

William Stallings Data and Computer Communications

Chapter 3 Data Transmission

Transmission Terminology

⌘ Transmission over transmission medium using electromagnetic waves.

⌘ Transmission media

☒ Guided media

☒ Waves guided along physical path

☒ e.g. twisted pair, coaxial cable, optical fiber

☒ Unguided media

☒ Waves not guided

☒ e.g. air, water, vacuum

Transmission Terminology

⌘ Direct link

- ☑ No intermediate devices other than amplifiers and repeaters

⌘ Point-to-point

- ☑ Direct link
- ☑ Only 2 devices share link

⌘ Multi-point

- ☑ More than two devices share the link

Transmission Terminology

⌘ Simplex

- ☑ One direction
 - ☑ e.g. Television

⌘ Half duplex

- ☑ Either direction, but only one way at a time
 - ☑ e.g. police radio

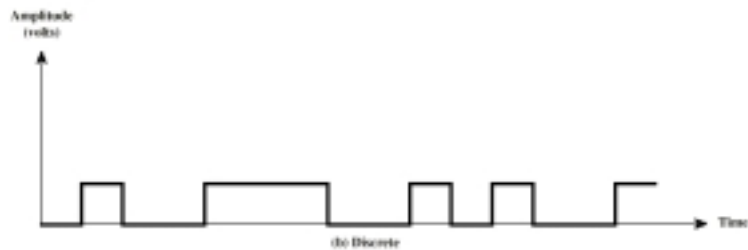
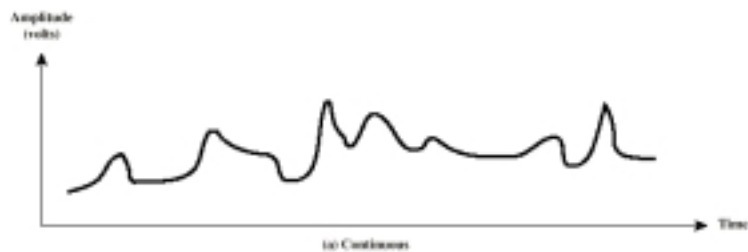
⌘ Full duplex

- ☑ Both directions at the same time
 - ☑ e.g. telephone

Frequency, Spectrum and Bandwidth

- ⌘ Electromagnetic signals used to transmit data
- ⌘ Signal is a function of time or frequency
- ⌘ Time domain concepts
 - ☑ Continuous signal
 - ☒ Varies in a smooth way over time
 - ☑ Discrete signal
 - ☒ Maintains a constant level then changes to another constant level
 - ☑ Periodic signal
 - ☒ Same signal pattern repeated over time
 - ☑ Aperiodic signal
 - ☒ Signal pattern not repeated over time

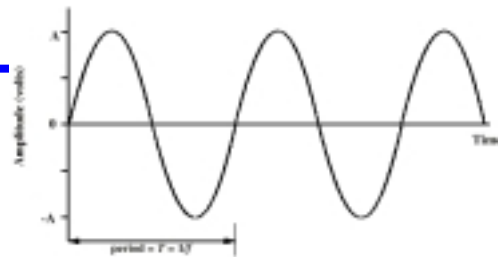
Continuous & Discrete Signals



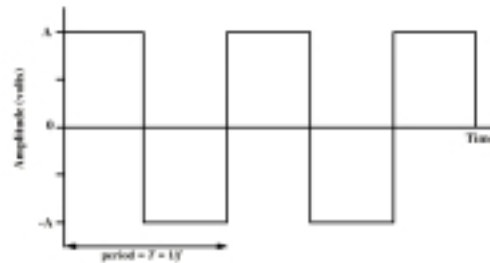
Periodic Signals

Signal is said to be Periodic if
 $S(t+T) = s(t)$ for all t

T is the period of signal



(a) Sine wave

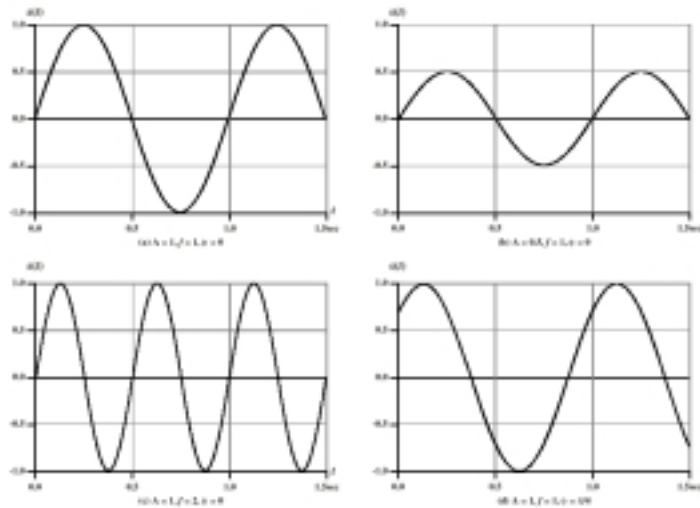


(b) Square wave

Sine Wave

- ⌘ Sine wave is fundamental periodic signal
- ⌘ Peak Amplitude (A)
 - ☑ maximum signal intensity over time, measured in volts
- ⌘ Frequency (f)
 - ☑ Rate at which signal repeats
 - ☑ Hertz (Hz) or cycles per second
 - ☑ Period = time for one repetition (T)
 - ☑ $T = 1/f$
- ⌘ Phase (ϕ)
 - ☑ Relative position in time within a single period
 - ☑ $s(t) = A \sin(2\pi f t + \phi)$

