

$$\frac{d^2 y_1}{dz^2} - \phi_1 y_1 y_2 = 0$$

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$$z = 0 \quad \frac{dy_1}{dz} = \frac{dy_2}{dz} = 0$$

$$z = 1 \quad y_1 = 1; \quad y_2 = 0.5$$

Unknowns

y_1	U_1
	U_2
	·
	·
	·
	U_{N1-1}
y_2	U_{N1}
	U_{N1+1}
	U_{N1+2}
	·
	·
	·
	U_{2N1-1}
	U_{2N1}

Residual Equations

BVP1	$i = 1$	$R(1) = \sum_{j=1}^{N1} D(1, j) U(j)$
	$i = 2$ · · · $i = N1-1$	$R(i) = \sum_{j=1}^{N1} DD(i, j) U(j) - \phi_1 U(i) U(N1+i)$
	$i = N1$	$R(N1) = U(N1) - 1$
	BVP2	$i = N1+1$
$i = N1+2$ · · · $i = 2N1-1$		$R(N1+i) = \sum_{j=1}^{N1} DD(i, j) U(N1+j) - \phi_2 U(i) U(N1+i)$
$i = 2N1$		$R(2N1) = U(2N1) - 0.5$

Residual Equations

$$\begin{aligned} R(1) &= \sum_{j=1}^{N1} D(1, j) U(j) & i = 0 \\ R(i) &= \sum_{j=1}^{N1} DD(i, j) U(j) - \phi_1 U(i) U(N1+i) & 2 \leq i \leq N1-1 \\ R(N1) &= U(N1) - 1 & i = N1 \\ R(N1+1) &= \sum_{j=1}^{N1} D(1, j) U(N1+j) & i = N1+1 \\ R(N1+i) &= \sum_{j=1}^{N1} DD(i, j) U(N1+j) - \phi_2 U(i) U(N1+i) & N1+2 \leq i \leq 2N1-1 \\ R(2N1) &= U(2N1) - 0.5 & i = 2N1 \end{aligned}$$

Jacobian Matrix

$$\begin{aligned} J(1, j) &= D(1, j) & 1 \leq j \leq N1 \\ J(i, j) &= DD(i, j) - \phi_1 U(N1+i) \delta_{i,j} & 2 \leq i \leq N1-1, 1 \leq j \leq N1 \\ J(i, N1+j) &= -\phi_1 U(i) \delta_{i,j} & 2 \leq i \leq N1-1, 1 \leq j \leq N1 \\ J(N1, j) &= \delta_{N1,j} & 1 \leq j \leq N1 \\ J(N1+1, N1+j) &= D(1, j) & 1 \leq j \leq N1 \\ J(N1+i, N1+j) &= DD(i, j) - \phi_2 U(i) \delta_{i,j} & 2 \leq i \leq N1-1, 1 \leq j \leq N1 \\ J(N1+i, j) &= -\phi_2 U(N1+i) \delta_{i,j} & 2 \leq i \leq N1-1, 1 \leq j \leq N1 \\ J(2N1, N1+j) &= \delta_{N1,j} & 1 \leq j \leq N1 \end{aligned}$$