COE 202, Term 052
Fundamentals of Computer Engineering
HW# 2

Q.1. Prove the identity of each of the following Boolean functions using algebraic manipulation:

(i) $A' + AB + AC' + AB'C' = A' + B + C'$

(ii) $(A + B + D)(A' + C)(B + C + D) = (A + B + D)(A' + C)$

(iii) $XZ + WY'Z' + W'YZ' + WX'Z' = XZ + WY'Z' + W'XY + X'YZ'$

Q.2. Simplify the following Boolean expressions to a minimum number of literals using algebraic manipulation:

(i) $W'X(Z' + Y'Z) + X(W + W'YZ)$

(ii) $[(CD)' + A]' + A + CD + AB$

(iii) $(A + B' + AB')(AB + A'C + BC)$

Q.3. Find the complement of the following Boolean functions and reduce them to a minimum number of literals:

(i) $WX(Y'Z + YZ') + W'X'(Y'+Z)(Y+Z')$

(ii) $ABC + A'CD$

Q.4. Using DeMorgan`s theorem, express the function $F=A'B' + AB + B'C$

(i) With only OR and complement operations.

(ii) With only AND and complement operations.

Q.5. Show that the dual of the equivalence function $F(A,B)=A'B' + AB$ is equal to its complement.

Q.6. A majority gate is a digital circuit whose output is equal to 1 if the majority of inputs are 1`s. The output is 0 otherwise.

(i) By means of a truth table, find the Boolean function implemented by a 3-input majority gate.

(ii) Express the 3-input majority gate as a sum of minterms and a product of maxterms.

Q.7. Express the following functions in a sum of minterms and a product of maxterms :

(i) $F(X,Y,Z) = (XY + Z)(Y + XZ)$

(ii) $F(A,B,C,D) = D(A' + B) + B'D$

HW#2

Q1 (i) $\bar{A} + AB + A\bar{C} = \bar{A} + B + \bar{C}$

$$
\begin{aligned}
\bar{A} + AB + A\bar{C} &= (\bar{A} + A) \cdot (\bar{A} + B) + A\bar{C} && \text{by distributivity} \\
&= 1 \cdot (\bar{A} + B) + A\bar{C} && \text{by complement} \\
&= \bar{A} + B + A\bar{C} && \text{by identity} \\
&= (\bar{A} + A) \cdot (\bar{A} + \bar{C}) + B && \text{by distributivity} \\
&= 1 \cdot (\bar{A} + \bar{C}) + B && \text{by complement} \\
&= \bar{A} + \bar{C} + B && \text{by identity} \\
&= \bar{A} + B + \bar{C} && \text{by commutativ.}
\end{aligned}
$$

(ii) $(A+B+D)(\bar{A}+C)(B+C+D) = (A+B+D)(\bar{A}+C)$

We can prove that this identity is true by proving that its dual is true. The dual of this identity is

$ABD + \bar{A}C + BCD = ABD + \bar{A}C$

$ABD + \bar{A}C + BCD$ is equal to $ABD + \bar{A}C$ by the consensus theorem. Let $X = BD$, then

$AX + \bar{A}C + XC = AX + \bar{A}C$ by Consensus

(iii) $XZ + W\bar{Y}\bar{Z} + \bar{W}Y\bar{Z} + W\bar{X}\bar{Z} = XZ + W\bar{Y}\bar{Z} + \bar{W}XY + \bar{X}Y\bar{Z}$

$$
\begin{aligned}
XZ + W\bar{Y}\bar{Z} + \bar{W}Y\bar{Z} + W\bar{X}\bar{Z} &= XZ + W\bar{Y}\bar{Z} + \bar{W}Y\bar{Z} \\
&\quad + W\bar{X}\bar{Z} + X\bar{W}Y
\end{aligned}
$$

This is because $XZ + \bar{W}Y\bar{Z} = X\underline{\underline{Z}} + \bar{W}Y\underline{\underline{\bar{Z}}} + X\bar{W}Y$ by consensus