Varying Sine Waves



Wavelength

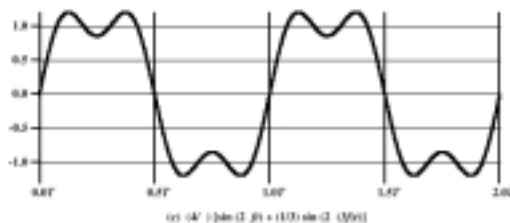
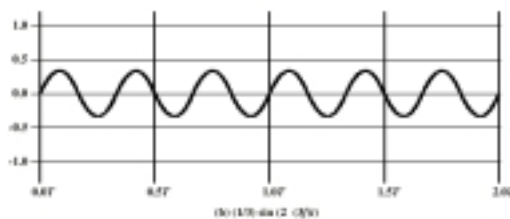
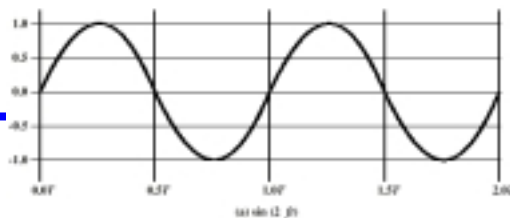
- ⌘ Distance occupied by one cycle, expressed as λ
- ⌘ Distance between two points of corresponding phase in two consecutive cycles
- ⌘ Assuming signal velocity v
 - ⊠ $\lambda = vT$
 - ⊠ $\lambda f = v$
 - ⊠ $c = 3 \times 10^8$ m/s (speed of light in free space)

Frequency Domain Concepts

- ⌘ Signal usually made up of many frequencies
- ⌘ Components are sinusoidal waves
- ⌘ Can be shown (Fourier analysis) that any signal is made up of component sinusoidal waves
- ⌘ Fundamental frequency
 - ☒ Base frequency such that frequency of all components expressed as its integer multiples
 - ☒ Period of aggregate signal is same as period of fundamental frequency

Addition of Frequency Components

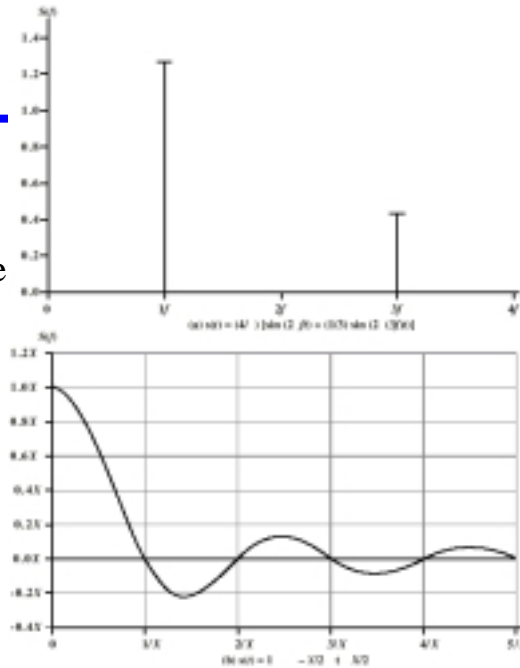
The signal
 $s(t) = 4/\pi [\sin(2\pi f t) + 1/3 \sin(2\pi (3f) t)]$
is made up of two frequency components



Frequency Domain

Time domain function $s(t)$ specifies a signal in terms of its amplitude at each instant of time.

Frequency domain function $S(f)$ specifies a signal in terms of its peak amplitude of constituent frequencies.



Spectrum & Bandwidth

⌘ Spectrum

☑ range of frequencies contained in signal

⌘ Absolute bandwidth

☑ width of spectrum

⌘ Effective bandwidth

☑ Often just *bandwidth*

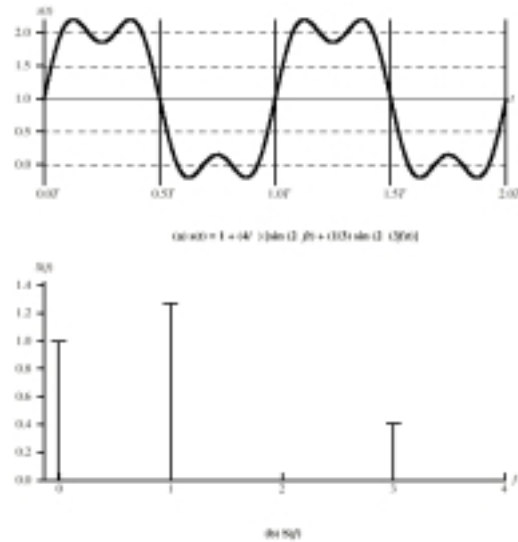
☑ Narrow band of frequencies containing most of the energy

⌘ DC Component

☑ Component of zero frequency

☑ Changes average amplitude of signal to non-zero

Signal with DC Component



Data Rate and Bandwidth

- ⌘ Any transmission system has a limited band of frequencies
 - ☑ Range of FM radio transmission is 88-108 MHz
- ⌘ This limits the data rate that can be carried
- ⌘ Increasing bandwidth increases data rate
- ⌘ A given bandwidth can support various data rates depending on receiver's ability to distinguish 1 and 0 signals.

Data Rate and Bandwidth

- ⌘ Any digital waveform has infinite bandwidth
 - ☒ Transmission system limits waveform as a signal over medium
 - ☒ Medium cost is directly proportional to transmission bandwidth
 - ☒ Signal of limited bandwidth preferable to reduce cost
 - ☒ Limiting bandwidth creates distortions making it difficult to interpret received signal

Frequency Components of a Square Wave

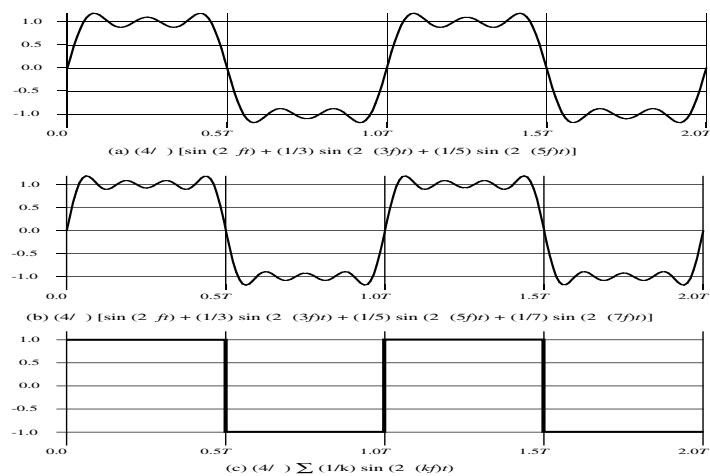


Figure 3.7 Frequency Components of Square Wave ($T = 1/f$)

Data Rate and Bandwidth

- ⌘ Assume digital transmission system has bandwidth of 4 MHz
- ⌘ Transmitting sequence of alternating 1's and 0's as a square wave
- ⌘ What data rate can be achieved?

