

- Beyond a certain distance attenuation becomes large
  - Use repeaters or amplifiers to strengthen signal
- Attenuation distorts received signal, reducing intelligibility
- Attenuation can be equalized over a band of frequencies
  - Using loading coils that change electrical properties of lines
  - Use amplifiers that can amplify higher frequencies more than lower frequencies
- Attenuation distortion has less effect on digital signals
  - Strength of digital signal falls off rapidly with frequency

Delay Distortion

- Only in guided media
- Signal propagation velocity varies with frequency
  - In bandlimited signal, velocity tends to be higher near center frequency and falls off towards two edges of band
  - Varying frequency components arrive at receiver at different times => phase shifts between different frequencies
- Critical for digital data transmission
  - Some signal components of one bit position spill over to other bit positions, causing intersymbol interference
  - Major limitation to maximum bit rate over transmission channel
- May be reduced by equalization techniques

Attenuation & Delay Distortion Curves for a Voice Channel



Noise

- Undesired signals inserted into real signal during transmission
- Four types of noise
- Thermal (white noise)
  - Due to thermal agitation of electrons
  - Uniformly distributed across frequency spectrum
  - Function of temperature; present in all electronic devices
  - Cannot be eliminated and places an upper bound on system performance

Thermal Noise

- Thermal noise in bandwidth of 1 Hz in any device

$$N_0 = k\,T\,(W/Hz)$$

  - N_0=noise power density in watts per 1 Hz of bandwidth
  - K=Boltzmann's constant=1.3803×10^{-23} J/K
  - T=temperature, degrees Kelvin (=T-273.15 degrees Celsius)
- Example
  - At room temperature, T=17 C or 290 K
  - Thermal noise power density $N_0 = (1.3803\times10^{-23})\times290$ $=4\times10^{-21}$ W/Hz
  - $=10 \log (4\times10^{-21})/1$ W = -204 dBW/HZ

Thermal Noise

- Thermal noise is assumed independent of frequency
- Thermal noise in watts in a bandwidth of B hertz

$$N = k\,T\,\mathrm{B}$$

- Thermal noise in decibel-watts

$$N = 10\log k + 10\log T + 10\log B$$

$$N = -228.6 dBW + 10\log T + 10\log B$$

Thermal Noise

- Example:
  - Given a receiver with effective noise temperature of 100 K and a 10 MHZ bandwidth
  - Thermal noise level at receiver's output

$$N = -228.6 \text{ dBW} + 10 \log 10^2 + 10 \log 10^7$$
$$= -228.6 + 20 + 70$$
$$= -138.6 \text{ dBW}$$

Intermodulation Noise

- Signals at different frequencies share the same transmission medium
- May result in signals that are sum or difference or multiples of original frequencies
- Occurs when there is nonlinearity in transmitter, receiver, transmission system
  - Nonlinearity caused by component malfunction or excessive signal strength

Crosstalk

- Unwanted coupling between signal paths
- Signal from one line is picked up by another
- Occurs due to
  - Electrical coupling between nearby twisted pairs,
  - Electrical coupling between multiple signals on coaxial cable,
  - Unwanted signals picked up by microwave antennas
- Same order of magnitude or less than thermal noise

Impulse Noise

- Noncontinuous noise; irregular pulses or spikes of short duration and high amplitude
- May be caused by lightning or flaws in communication system
- Not a major problem for analog data but can be significant for digital data
  - A spike of 0.01 s will not destroy any voice data but will destroy 560 bits transmitted at 56 kbps

Effect of Noise on Digital Data

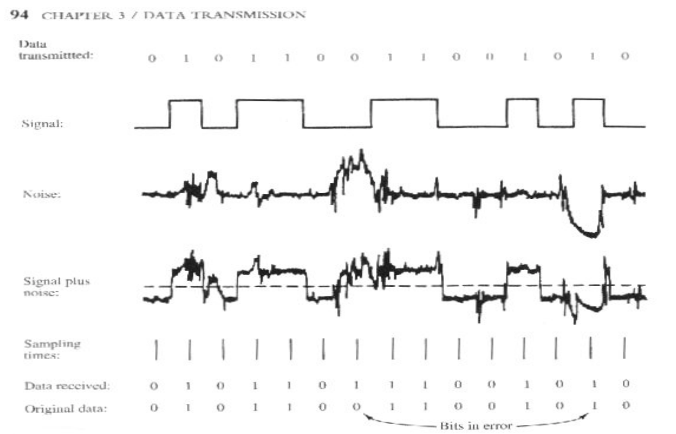


Figure 3.13 Effect of Noise on a Digital Signal

Channel Capacity

- Maximum rate at which data can be transmitted over communication channel
- Data rate
  - In bits per second
  - Rate at which data can be communicated
- Bandwidth
  - Bandwidth of transmitted signal
  - In cycles per second or Hertz
  - Constrained by transmitter and medium