$= XZ + W\bar{Y}\bar{Z} + \bar{W}Y\bar{Z} + W\bar{X}\bar{Z} + X\bar{W}Y + \bar{X}Y\bar{Z}$

This is also by consensus since $\underline{\underline{\bar{W}}}Y\bar{Z} + \underline{\underline{W}}\bar{X}\bar{Z}$

$= \bar{W}Y\bar{Z} + W\bar{X}\bar{Z} + \bar{X}Y\bar{Z}$

$= xz + w\bar{y}\bar{z} + w\bar{x}\bar{z} + x\bar{w}y + \bar{x}y\bar{z}$

since $\underline{\underline{x}}\bar{w}y + \underline{\underline{\bar{x}}}y\bar{z} + \bar{w}y\bar{z} = x\bar{w}y + \bar{x}y\bar{z}$ by consensus

$= xz + w\bar{y}\bar{z} + x\bar{w}y + \bar{x}y\bar{z} = RHS$

since $w\underline{\underline{\bar{y}}}\bar{z} + w\bar{x}\bar{z} + \bar{x}\underline{\underline{y}}\bar{z} = w\bar{y}\bar{z} + \bar{x}y\bar{z}$

Q2 (i) $\bar{w}x(\bar{z} + \bar{y}z) + x(w + \bar{w}yz)$

$= \bar{w}x(\bar{z} + \bar{y}) + x(w + yz)$

$= \bar{w}x\bar{z} + \bar{w}x\bar{y} + xw + xyz$

$= x(w + \bar{w}\bar{z}) + \bar{w}x\bar{y} + xyz$

$= x(w + \bar{z}) + \bar{w}x\bar{y} + xyz$

$= xw + x\bar{z} + \bar{w}x\bar{y} + xyz$

$= x(w + \bar{w}\bar{y}) + x(\bar{z} + yz)$

$= x(w + \bar{y}) + x(\bar{z} + y)$

$= xw + x\bar{y} + x\bar{z} + xy$

$= x(\bar{y} + y) + xw + x\bar{z}$

$= x + xw + x\bar{z}$

$= x$ one literal

(ii) $[(CD)' + A]' + A + CD + AB$

$= CD\bar{A} + A + CD + AB$

$= CD + A + AB$

$= CD + A$ three literals

(iii) $(A+\bar{B}+A\bar{B})(AB+\bar{A}C+BC)$

$= AB + ABC + \bar{A}\bar{B}C$

$= AB + \bar{A}\bar{B}C$ 5 literals

Q3

(i) $wx(\bar{y}z+y\bar{z}) + \bar{w}\bar{x}(\bar{y}+z)(y+\bar{z})$

$= wx\bar{y}z + wxy\bar{z} + \bar{w}\bar{x}(\bar{y}\bar{z}+zy)$

$= wx\bar{y}z + wxy\bar{z} + \bar{w}\bar{x}\bar{y}\bar{z} + \bar{w}\bar{x}zy$

It can be seen that this expression can not be simplified

$= wx(\bar{y}z+y\bar{z}) + \bar{w}\bar{x}(\bar{y}\bar{z}+yz)$

The complement of this function is:

$= [wx(\bar{y}z+y\bar{z}) + \bar{w}\bar{x}(\bar{y}\bar{z}+yz)]'$

$= [(wx)' + (\bar{y}z+y\bar{z})'][(\bar{w}\bar{x})' + (\bar{y}\bar{z}+yz)']$

$= [\bar{w}+\bar{x} + (y+\bar{z})(\bar{y}+z)][w+x+(y+z)(\bar{y}+\bar{z})]$

$= [\bar{w}+\bar{x} + yz+\bar{y}\bar{z}][w+x+y\bar{z}+\bar{y}z]$ 12 literals

(ii) $ABC + \bar{A}CD$

$= [AB+\bar{A}D]\cdot C$

The complement of this function is

$= \{[AB+\bar{A}D]\cdot C\}'$

$= [(\bar{A}+\bar{B})(A+\bar{D})] + \bar{C}$

$= \bar{A}\bar{D} + A\bar{B} + \bar{C}$ 5 literals

Q4 $F = \bar{A}\bar{B} + AB + \bar{B}C$

(i) with only OR and complement operations

$$F = (\bar{A}\bar{B})'' + (AB)'' + (\bar{B}C)''$$
$$= (A+B)' + (\bar{A}+\bar{B})' + (B+\bar{C})'$$