Case 1

- ⌘ Approximate square wave with waveform of first three sinusoidal components
 - ☑ $s(t) = 4/\pi [\sin(2\pi f t) + 1/3 \sin(2\pi(3f)t) + 1/5 \sin(2\pi(5f)t)]$
- ⌘ Bandwidth = 4 MHz = $5f - f = 4f$
 - ☑ $\Rightarrow f = 1 \text{ MHz} = 10^6 \text{ cycles/second}$
- ⌘ For $f = 1 \text{ MHz}$, period of fundamental frequency
 - $T = 1/10^6 = 10^{-6} = 1 \text{ us}$
 - ☑ One bit occurs every 0.5 us
 - ☑ Data rate is 2×10^6 bps or 2 Mbps

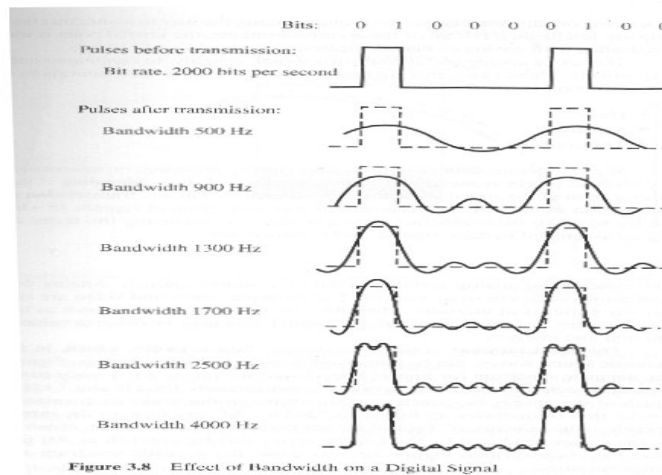
Case 2

- ⌘ Assume bandwidth is 8 MHz
- ⌘ Bandwidth = 8 MHz = $5f - f = 4f$
 - ☑ $\Rightarrow f = 2 \text{ MHz} = 2 \times 10^6 \text{ cycles/second}$
- ⌘ For $f = 2 \text{ MHz}$, period of fundamental frequency
 $T = 1/(2 \times 10^6) = 0.5 \times 10^{-6} = 0.5 \text{ us}$
 - ☑ One bit occurs every 0.25 us
 - ☑ Data rate is $4 \times 10^6 \text{ bps}$ or 4 Mbps
- ⌘ Bandwidth = 4 MHz Data Rate = 2 Mbps
- ⌘ Bandwidth = 8 MHz Data Rate = 4 Mbps

Case 3

- ⌘ Approximate square wave with waveform of first two sinusoidal components
 $s(t) = 4/\pi [\sin(2\pi f t) + 1/3 \sin(2\pi(3f)t)]$
- ⌘ Assume bandwidth = 4 MHz = $3f - f = 2f$
 - ☑ $\Rightarrow f = 2 \text{ MHz} = 2 \times 10^6 \text{ cycles/second}$; period $T = 0.5 \text{ us}$
 - ☑ One bit occurs every 0.25 us
 - ☑ Data rate is $4 \times 10^6 \text{ bps}$ or 4 Mbps
- ⌘ Bandwidth = 4 MHz Data Rate = 4 Mbps
- ⌘ A given bandwidth can support various data rates depending on ability of receiver to distinguish 0 & 1

Effect of Bandwidth on Digital Signal



Analog and Digital Data Transmission

⌘ Data

- ☑ Entities that convey meaning or information

⌘ Signals

- ☑ Electric or electromagnetic representations of data

⌘ Transmission

- ☑ Communication of data by propagation and processing of signals

Data

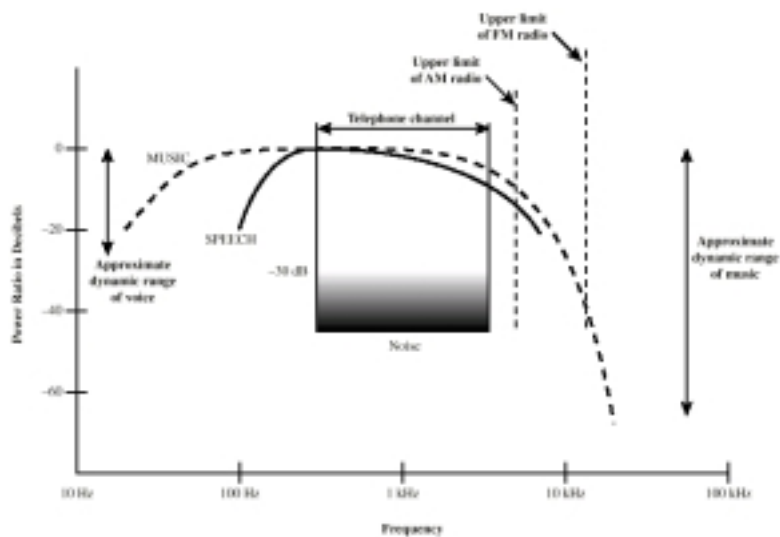
⌘ Analog

- ☑ Continuous values within some interval
- ☑ e.g. sound, video, data collected by sensors

⌘ Digital

- ☑ Discrete values
- ☑ e.g. text, integers

Acoustic Spectrum (Analog)



Signals

⌘ Means by which data are propagated

⌘ Analog

▣ Continuously variable

▣ Various media

▣ wire, fiber optic, space

▣ Speech bandwidth 100Hz to 7kHz

▣ Telephone bandwidth 300Hz to 3400Hz

▣ Video bandwidth 4MHz

⌘ Digital

▣ Use two DC components

Data and Signals

⌘ Usually use digital signals for digital data and analog signals for analog data