Channel Capacity

- Noise
  - Average level of noise over communication path
- Error Rate
  - Rate at which error occurs
  - Error occurs when
    - reception of 1 when 0 transmitted
    - reception of 0 when 1 transmitted

Nyquist Theorem

- Assume channel is noise free
- If rate of signal transmission is 2B, a signal with frequencies no greater than B sufficient to carry signal rate
- Given a bandwidth of B, highest signal rate that can be carried is 2B
- Channel capacity

$$C = 2B \log_2 M$$

  - M is number of discrete signals or voltage levels

Nyquist Theorem

- Example
  - Assume voice channel used via modem to transmit digital data
  - Assume bandwidth=3100Hz
  - If M=2 (binary signals), C=2B=6200 bps
  - If M=8, C=6B=18,600 bps
- For given bandwidth, data rate increased by increasing number of different signal elements
- Noise and transmission impairments limit practical value of M

Signal-to-Noise Ratio (SNR)

- Important parameter in determining performance of transmission system
- Relative, not absolute measure
- Measured in decibel (dB)
- A high signal to noise ratio means high quality signal reception

$$(SNR)_{dB} = 10\log_{10}\frac{signal\ \ power}{noise\ \ power}$$

Shannon Theorem

- Maximum channel capacity

$$C = B\log_2(1+SNR)$$

- Represents theoretical maximum data rate (bps)
- In practice much lower rates achieved
  - Assumes white noise
- As signal strength increases, effects of nonlinearities increase => intermodulation noise
- As B increases, white noise increases, SNR decreases

Example

- Assume channel spectrum 3MHZ-4MHZ
- Assume SNR is 24 dB
- B=4 MHZ-3 MHZ=1 MHZ
- SNR_{dB} = 24 dB = 10 log (SNR) => SNR =251
- Using Shannon's formula
  - C= 10^6 x $\log_2$ (1+251) = 10^6 x 8 = 8 Mbps
- Assume this rate is achieved, we compute signaling levels required using Nyquist theorem
  - C=2B $\log_2$ M => 8x 10^6 = 2 x 10^6 x $\log_2$ M
  - M=16

E_b/N_0 Ratio

- Ratio of signal energy per bit to noise power density per hertz
- Energy per bit in a signal $E_b = S\,T_b$
  - S is signal power
  - T_b is time required to send one bit
- Data rate $R = 1/T_b$

$$\frac{E_b}{N_0} = \frac{S/R}{N_0} = \frac{S}{kTR}$$

$$\left(\frac{E_b}{N_0}\right)_{dB} = S_{dBW} - 10\log R - 10\log K - 10\log T$$

E_b/N_0 Ratio

- Bit error rate for digital data is a decreasing function of the ratio E_b/N_0
- Given a value of E_b/N_0 needed to achieve a desired error rate
  - As bit rate increases, transmitted signal power relative to noise must increase
- Example: For binary phase-shift keying
  - E_b/N_0 =8.4 dB for a bit error rate of 10^{-4}
  - Effective noise temperature is 290 K (room temperature)
  - Data rate is 2400 bps, required received signal level?
  - 8.4=S(dBW)-10 log 2400 + 228.6 dBW – 10 log 290
    => S = -161.8 dBW

Fourier Series

⌘ Any periodic signal can be represented as sum of sinusoids, known as Fourier Series

$$x(t) = \frac{A_0}{2} + \sum_{n=1}^{\infty} \left[A_n \cos(2\pi n f_0 t) + B_n \sin(2\pi n f_0 t) \right]$$

$$A_0 = \frac{2}{T} \int_0^T x(t) dt$$

$$A_n = \frac{2}{T} \int_0^T x(t) \cos(2\pi n f_0 t) dt$$

If A_0 is not 0, $x(t)$ has a DC component

$$B_n = \frac{2}{T} \int_0^T x(t) \sin(2\pi n f_0 t) dt$$

Fourier Series

⌘ Amplitude-phase representation

$$x(t) = \frac{C_0}{2} + \sum_{n=1}^{\infty} \left[C_n \cos(2\pi n f_0 t + \theta_n) \right]$$

