King Fahd University of Petroleum & Minerals
Chemical Engineering Department
CHE 501 – Advanced Transport Phenomena
First Semester, 2013 - 2014 (131)

HW#4
Due: Mon. 4-Nov.-2013

Solve the following problems:

1. 4B.1
2. 4B.2
3. 4B.3 a, b, c
4. For Example 4.1-1 we presented in class the following similarity solution for the dimensionless velocity:

$$\phi(\eta) = 1 - \frac{\int_0^{\eta} \exp(-\bar{\eta}^2)\, d\bar{\eta}}{\int_0^{\eta} \exp(-\bar{\eta}^2)\, d\bar{\eta}} = 1 - \frac{2}{\sqrt{\pi}} \int_0^{\eta} \exp(-\bar{\eta}^2)\, d\bar{\eta} = 1 - \text{erf}\, \eta$$

   Use computer software to plot ϕ as function of η (from 0 to 5).

5. For Example 4.1-2 we presented in class the following series solution for the dimensionless velocity:

$$\phi(\eta, \tau) = (1 - \eta) - \sum_{n=1}^{\infty} \left(\frac{2}{n\pi}\right) \exp(-n^2\pi^2\tau)\ \sin\ n\pi\eta \qquad (4.1\text{-}40)$$

   Use computer software to plot ϕ as function of η for different values of τ = 0.0, 0.05, 0.1, 0.5, 1.0 and 10. Your results should be presented in one chart.

6. For Example 4.1-3 we presented in class the following solution for the velocity:

$$v_x(y, t) = v_0 e^{-\sqrt{\omega/2\nu}y} \cos(\omega t - \sqrt{\omega/2\nu}y) \qquad (4.1\text{-}57)$$

   Put the above equation in dimensionless form and computer software to plot the dimensionless velocity as function of dimensionless distance and time.

7. For Example 4.2-1, the boundary condition far from the sphere was given as:

   B.C. 3: $\quad$ as $r \to \infty$, $\quad \psi \to -\frac{1}{2} v_\infty r^2 \sin^2\theta \qquad (4.2\text{-}6)$

   This was derived based on the argument that far from the sphere the velocities can be written such that they satisfy the continuity equation: $v_r = v_\infty \cos\theta$ and $v_\theta = -v_\infty \sin\theta$.

   (a) Show that the velocities far from the sphere satisfy the continuity equation.

   (b) Using the definition of the stream function and derive BC3 from v_r and v_θ.

8. Use Mathematica© to:
   a) Derive Eq. 4.2-8 using 4.2-7 and 4.2-3 as well as the simplified B.6-7 and B.6-8.
   b) Derive equations 4.2-15 & 4.2-16 using 4.2-13 and 4.2-14.
   c) Derive equation 4.2-20 by evaluating the integral 4.2-19.

Note for 8. You have to prove the results and provide the input and output files as well.