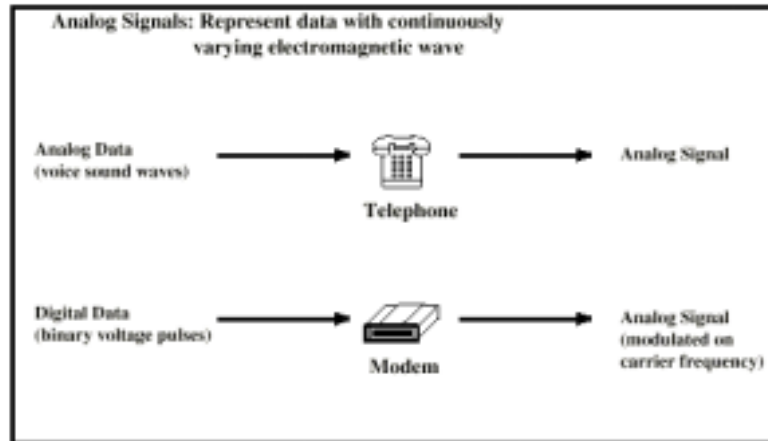
⌘ Can use analog signal to carry digital data

▣ Modem

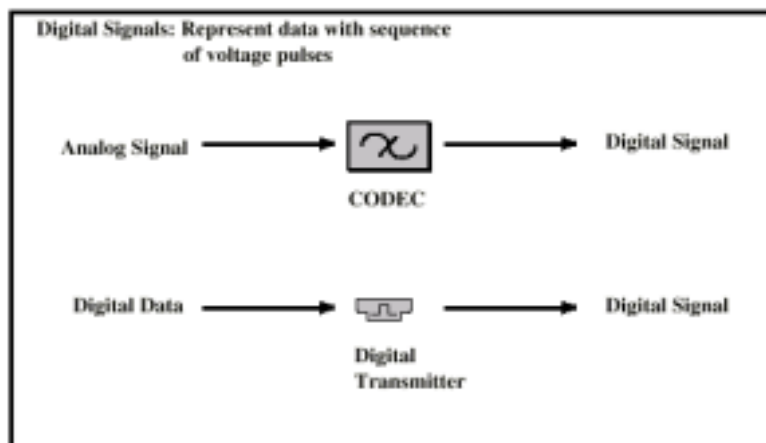
⌘ Can use digital signal to carry analog data

▣ Compact Disc audio

Analog Signals Carrying Analog and Digital Data



Digital Signals Carrying Analog and Digital Data



Analog Transmission

- ⌘ Analog signal transmitted without regard to content
- ⌘ May be analog or digital data
- ⌘ Attenuated over distance
- ⌘ Use amplifiers to boost signal
- ⌘ Also amplifies noise
- ⌘ With amplifiers cascaded to achieve long distances, the signal becomes more distorted.

Digital Transmission

- ⌘ Concerned with content
- ⌘ Integrity endangered by noise, attenuation etc.
- ⌘ Repeaters used
 - ☑ Repeater receives signal
 - ☑ Extracts bit pattern
 - ☑ Retransmits
 - ☑ Attenuation is overcome
- ⌘ Noise is not amplified

Advantages of Digital Transmission

⌘ Digital technology

- ☑ Low cost LSI/VLSI technology

⌘ Data integrity

- ☑ Longer distances over lower quality lines

⌘ Capacity utilization

- ☑ High bandwidth links economical
- ☑ High degree of multiplexing easier with digital techniques

⌘ Security & Privacy

- ☑ Encryption

⌘ Integration

- ☑ Can treat analog and digital data similarly

Decibels and Signal Strength

⌘ Decibel is a measure of ratio between two signal levels

- ☑ N_{dB} = number of decibels

- ☑ P_1 = input power level

- ☑ P_2 = output power level

$$N_{dB} = 10 \log_{10} \frac{P_2}{P_1}$$

⌘ Example:

- ☑ A signal with power level of 10mW inserted onto a transmission line

- ☑ Measured power some distance away is 5mW

- ☑ Loss expressed as $N_{dB} = 10 \log(5/10) = 10(-0.3) = -3 \text{ dB}$

Decibels and Signal Strength

⌘ Decibel is a measure of relative not absolute difference

- ☒ A loss from 1000 mW to 500 mW is a loss of 3dB
- ☒ A loss of 3 dB halves the power
- ☒ A gain of 3 dB doubles the power

⌘ Example:

- ☒ Input to transmission system at power level of 4 mW
- ☒ First element is transmission line with a 12 dB loss
- ☒ Second element is amplifier with 35 dB gain
- ☒ Third element is transmission line with 10 dB loss
- ☒ Output power P₂
 - ☒ $(-12+35-10)=13 \text{ dB} = 10 \log (P_2 / 4\text{mW})$
 - ☒ $P_2 = 4 \times 10^{1.3} \text{ mW} = 79.8 \text{ mW}$

Relationship Between Decibel Values and Powers of 10

Power Ratio	dB	Power Ratio	dB
10^1	10	10^{-1}	-10
10^2	20	10^{-2}	-20
10^3	30	10^{-3}	-30
10^4	40	10^{-4}	-40
10^5	50	10^{-5}	-50
10^6	60	10^{-6}	-60

Decibel-Watt (dBW)

- ⌘ Absolute level of power in decibels
- ⌘ Value of 1 W is a reference defined to be 0 dBW

$$Power_{dBW} = 10 \log_{10} \frac{Power_W}{1W}$$

- ⌘ Example:
 - ☑ Power of 1000 W is 30 dBW
 - ☑ Power of 1 mW is -30 dBW

Decibel & Difference in Voltage

- ⌘ Decibel is used to measure difference in voltage.
- ⌘ Power $P = V^2/R$

$$N_{dB} = 10 \log \frac{P_2}{P_1} = 10 \log \frac{V_2^2 / R}{V_1^2 / R} = 20 \log \frac{V_2}{V_1}$$