$$C_0 = A_0$$

$$C_n = \sqrt{A_n^2 + B_n^2}$$

$$\theta_n = \tan^{-1}\left(\frac{-B_n}{A_n} \right)$$

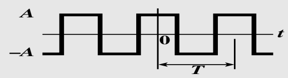
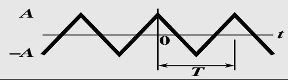
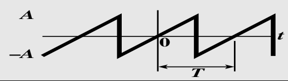
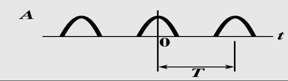
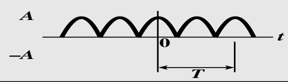
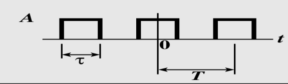
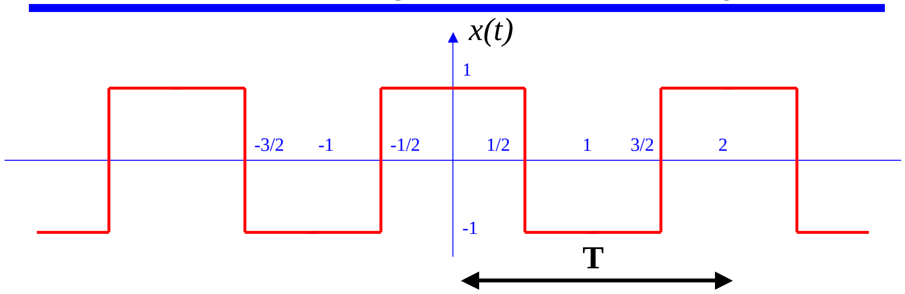
Signal	Fourier Series
Square wave 	$(4A/\ ) \times [\cos(2\ f_1 t) - (1/3)\cos(2\ (3f_1)t) + (1/5)\cos(2\ (5f_1)t) - (1/7)\cos(2\ (7f_1)t) + \ldots]$
Triangular wave 	$(8A/\ ^2) \times [\cos(2\ f_1 t) + (1/9)\cos(2\ (3f_1)t) + (1/25)\cos(2\ (5f_1)t) + \ldots]$
Sawtooth wave 	$(2A/\ ) \times [\sin(2\ f_1 t) - (1/2)\sin(2\ (2f_1)t) + (1/3)\sin(2\ (3f_1)t) - (1/4)\sin(2\ (4f_1)t) + \ldots]$
Half-wave rectified cosine 	$C_0 = A/$ $C_n = 0$ for n odd $C_n = (A/\ ) \times (-1)^{(1+n/2)} \times (2/(n^2-1))$ for n even
Full-wave rectified cosine 	$C_0 = 2A/$ $C_n = (2A/\ ) \times (-1)^n \times (1/(4n^2-1))$
Pulse Train 	$C_n = A \times \left\lvert \frac{\sin(n\ \tau/T)}{n\ \tau/T} \right\rvert$

Figure 3.15 Some Common Periodic Signals and Their Fourier Series

Fourier Series Representation of Periodic Signals - Example



Note that x(-t)=x(t) => x(t) is an even function

$$A_0 = \frac{2}{T}\int_0^T x(t)dt = \frac{2}{2}\int_0^2 x(t)dt = 2\int_0^1 x(t)dt = 2\int_0^{1/2} 1dt + 2\int_{1/2}^1 -1dt = 1 - 1 = 0$$

Fourier Series Representation of Periodic Signals - Example

$$A_n = \frac{2}{T}\int_0^T x(t)\cos(2\pi n f_0 t)dt = \frac{4}{T}\int_0^{T/2} x(t)\cos(2\pi n f_0 t)dt = 2\int_0^1 x(t)\cos(2\pi n f_0 t)dt$$

$$= 2\int_0^{1/2}\cos(2\pi n f_0 t)dt + 2\int_{1/2}^1 -\cos(2\pi n f_0 t)dt = \frac{4}{n\pi}\sin\frac{n\pi}{2}$$

$$B_n = \frac{2}{T}\int_0^T x(t)\sin(2\pi n f_0 t)dt = \frac{2}{T}\int_{-T/2}^{T/2} x(t)\sin(2\pi n f_0 t)dt$$

$$= \frac{2}{T}\int_{-T/2}^{0} x(t)\sin(2\pi n f_0 t)dt + \frac{2}{T}\int_0^{T/2} x(t)\sin(2\pi n f_0 t)dt$$

$$= -\frac{2}{T}\int_0^{T/2} x(-t)\sin(2\pi n f_0 t)dt + \frac{2}{T}\int_0^{T/2} x(t)\sin(2\pi n f_0 t)dt$$

