$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} - \phi\, u^2 = 0$$

$$x = 0 \quad u = 1 \qquad 0 \le y \le 1$$

$$x = 1 \quad u = 1 \qquad 0 \le y \le 1$$

$$y = 0 \quad u = 1 \qquad 0 \le x \le 1$$

$$y = 1 \quad u = 0 \qquad 0 < x < 1$$

$x \rightarrow$

$y \downarrow$

	i = 1	**i = 2**	**i = 3**	**i = 4**	**i = 5**	**i = 6**	**…..**	**i = NX1**
j = 1	1	1	1	1	1	1	1	1
j = 2	1							1
j = 3	1							1
j = 4	1							1
j = 5	1							1
j = 6	1							1
….	1							1
j = NY1	1	0	0	0	0	0	0	1

j = 1	BC1Y	i = 1
j = 2	BC1X	i = 1
j = 3	BC1X	i = 1
…	BC1X	i = 1
j = NY1	BC2Y	i = 1
j = 1	BC1Y	i = 2
j = 2		i = 2
j = 3		i = 2
…		i = 2
j = NY1	BC2Y	i = 2
j = 1	BC1Y	i = 3
j = 2		i = 3
j = 3		i = 3
…		i = 3
j = NY1	BC2Y	i = 3
	•	
	•	
	•	
j = 1	BC1Y	i = NX1
j = 2	BC1X	i = NX1
j = 3	BC1X	i = NX1
…	BC1X	i = NX1
j = NY1	BC2Y	i = NX1

Residual Equations

$$\boxed{R((i-1)*NY1+j)=\sum_{n=1}^{NX1}DDX(i,n)u((n-1)*NY1+j)+\sum_{m=1}^{NY1}DDY(j,m)u((i-1)*NY1+m)-\phi\left(u((i-1)*NY1+j)\right)^2}\qquad \begin{array}{l} i=2,\ \ldots\ ,NX1-1 \\ j=2,\ \ldots\ ,NY1-1 \end{array}$$

$$\boxed{R((1-1)*NY1+j)=u((1-1)*NY1+j)-1}\qquad (j=1,\ \ldots\ ,NY1)$$

$$\boxed{R((NX1-1)*NY1+j)=u((NX1-1)*NY1+j)\ \text{-}1}\qquad (j=2,\ \ldots\ ,NY1-1)$$

$$\boxed{R((i-1)*NY1+1)=u((i-1)*NY1+1)-1}\qquad (i=1,\ \ldots\ ,NX1)$$

$$\boxed{R((i-1)*NY1+NY1)=u((i-1)*NY1+NY1)\text{-}0}\qquad (i=2,\ \ldots\ ,NX1-1)$$

Jacobean Matrix

$$\boxed{\begin{array}{l} J((i-1)*NY1+j,(n-1)*NY1+m)= \\ DDX(i,n)\delta_{jm}+DDY(j,m)\delta_{in}-2\phi\,u((i-1)*NY1+j)\delta_{in}\delta_{jm} \end{array}}\qquad \begin{pmatrix} i=2,\ \ldots\ ,NX1-1,\ j=2,\ \ldots\ ,NY1-1, \\ n=1,\ \ldots\ ,NX1, m=1,\ \ldots\ ,NY1 \end{pmatrix}$$

$$\boxed{J((1-1)*NY1+j,(1-1)*NY1+j)=1}\qquad (i=1,\ \ldots\ ,NX1,\ j=1,\ \ldots\ ,NY1)$$

$$\boxed{J((NX1-1)*NY1+j,(NX1-1)*NY1+j)=1}\qquad (i=1,\ \ldots\ ,NX1,\ j=2,\ \ldots\ ,NY1-1)$$

$$\boxed{J((i-1)*NY1+1,(i-1)*NY1+1)=1}\qquad (i=1,\ \ldots\ ,NX1,\ j=1,\ \ldots\ ,NY1)$$

$$\boxed{J((i-1)*NY1+NY1,(i-1)*NY1+NY1)=1}\qquad (i=2,\ \ldots\ ,NX1-1,\ j=1,\ \ldots\ ,NY1)$$