- ⌘ Decibel-millivolt (dBmV) is an absolute unit with 0 dBmV equivalent to 1mV.
 - ☑ Used in cable TV and broadband LAN

$$Voltage_{dBmV} = 20 \log \frac{Voltage_{mV}}{1mV}$$

Transmission Impairments

- ⌘ Signal received may differ from signal transmitted
- ⌘ Analog - degradation of signal quality
- ⌘ Digital - bit errors
- ⌘ Caused by
 - ☒ Attenuation and attenuation distortion
 - ☒ Delay distortion
 - ☒ Noise

Attenuation

- ⌘ Signal strength falls off with distance
- ⌘ Depends on medium
 - ☒ Logarithmic for guided media; constant number of decibels per unit distance
 - ☒ For unguided media, complex function of distance and atmospheric conditions
- ⌘ Received signal strength:
 - ☒ must be strong enough to be detected
 - ☒ must be sufficiently higher than noise to be received without error
- ⌘ **Attenuation is an increasing function of frequency**

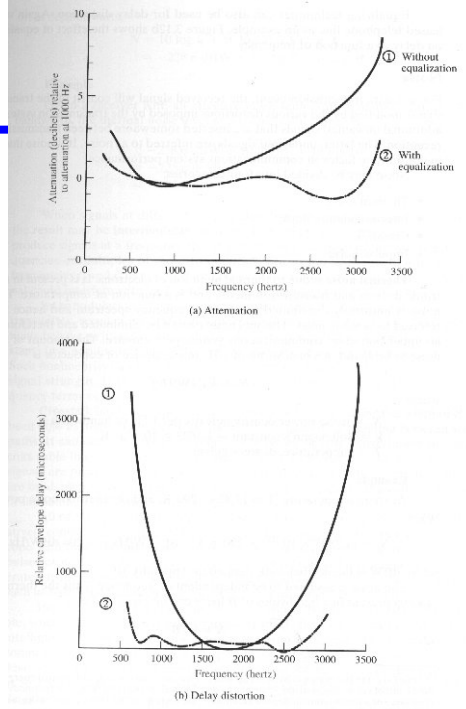
Attenuation Distortion

- ⌘ Beyond a certain distance attenuation becomes large
 - ☒ Use repeaters or amplifiers to strengthen signal
- ⌘ Attenuation distorts received signal, reducing intelligibility
- ⌘ Attenuation can be equalized over a band of frequencies
 - ☒ Using loading coils that change electrical properties of lines
 - ☒ Use amplifiers that can amplify higher frequencies more than lower frequencies
- ⌘ Attenuation distortion has less effect on digital signals
 - ☒ Strength of digital signal falls off rapidly with frequency

Delay Distortion

- ⌘ Only in guided media
- ⌘ **Signal propagation velocity varies with frequency**
 - ☒ In bandlimited signal, velocity tends to be higher near center frequency and falls off towards two edges of band
 - ☒ Varying frequency components arrive at receiver at different times => phase shifts between different frequencies
- ⌘ Critical for digital data transmission
 - ☒ Some signal components of one bit position spill over to other bit positions, causing **intersymbol interference**
 - ☒ Major limitation to maximum bit rate over transmission channel
- ⌘ May be reduced by equalization techniques

Attenuation & Delay Distortion Curves for a Voice Channel



Noise

- ⌘ Undesired signals inserted into real signal during transmission
- ⌘ Four types of noise
 - ⌘ Thermal (white noise)
 - ☒ Due to thermal agitation of electrons
 - ☒ Uniformly distributed across frequency spectrum
 - ☒ Function of temperature; present in all electronic devices
 - ☒ Cannot be eliminated and places an upper bound on system performance

Thermal Noise

⌘ Thermal noise in bandwidth of 1 Hz in any device

$$N_0 = k T \text{ (W/Hz)}$$

☒ N_0 =noise power density in watts per 1 Hz of bandwidth

☒ k =Boltzmann's constant= 1.3803×10^{-23} J/K

☒ T =temperature, degrees Kelvin (=T-273.15 degrees Celsius)

⌘ Example

☒ At room temperature, $T=17$ C or 290 K

☒ Thermal noise power density $N_0 = (1.3803 \times 10^{-23}) \times 290$
 $= 4 \times 10^{-21}$ W/Hz

☒ $= 10 \log (4 \times 10^{-21}) / 1 \text{ W} = -204$ dBW/Hz

Thermal Noise

⌘ Thermal noise is assumed independent of frequency

⌘ Thermal noise in watts in a bandwidth of B hertz

$$N = k T B$$

⌘ Thermal noise in decibel-watts

$$N = 10 \log k + 10 \log T + 10 \log B$$

$$N = -228.6 \text{ dBW} + 10 \log T + 10 \log B$$

Thermal Noise

⌘ Example:

☒ Given a receiver with effective noise temperature of 100 K and a 10 MHz bandwidth

☒ Thermal noise level at receiver's output

$$\begin{aligned} N &= -228.6 \text{ dBW} + 10 \log 10^2 + 10 \log 10^7 \\ &= -228.6 + 20 + 70 \\ &= -138.6 \text{ dBW} \end{aligned}$$

Intermodulation Noise

⌘ Signals at different frequencies share the same transmission medium

⌘ May result in signals that are sum or difference or multiples of original frequencies

⌘ Occurs when there is nonlinearity in transmitter, receiver, transmission system

☒ Nonlinearity caused by component malfunction or excessive signal strength

Crosstalk

- ⌘ Unwanted coupling between signal paths
- ⌘ Signal from one line is picked up by another
- ⌘ Occurs due to
 - ☒ Electrical coupling between nearby twisted pairs,
 - ☒ Electrical coupling between multiple signals on coaxial cable,
 - ☒ Unwanted signals picked up by microwave antennas
- ⌘ Same order of magnitude or less than thermal noise

Impulse Noise

- ⌘ Noncontinuous noise; irregular pulses or spikes of short duration and high amplitude
- ⌘ May be caused by lightning or flaws in communication system
- ⌘ Not a major problem for analog data but can be significant for digital data
 - ☒ A spike of 0.01 s will not destroy any voice data but will destroy 560 bits transmitted at 56 kbps

