Numerical Integration of Initial Value Problems IVP's
Linear Multistep Methods

General form of IVP's:

$\frac{d\underline{y}}{dt} = \underline{f}\left(\underline{y}\right)$ Subject to initial condition $t = 0, \quad \underline{y} = \underline{y}^0$

Explicit Methods:

Euler Method (First order):

$$\frac{\underline{y}^{n+1} - \underline{y}^n}{h} = \underline{f}\left(\underline{y}^n\right) = \underline{f}^n$$

Rearranging $\Longrightarrow \underline{y}^{n+1} = \underline{y}^n + h\,\underline{f}^n$

Adams-Bashford (Second order):

$$\frac{\underline{y}^{n+1} - \underline{y}^n}{h} = \frac{3}{2}\underline{f}^n - \frac{1}{2}\underline{f}^{n-1}$$

Rearranging $\Longrightarrow \underline{y}^{n+1} = \underline{y}^n + h\left(\frac{3}{2}\underline{f}^n - \frac{1}{2}\underline{f}^{n-1}\right)$

Adams-Bashford (fourth order):

$$\frac{\underline{y}^{n+1} - \underline{y}^n}{h} = \frac{1}{24}\left(55\underline{f}^n - 59\underline{f}^{n-1} + 37\underline{f}^{n-2} - 9\underline{f}^{n-3}\right)$$

Rearranging $\Longrightarrow \underline{y}^{n+1} = \underline{y}^n + \frac{h}{24}\left(55\underline{f}^n - 59\underline{f}^{n-1} + 37\underline{f}^{n-2} - 9\underline{f}^{n-3}\right)$

Implicit Methods:

Backward Euler Method (First order):

$$\frac{\underline{y}^{n+1} - \underline{y}^n}{h} = \underline{f}\left(\underline{y}^{n+1}\right) = \underline{f}^{n+1}$$

Rearranging $\Longrightarrow \underline{y}^{n+1} - h\,\underline{f}^{n+1} = \underline{y}^{n}$

Adams-Moulton (Second order):

$$\frac{\underline{y}^{n+1}-\underline{y}^{n}}{h} = \frac{\underline{f}^{n+1}+\underline{f}^{n}}{2}$$

Rearranging $\Longrightarrow \underline{y}^{n+1} - \frac{h}{2}\,\underline{f}^{n+1} = \underline{y}^{n} + \frac{h}{2}\,\underline{f}^{n}$

Adams-Moulton (fourth order):

$$\frac{\underline{y}^{n+1}-\underline{y}^{n}}{h} = \frac{1}{24}\left(9\underline{f}^{n+1} + 19\underline{f}^{n} - 5\underline{f}^{n-1} + \underline{f}^{n-2}\right)$$

Rearranging $\Longrightarrow \underline{y}^{n+1} - \frac{9\,h}{24}\underline{f}^{n+1} = \underline{y}^{n} + \frac{h}{24}\left(19\underline{f}^{n} - 5\underline{f}^{n-1} + \underline{f}^{n-2}\right)$

Predictor Corrector Methods:

First Order Predictor Corrector:

Predictor: $\underline{\tilde{y}}^{n+1} = \underline{y}^{n} + h\,\underline{f}\left(\underline{y}^{n}\right)$

Corrector: $\underline{y}^{n+1} = \underline{y}^{n} + h\underline{f}\left(\underline{\tilde{y}}^{n+1}\right)$

Second Order Predictor Corrector:

Predictor: $\underline{\tilde{y}}^{n+1} = \underline{y}^{n} + h\left(\frac{3}{2}\underline{f}^{n} - \frac{1}{2}\underline{f}^{n-1}\right)$

Corrector: $\underline{y}^{n+1} = \underline{y}^{n} + h\frac{\underline{f}\left(\underline{\tilde{y}}^{n+1}\right)+\underline{f}\left(\underline{y}^{n}\right)}{2}$