Replacing t by $-t$ in the first integral

sin(-2πnf t)=
- sin(2πnf t)

Fourier Series Representation of Periodic Signals - Example

Since *x(-t)=x(t)* as *x(t)* is an even function, then

$B_n = 0$ for n=1, 2, 3, …

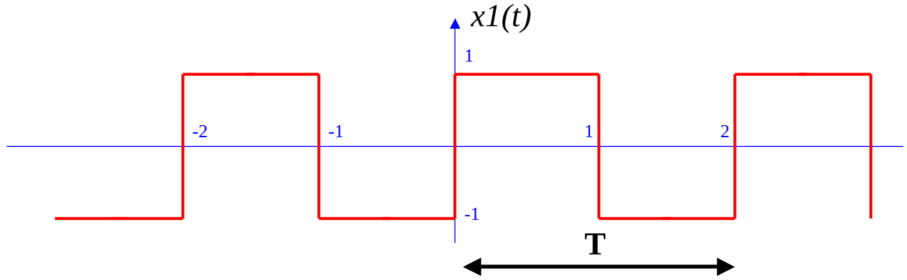
$$x(t) = \frac{A_0}{2} + \sum_{n=1}^{\infty}\left[A_n\cos(2\pi n f_0 t) + B_n\sin(2\pi n f_0 t)\right]$$

$$x(t) = \sum_{n=1}^{\infty}\frac{4}{n\pi}\sin\frac{n\pi}{2}\cos n\pi t$$

$$x(t) = \frac{4}{\pi}\cos\pi t - \frac{4}{3\pi}\cos 3\pi t + \frac{4}{5\pi}\cos 5\pi t - \frac{4}{7\pi}\cos 7\pi t$$

$$x(t) = \frac{4}{\pi}\left[\cos\pi t - \frac{1}{3}\cos 3\pi t + \frac{1}{5}\cos 5\pi t - \frac{1}{7}\cos 7\pi t\right]$$

Another Example



Note that x1(-t)= -x1(t) => x(t) is an odd function

Also, x1(t)=x(t-1/2)

$$x1(t)=\frac{4}{\pi}\left[\cos\pi\left(t-\frac{1}{2}\right)-\frac{1}{3}\cos3\pi\left(t-\frac{1}{2}\right)+\frac{1}{5}\cos5\pi\left(t-\frac{1}{2}\right)-\frac{1}{7}\cos7\pi\left(t-\frac{1}{2}\right)\right]$$

Another Example

$$x1(t)=\frac{4}{\pi}\left[\cos\left(\pi t-\frac{\pi}{2}\right)-\frac{1}{3}\cos\left(3\pi t-\frac{3\pi}{2}\right)+\frac{1}{5}\cos\left(5\pi t-\frac{5\pi}{2}\right)-\frac{1}{7}\cos\left(7\pi t-\frac{7\pi}{2}\right)\right]$$

$$x1(t)=\frac{4}{\pi}\left[\sin\pi t+\frac{1}{3}\sin3\pi t+\frac{1}{5}\sin5\pi t+\frac{1}{7}\sin7\pi t\right]$$

$$\cos\left(\pi t-\frac{\pi}{2}\right)=\sin\pi t \qquad \cos\left(3\pi t-\frac{3\pi}{2}\right)=-\sin3\pi t$$

$$\cos\left(5\pi t-\frac{5\pi}{2}\right)=\sin5\pi t \qquad \cos\left(7\pi t-\frac{7\pi}{2}\right)=-\sin7\pi t$$

Fourier Transform

- For a periodic signal, spectrum consists of discrete frequency components at fundamental frequency & its harmonics.
- For an aperiodic signal, spectrum consists of a continuum of frequencies.
  - Spectrum can be defined by Fourier transform
  - For a signal x(t) with spectrum X(f), the following relations hold

$$x(t) = \int_{-\infty}^{\infty} X(f) e^{j2\pi ft} df \qquad X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt$$