Effect of Noise on Digital Data

94 CHAPTER 3 / DATA TRANSMISSION

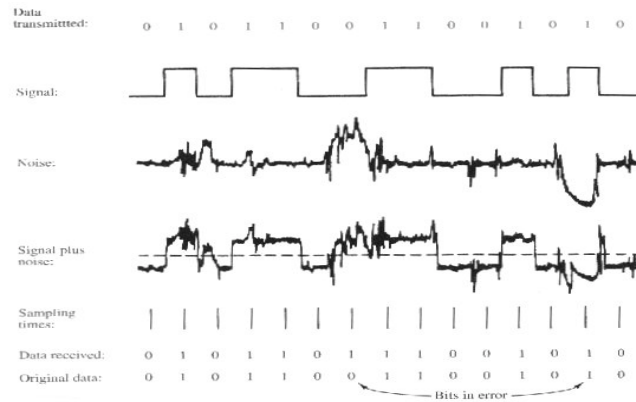


Figure 3.13 Effect of Noise on a Digital Signal

Channel Capacity

- ⌘ Maximum rate at which data can be transmitted over communication channel
- ⌘ Data rate
 - ☑ In bits per second
 - ☑ Rate at which data can be communicated
- ⌘ Bandwidth
 - ☑ Bandwidth of transmitted signal
 - ☑ In cycles per second or Hertz
 - ☑ Constrained by transmitter and medium

Channel Capacity

⌘ Noise

- ⊞ Average level of noise over communication path

⌘ Error Rate

- ⊞ Rate at which error occurs
- ⊞ Error occurs when
 - ⊞ reception of 1 when 0 transmitted
 - ⊞ reception of 0 when 1 transmitted

Nyquist Theorem

⌘ Assume channel is noise free

- ⌘ If **rate of signal** transmission is $2B$, a signal with frequencies no greater than B sufficient to carry signal rate

- ⌘ Given a bandwidth of B , highest **signal rate** that can be carried is $2B$

⌘ Channel capacity

$$C = 2B \log_2 M$$

- ⊞ M is number of discrete signals or voltage levels

Nyquist Theorem

⌘ Example

☒ Assume voice channel used via modem to transmit digital data

☒ Assume bandwidth=3100Hz

☒ If M=2 (binary signals), C=2B=6200 bps

☒ If M=8, C=6B=18,600 bps

⌘ For given bandwidth, data rate increased by increasing number of different signal elements

⌘ Noise and transmission impairments limit practical value of M

Signal-to-Noise Ratio (SNR)

⌘ Important parameter in determining performance of transmission system

⌘ Relative, not absolute measure

⌘ Measured in decibel (dB)

⌘ A high signal to noise ratio means high quality signal reception

$$(SNR)_{dB} = 10 \log_{10} \frac{\text{signal power}}{\text{noise power}}$$

Shannon Theorem

- ⌘ Maximum channel capacity

$$C = B \log_2(1 + SNR)$$

- ⌘ Represents theoretical maximum data rate (bps)
- ⌘ In practice much lower rates achieved
 - ☑ Assumes white noise
- ⌘ As signal strength increases, effects of nonlinearities increase => intermodulation noise
- ⌘ As B increases, white noise increases, SNR decreases

Example

- ⌘ Assume channel spectrum 3MHZ-4MHZ
- ⌘ Assume SNR is 24 dB
- ⌘ $B = 4 \text{ MHz} - 3 \text{ MHz} = 1 \text{ MHz}$
- ⌘ $SNR_{dB} = 24 \text{ dB} = 10 \log(SNR) \Rightarrow SNR = 251$
- ⌘ Using Shannon's formula
 - ☑ $C = 10^6 \times \log_2(1 + 251) = 10^6 \times 8 = 8 \text{ Mbps}$
- ⌘ Assume this rate is achieved, we compute signaling levels required using Nyquist theorem
 - ☑ $C = 2B \log_2 M \Rightarrow 8 \times 10^6 = 2 \times 10^6 \times \log_2 M$
 - ☑ $M = 16$

E_b/N_0 Ratio

- ⌘ Ratio of signal energy per bit to noise power density per hertz
- ⌘ Energy per bit in a signal $E_b = S T_b$
 - ☑ S is signal power
 - ☑ T_b is time required to send one bit
- ⌘ Data rate $R = 1/T_b$

$$\frac{E_b}{N_0} = \frac{S/R}{N_0} = \frac{S}{kTR}$$

$$\left(\frac{E_b}{N_0} \right)_{dB} = S_{dBW} - 10 \log R - 10 \log K - 10 \log T$$

E_b/N_0 Ratio

- ⌘ Bit error rate for digital data is a decreasing function of the ratio E_b/N_0
- ⌘ Given a value of E_b/N_0 needed to achieve a desired error rate
 - ☑ As bit rate increases, transmitted signal power relative to noise must increase
- ⌘ Example: For binary phase-shift keying
 - ☑ $E_b/N_0 = 8.4$ dB for a bit error rate of 10^{-4}
 - ☑ Effective noise temperature is 290 K (room temperature)
 - ☑ Data rate is 2400 bps, required received signal level?
 - ☑ $8.4 = S(\text{dBW}) - 10 \log 2400 + 228.6 \text{ dBW} - 10 \log 290$
 $\Rightarrow S = -161.8 \text{ dBW}$

