King Fahd University of Petroleum & Minerals
Chemical Engineering Department
CHE 560 –Numerical Methods in Chemical Engineering
2010 - 2011 (102)

HW#5
Due: Sunday: 17-April-2011

Using order of magnitude analysis, Blasius proposed the following simplified equations for the boundary layer flow over a flat plate:

$$v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} = \nu \frac{\partial^2 v_x}{\partial y^2}$$
$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0$$

subject to the following boundary conditions:

$$y = 0: \qquad v_x = 0$$
$$y = 0: \qquad v_y = 0$$
$$y = \infty: \qquad v_x = v_\infty$$

The above PDE's can be reduced to a BVP using the similarity transformation method by introducing a new space variable, ζ, that combines x and y as follows:

$$\zeta \equiv y\sqrt{\frac{v_\infty}{\nu\, x}}$$

Also by introducing the stream function, ψ, such that:

$$v_x = \frac{\partial \psi}{\partial y},\; v_y = -\frac{\partial \psi}{\partial x}$$

which automatically satisfy the boundary conditions. Introducing the following separation of variables of the form:

$$\psi = g(x)\, f(\zeta).$$

one can show that:

$$v_x = \frac{\partial \psi}{\partial y} = \sqrt{\frac{v_\infty}{\nu\, x}}\, g(x) \frac{df}{d\zeta}$$
$$v_y = -\frac{\partial \psi}{\partial x} = -\frac{dg}{dx}\, f(\zeta) + \frac{1}{2x} \zeta\, g(x) \frac{df}{d\zeta}$$

Perform the following:

(a) (10 points) Introduce $g(x) = \sqrt{\nu\, x\, v_\infty}$, and show that the system of PDE's reduce to the following BVP:

$$f\frac{d^2 f}{d\zeta^2} + 2\frac{d^3 f}{d\zeta^3} = 0$$

also show that the following boundary conditions can be obtained:

$$\zeta = 0: \qquad f = 0$$
$$\zeta = 0: \qquad \frac{df}{d\zeta} = 0$$
$$\zeta = \infty: \qquad \frac{df}{d\zeta} = 1$$

(b) (10 points) Introducing: $y_1 = f$, $y_2 = \frac{df}{d\zeta}$, $y_3 = \frac{d^2 f}{d\zeta^2}$ <u>show that</u> the BVP in part (a) can be represented by the following system IVP's:

$$\frac{dy_1}{d\zeta} = y_2$$
$$\frac{dy_2}{d\zeta} = y_3$$
$$\frac{dy_3}{d\zeta} = -\frac{1}{2} y_1\, y_3$$

Also show that the following initial conditions can be obtained:

$$\zeta = 0: \qquad y_1 = 0$$
$$\zeta = 0: \qquad y_2 = 0$$
$$\zeta = 0: \qquad y_3 = \text{unknown}$$

Moreover, by utilizing the boundary condition: $y = \infty:\ v_x = v_\infty$, show that:

$$\zeta = \infty: \qquad y_2 = 1$$

which can be utilized to evaluate the unknown initial condition for y_3.

(c) Solve the system of IVP's derived in part (b) using the shooting method algorithm by integrating using Adams-Bashford 2nd-order method starting from $\zeta = 0$ to $\zeta = 10\ (10 \approx \infty)$ and plot the dimensionless velocities $V_x \equiv \frac{v_x}{v_\infty}$ and $V_y \equiv v_y \sqrt{x/\nu\, v_\infty}$ as functions of ζ . Set up the equations that allow the numerical solution of the IVP's showing all required details (algorithm, recursive formulas, residual equations, Jacobian matrix ... etc.). Send your program by e-mail yourname-hw6.f