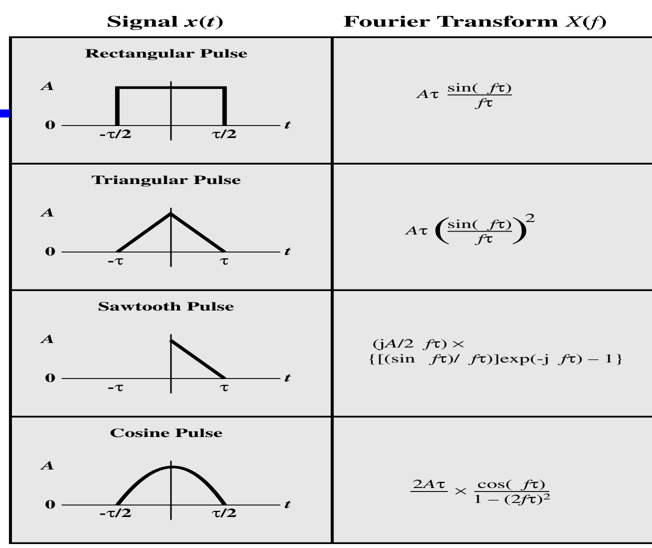
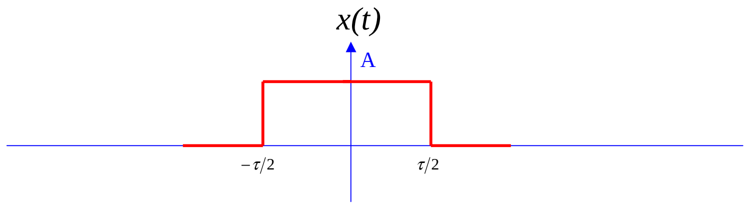


Figure 3.16 Some Common Aperiodic Signals and Their Fourier Transforms

Fourier Transform Example



$$X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi ft}\, dt$$

$$X(f) = \int_{-\tau/2}^{\tau/2} A\, e^{-j2\pi ft}\, dt = -\frac{A}{j2\pi f} e^{-j2\pi ft} \Big|_{-\tau/2}^{\tau/2}$$

Fourier Transform Example

$$= \frac{2A}{2\pi f}\left[\frac{e^{j2\pi f\tau/2} - e^{-j2\pi f\tau/2}}{2j}\right] = \frac{2A}{2\pi f}\left(\frac{2\pi f\tau}{2}\right)\left(\frac{\sin(2\pi f\tau/2)}{2\pi f\tau/2}\right)$$

$$X(f) = A\tau\frac{\sin(2\pi f\tau/2)}{2\pi f\tau/2} = A\tau\frac{\sin(\pi f\tau)}{\pi f\tau}$$

$$\sin\theta = \left[\frac{e^{j\theta} - e^{-j\theta}}{2j}\right] \qquad \cos\theta = \left[\frac{e^{j\theta} + e^{-j\theta}}{2}\right]$$

Signal Power

- ⌘A function x(t) specifies a signal in terms of either voltage or current
- ⌘Instantaneous power of a signal is related to $|x(t)|^2$
- ⌘Average power of a time limited signal is

$$\frac{1}{t1 - t2}\int_{t1}^{t2} |x(t)|^2 \, dt$$

- ⌘For a periodic signal, the average power in one period is

$$\frac{1}{T}\int_{0}^{T} |x(t)|^2 \, dt$$

Power Spectral Density & Bandwidth

- ⌘Absolute bandwidth of any time-limited signal is infinite.
- ⌘Most power in a signal is concentrated in finite band.
- ⌘Effective bandwidth is the spectrum portion containing most of the power.
- ⌘Power spectral density (PSD) describes power content of a signal as a function of frequency.

Power Spectral Density & Bandwidth

- For a continuous valued function *S(f),* power contained in a band of frequencies $f_1<f<f_2$

$$P = 2\int_{f_1}^{f_2} S(f)\,df$$

- For a periodic waveform, the power through the first *j* harmonics is

$$P = C_0^2 + \frac{1}{2}\sum_{n=1}^{j} C_n^2$$

Power Spectral Density & Bandwidth - Example

- Consider the following signal

$$x(t) = \left[\sin \pi t + \frac{1}{3}\sin 3\pi t + \frac{1}{5}\sin 5\pi t + \frac{1}{7}\sin 7\pi t\right]$$

- The signal power is

$$Power = \frac{1}{2}\left[1 + \frac{1}{9} + \frac{1}{25} + \frac{1}{49}\right] = 0.586 \text{ watt}$$