Fourier Series

⌘ Any periodic signal can be represented as sum of sinusoids, known as **Fourier Series**

$$x(t) = \frac{A_0}{2} + \sum_{n=1}^{\infty} [A_n \cos(2\pi n f_0 t) + B_n \sin(2\pi n f_0 t)]$$

$$A_0 = \frac{2}{T} \int_0^T x(t) dt$$

$$A_n = \frac{2}{T} \int_0^T x(t) \cos(2\pi n f_0 t) dt$$

**If A_0 is not 0,
 $x(t)$ has a DC
component**

$$B_n = \frac{2}{T} \int_0^T x(t) \sin(2\pi n f_0 t) dt$$

Fourier Series

⌘ Amplitude-phase representation

$$x(t) = \frac{C_0}{2} + \sum_{n=1}^{\infty} [C_n \cos(2\pi n f_0 t + \theta_n)]$$

$$C_0 = A_0$$

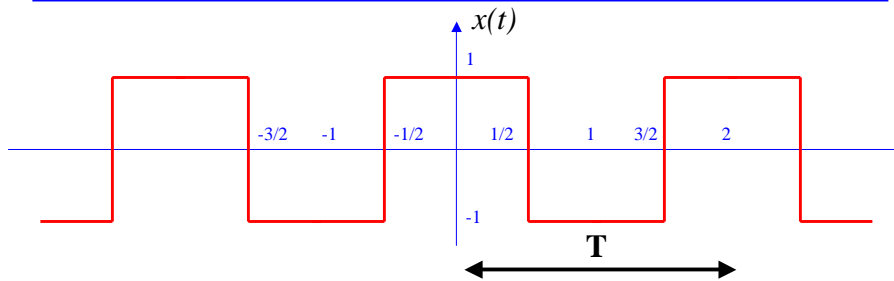
$$C_n = \sqrt{A_n^2 + B_n^2}$$

$$\theta_n = \tan^{-1} \left(\frac{-B_n}{A_n} \right)$$

Signal	Fourier Series
Square wave 	$(4A/\pi) \times [\cos(2 f_1 t) - (1/3) \cos(2 (3f_1)t) + (1/5) \cos(2 (5f_1)t) - (1/7) \cos(2 (7f_1)t) + \dots]$
Triangular wave 	$(8A/\pi^2) \times [\cos(2 f_1 t) + (1/9) \cos(2 (3f_1)t) + (1/25) \cos(2 (5f_1)t) + \dots]$
Sawtooth wave 	$(2A/\pi) \times [\sin(2 f_1 t) - (1/2) \sin(2 (2f_1)t) + (1/3) \sin(2 (3f_1)t) - (1/4) \sin(2 (4f_1)t) + \dots]$
Half-wave rectified cosine 	$C_0 = A/\pi$ $C_n = 0$ for n odd $C_n = (A/\pi) \times (-1)^{(1+n/2)} \times (2/(n^2 - 1))$ for n even
Full-wave rectified cosine 	$C_0 = 2A/\pi$ $C_n = (2A/\pi) \times (-1)^n \times (1/(4n^2 - 1))$
Pulse Train 	$C_n = A \times \left \frac{\sin(n \pi \tau/T)}{n \pi \tau/T} \right $

Figure 3.15 Some Common Periodic Signals and Their Fourier Series

Fourier Series Representation of Periodic Signals - Example



Note that $x(-t)=x(t) \Rightarrow x(t)$ is an *even function*

$$A_0 = \frac{2}{T} \int_0^T x(t) dt = \frac{2}{2} \int_0^2 x(t) dt = 2 \int_0^1 x(t) dt = 2 \int_0^{1/2} 1 dt + 2 \int_{1/2}^1 -1 dt = 1 - 1 = 0$$

Fourier Series Representation of Periodic Signals - Example

$$A_n = \frac{2}{T} \int_0^T x(t) \cos(2\pi n f_0 t) dt = \frac{4}{T} \int_0^{T/2} x(t) \cos(2\pi n f_0 t) dt = 2 \int_0^1 x(t) \cos(2\pi n f_0 t) dt$$

$$= 2 \int_0^{1/2} \cos(2\pi n f_0 t) dt + 2 \int_{1/2}^1 -\cos(2\pi n f_0 t) dt = \frac{4}{n\pi} \sin \frac{n\pi}{2}$$

$$B_n = \frac{2}{T} \int_0^T x(t) \sin(2\pi n f_0 t) dt = \frac{2}{T} \int_{-T/2}^{T/2} x(t) \sin(2\pi n f_0 t) dt$$

$$= \frac{2}{T} \int_{-T/2}^0 x(t) \sin(2\pi n f_0 t) dt + \frac{2}{T} \int_0^{T/2} x(t) \sin(2\pi n f_0 t) dt$$

Replacing t by $-t$
in the first integral
 $\sin(-2\pi n f_0 t) =$
 $-\sin(2\pi n f_0 t)$

$$= -\frac{2}{T} \int_0^{T/2} x(-t) \sin(2\pi n f_0 t) dt + \frac{2}{T} \int_0^{T/2} x(t) \sin(2\pi n f_0 t) dt$$

Fourier Series Representation of Periodic Signals - Example

Since $x(-t) = x(t)$ as $x(t)$ is an even function, then

$B_n = 0$ for $n=1, 2, 3, \dots$

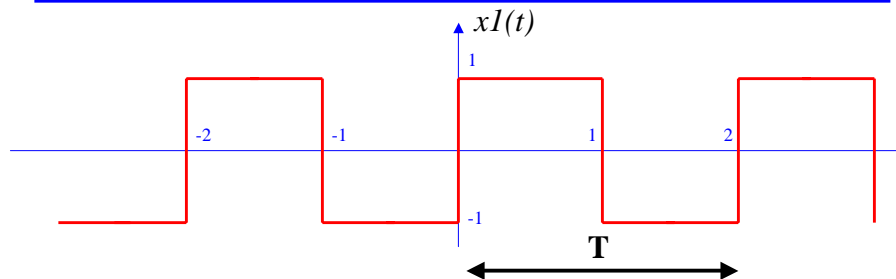
$$x(t) = \frac{A_0}{2} + \sum_{n=1}^{\infty} [A_n \cos(2\pi n f_0 t) + B_n \sin(2\pi n f_0 t)]$$

$$x(t) = \sum_{n=1}^{\infty} \frac{4}{n\pi} \sin \frac{n\pi}{2} \cos n\pi t$$

$$x(t) = \frac{4}{\pi} \cos \pi t - \frac{4}{3\pi} \cos 3\pi t + \frac{4}{5\pi} \cos 5\pi t - \frac{4}{7\pi} \cos 7\pi t$$

$$x(t) = \frac{4}{\pi} \left[\cos \pi t - \frac{1}{3} \cos 3\pi t + \frac{1}{5} \cos 5\pi t - \frac{1}{7} \cos 7\pi t \right]$$

