King Fahd University of Petroleum & Minerals
Chemical Engineering Department
CHE 501 – Advanced Transport Phenomena
First Semester, 2013 - 2014 (131)

HW#5
Due: Mon. 25-Nov.-2013

Solve the following problems:
1. 4B.5
2. 12B.4

And solve the following two problems:

Problem 1.

We have shown in class for the case when $v_e = v_\infty$ and $dv_e/dx = 0$, the following equations for the boundary layer flow over a flat plate where derived:

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0$$

$$v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} = \nu \frac{\partial^2 v_x}{\partial y^2}$$

subject to the following boundary conditions:

$$y = 0: \qquad v_x = 0$$
$$y = 0: \qquad v_y = 0$$
$$y = \infty: \qquad v_x = v_\infty$$

using the similarity variable:

$$\zeta \equiv y\sqrt{\frac{v_\infty}{\nu\, x}}$$

and by introducing the stream function, ψ, such that:

$$v_x = -\frac{\partial \psi}{\partial y},\ v_y = \frac{\partial \psi}{\partial x}$$

Introducing the following separation of variables of the form:

$$\psi = g(x)\, f(\zeta).$$

Perform the following:

(a) one can show that:

$$v_x = -\frac{\partial \psi}{\partial y} = -\sqrt{\frac{v_\infty}{\nu\, x}}\; g(x)\frac{df}{d\zeta}$$
$$v_y = \frac{\partial \psi}{\partial x} = \frac{dg}{dx}\, f(\zeta) - \frac{1}{2x}\zeta\, g(x)\frac{df}{d\zeta}$$

(b) Introducing $g(x) = \sqrt{\nu\, x\, v_\infty}$, show that the system of PDE's reduce to the following boundary value problem (BVP):

$$f\frac{d^2 f}{d\zeta^2} + 2\frac{d^3 f}{d\zeta^3} = 0$$

Subject to the following boundary conditions:

$$\zeta = 0: \qquad f = 0$$
$$\zeta = 0: \qquad \frac{df}{d\zeta} = 0$$
$$\zeta = \infty: \qquad \frac{df}{d\zeta} = 1$$

(c) Introducing: $y_1 = f,\ y_2 = \frac{df}{d\zeta},\ y_3 = \frac{d^2 f}{d\zeta^2}$ show that the BVP can be represented by the following system initial value problems IVP's:

$$\frac{dy_1}{d\zeta} = y_2$$
$$\frac{dy_2}{d\zeta} = y_3$$
$$\frac{dy_3}{d\zeta} = -\frac{1}{2} y_1\, y_3$$

Subject to the following initial conditions:

$$\zeta = 0: \qquad y_1 = 0$$
$$\zeta = 0: \qquad y_2 = 0$$
$$\zeta = 0: \qquad y_3 = \text{unknown}$$

(d) Solve the IVP's listed in (c) numerically using an appropriate integration method. Note this procedure involves trial and error. Hence you must guess the value of y_3 by utilizing the boundary condition: $y = \infty:\ v_x = v_\infty$, we can show that:

$$\zeta = \infty: \qquad y_2 = 1$$

In your case use $\zeta = 10 \approx \infty$. Hence you update you guess until $\zeta = 10: \qquad y_2 = 1$.
Tabulate the values of y_1, y_2 and y_3, as functions of ξ.

Problem 2.

For the analysis of the approximate solution of the boundary layer equations (also known as von Karman method), the following momentum and energy balances are derived

$$\mu \frac{\partial v_x}{\partial y}\bigg|_{y=0} = \frac{d}{dx}\int_0^\infty \rho v_x (v_e - v_x)dy + \frac{dv_e}{dx}\int_0^\infty \rho (v_e - v_x)dy \qquad (4.4\text{-}13)$$

$$k\frac{\partial T}{\partial y}\bigg|_{y=0} = \frac{d}{dx}\int_0^\infty \rho \hat{C}_p v_x (T_\infty - T)dy \qquad (12.4\text{-}5)$$

The method is implemented by assuming velocity and temperature profiles that satisfy the boundary conditions ($y = 0$, $v_x = 0$, $T = T_0$, $y \rightarrow \infty$, $v_x = 0$, T= T∞, $dv_x/dy = 0$, $dT/dy = 0$, … etc.) then substituting these profiles into the above equations then solving for the thickness of the boundary layers, $\delta(x)$ and $\delta_T(x)$, see examples 4.4-1 and 12.4-1 for more details. For the case when $v_e = v_\infty$ (constant) and using:

$$\frac{v_x}{v_\infty} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$$

$$\frac{T_0 - T}{T_0 - T_\infty} = 2\left(\frac{y}{\delta_T}\right) - \left(\frac{y}{\delta_T}\right)^2$$

(a) Substitute the above profiles into the momentum and energy balances and derive formulae for $\delta(x)$ and $\delta_T(x)$.
(b) Derive formula for the wall shear stress and the wall heat flux.