(ii) with only AND and complement operations

$$F = [\bar{A}\bar{B} + AB + \bar{B}C]''$$
$$= [(\bar{A}\bar{B})' \cdot (AB)' \cdot (\bar{B}C)']'$$

Q5 $F = \bar{A}\bar{B} + AB$

The dual of the function $F^d = (\bar{A}+\bar{B})(A+B)$

The complement of the function $\bar{F} = [\bar{A}\bar{B} + AB]'$

$= (A+B) \cdot (\bar{A}+\bar{B})$

Thus, $\bar{F} = F^d$

Q6 (i) 3-input majority gate

A	B	C	M
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

(ii) $M(A,B,C) = \Sigma(m_3, m_5, m_6, m_7)$

$= \bar{A}BC + A\bar{B}C + AB\bar{C} + ABC$

$M(A,B,C) = \Pi(M_0, M_1, M_2, M_4)$

$= (A+B+C)(A+B+\bar{C})(A+\bar{B}+C)(\bar{A}+B+C)$

Q7 (i)

$$F(X,Y,Z) = (XY+Z)(Y+XZ)$$
$$= XY + XYZ + YZ + XZ$$
$$= XY + XZ + YZ$$
$$= XY(Z+\bar{Z}) + XZ(Y+\bar{Y}) + YZ(X+\bar{X})$$
$$= XYZ + XY\bar{Z} + XYZ + X\bar{Y}Z + XYZ + \bar{X}YZ$$
$$= XYZ + XY\bar{Z} + X\bar{Y}Z + \bar{X}YZ$$
$$= \Sigma(m3, m5, m6, m7)$$
$$= \Pi(M_0, M1, M2, M4)$$
$$= (X+Y+Z)(X+Y+\bar{Z})(X+\bar{Y}+Z)(\bar{X}+Y+Z)$$

(ii)

$$F(A,B,C,D) = D(\bar{A}+B) + \bar{B}D$$
$$= \bar{A}D + BD + \bar{B}D$$
$$= \bar{A}D(C+\bar{C}) + BD(A+\bar{A}) + \bar{B}D(A+\bar{A})$$
$$= \bar{A}BD + \bar{A}\bar{B}D + ABD + \bar{A}BD + A\bar{B}D + \bar{A}\bar{B}D$$
$$= \bar{A}BD + \bar{A}\bar{B}D + ABD + A\bar{B}D$$
$$= \bar{A}BD(C+\bar{C}) + \bar{A}\bar{B}D(C+\bar{C}) + ABD(C+\bar{C}) + A\bar{B}D(C+\bar{C})$$
$$= \bar{A}BCD + \bar{A}B\bar{C}D + \bar{A}\bar{B}CD + \bar{A}\bar{B}\bar{C}D + ABCD + AB\bar{C}D + A\bar{B}CD + A\bar{B}\bar{C}D$$
$$= \Sigma(m1, m3, m5, m7, m9, m11, m13, m15)$$
$$= \Pi(M_0, M2, M4, M6, M8, M10, M12, M14)$$
$$= (A+B+C+D)(A+B+\bar{C}+D)(A+\bar{B}+C+D)(A+\bar{B}+\bar{C}+D)(\bar{A}+B+C+D)(\bar{A}+B+\bar{C}+D)(\bar{A}+\bar{B}+C+D)(\bar{A}+\bar{B}+\bar{C}+D)$$