Another Example



Note that $x1(-t) = -x1(t) \Rightarrow x(t)$ is an *odd function*

Also, $x1(t) = x(t-1/2)$

$$x1(t) = \frac{4}{\pi} \left[\cos\pi \left(t - \frac{1}{2} \right) - \frac{1}{3} \cos 3\pi \left(t - \frac{1}{2} \right) + \frac{1}{5} \cos 5\pi \left(t - \frac{1}{2} \right) - \frac{1}{7} \cos 7\pi \left(t - \frac{1}{2} \right) \right]$$

Another Example

$$x1(t) = \frac{4}{\pi} \left[\cos \left(\pi - \frac{\pi}{2} \right) - \frac{1}{3} \cos \left(3\pi - \frac{3\pi}{2} \right) + \frac{1}{5} \cos \left(5\pi - \frac{5\pi}{2} \right) - \frac{1}{7} \cos \left(7\pi - \frac{7\pi}{2} \right) \right]$$

$$x1(t) = \frac{4}{\pi} \left[\sin\pi + \frac{1}{3} \sin 3\pi + \frac{1}{5} \sin 5\pi + \frac{1}{7} \sin 7\pi \right]$$

$$\cos \left(\pi - \frac{\pi}{2} \right) = \sin\pi$$

$$\cos \left(3\pi - \frac{3\pi}{2} \right) = -\sin 3\pi$$

$$\cos \left(5\pi - \frac{5\pi}{2} \right) = \sin 5\pi$$

$$\cos \left(7\pi - \frac{7\pi}{2} \right) = -\sin 7\pi$$

Fourier Transform

- ⌘ For a **periodic signal**, spectrum consists of discrete frequency components at fundamental frequency & its harmonics.
- ⌘ For an **aperiodic signal**, spectrum consists of a continuum of frequencies.
 - ☑ Spectrum can be defined by Fourier transform
 - ☑ For a signal $x(t)$ with spectrum $X(f)$, the following relations hold

$$x(t) = \int_{-\infty}^{\infty} X(f) e^{j2\pi ft} df$$

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt$$





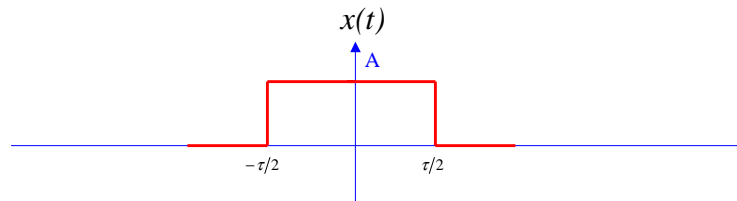
Signal $x(t)$	Fourier Transform $X(f)$
Rectangular Pulse 	$A\tau \frac{\sin(\pi f\tau)}{f\tau}$
Triangular Pulse 	$A\tau \left(\frac{\sin(\pi f\tau)}{f\tau}\right)^2$
Sawtooth Pulse 	$(jA/2 \pi f\tau) \times \{[\sin(\pi f\tau)/\pi f\tau] \exp(-j\pi f\tau) - 1\}$
Cosine Pulse 	$\frac{2A\tau}{\pi} \times \frac{\cos(\pi f\tau)}{1 - (2f\tau)^2}$

Figure 3.16 Some Common Aperiodic Signals and Their Fourier Transforms

Fourier Transform Example



$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt$$

$$X(f) = \int_{-\tau/2}^{\tau/2} A e^{-j2\pi ft} dt = -\frac{A}{j2\pi f} e^{-j2\pi ft} \Big|_{-\tau/2}^{\tau/2}$$

Fourier Transform Example

$$= \frac{2A}{2\pi f} \left[\frac{e^{j2\pi f\tau/2} - e^{-j2\pi f\tau/2}}{2j} \right] = \frac{2A}{2\pi f} \left(\frac{2\pi f\tau}{2} \right) \left(\frac{\sin(2\pi f\tau/2)}{2\pi f\tau/2} \right)$$

$$X(f) = A\tau \frac{\sin(2\pi f\tau/2)}{2\pi f\tau/2} = A\tau \frac{\sin(\pi f\tau)}{\pi f\tau}$$

$$\sin \theta = \left[\frac{e^{j\theta} - e^{-j\theta}}{2j} \right]$$

$$\cos \theta = \left[\frac{e^{j\theta} + e^{-j\theta}}{2} \right]$$

Signal Power

- ⌘ A function $x(t)$ specifies a signal in terms of either voltage or current
- ⌘ Instantaneous power of a signal is related to $|x(t)|^2$
- ⌘ Average power of a time limited signal is

$$\frac{1}{t_1 - t_2} \int_{t_1}^{t_2} |x(t)|^2 dt$$

- ⌘ For a periodic signal, the average power in one period is

$$\frac{1}{T} \int_0^T |x(t)|^2 dt$$

Power Spectral Density & Bandwidth

- ⌘ Absolute bandwidth of any time-limited signal is infinite.
- ⌘ Most power in a signal is concentrated in finite band.
- ⌘ Effective bandwidth is the spectrum portion containing most of the power.
- ⌘ Power spectral density (PSD) describes power content of a signal as a function of frequency.

Power Spectral Density & Bandwidth

⌘ For a continuous valued function $S(f)$, power contained in a band of frequencies $f_1 < f < f_2$

$$P = 2 \int_{f_1}^{f_2} S(f) df$$

⌘ For a periodic waveform, the power through the first j harmonics is

$$P = C_0^2 + \frac{1}{2} \sum_{n=1}^j C_n^2$$

Power Spectral Density & Bandwidth - Example

⌘ Consider the following signal

$$x(t) = \left[\sin \pi t + \frac{1}{3} \sin 3\pi t + \frac{1}{5} \sin 5\pi t + \frac{1}{7} \sin 7\pi t \right]$$

⌘ The signal power is

$$Power = \frac{1}{2} \left[1 + \frac{1}{9} + \frac{1}{25} + \frac{1}{49} \right] = 0.586 \text{ watt}$$