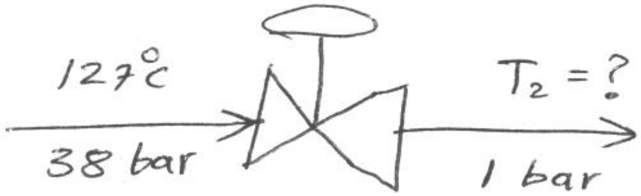
Chemica Engineering Dept., KFUPM, CHE303, Handout_11,
Application of residual properties to process equipment

Example

Propylene gas



Find T_2 and ΔS for this process

For a throttling valve $\Delta H = 0$

$$\Rightarrow \left(H_2^{ig} + H_2^{R}\right) - \left(H_1^{ig} + H_1^{R}\right) = 0$$

at state 2 $P = 1$ bar $\Rightarrow$ propylene gas may be assumed ideal gas $\Rightarrow H_2^R = 0$

$$\Rightarrow \left(H_2^{ig} - H_1^{ig}\right) - H_1^R = 0$$

$$\int_{T_1}^{T_2} C_p^{ig}\, dT - H_1^R = 0$$

For propylene :

$\omega = 0.14$ $\quad T_c = 365.6\,K$ $\quad P_c = 46.65$ bar

$$C_p^{ig} = R\left(1.637 + 22.706 * 10^{-3}\, T - 6.915 * 10^{-6}\, T^2\right)$$

$Tr_1 = 1.095$ $\quad Pr_1 = 0.815$

using Lee/Kesler generalized correlations at T_{r_1} & P_{r_1}

$$\left(\frac{H^R}{RT_c}\right)^0 = -0.863$$

$$\left(\frac{H^R}{RT_c}\right)^1 = -0.534$$

by interpolation (verify yourself)

$$\left(\frac{S^R}{R}\right)^0 = -0.565$$

$$\left(\frac{S^R}{R}\right)^1 = -0.496$$

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$$H_1^R = (H^R)^0 + \omega (H^R)^1 = -2.85 * 10^3 \ \frac{J}{mol}$$

$$S_1^R = (S^R)^0 + \omega (S^R)^1 = -5.275 \ \frac{J}{mol \cdot K}$$

$$\Rightarrow \int_{127+273.15}^{T_2} R(1.637 + 22.706 * 10^{-3} T - 6.915 * 10^{-6} T^2)\, dT$$

$$- (-2.85 * 10^3) = 0$$

$$R\left[1.637 (T_2 - 400.15) + 22.706 * 10^{-3} \left(\frac{T_2^2}{2} - \frac{(400.15)^2}{2}\right) - 6.915 * 10^{-6} \left(\frac{T_2^3}{3} - \frac{(400.15)^3}{3}\right)\right] + 2.85 * 10^{+3} = 0$$

by trail and error $T_2 \simeq 363.3\ K$ ③

$$\Delta S = \left(S_2^{ig} + \cancel{S_2^R}\right) - \left(S_1^{ig} + S_1^R\right)$$

0
i.g.

$$= \Delta S^{ig} - S_1^R$$

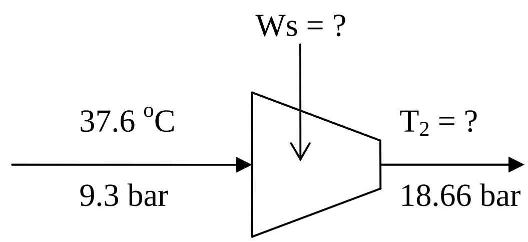
$$= \int_{T_1}^{T_2} \frac{C_p^{ig}}{T}\, dT - (-5.275)$$

- R ln(P2/P1)

$$= 22.774 + 5.275 = 28.049\ \frac{J}{mol \cdot K}$$

Example:

Propylene gas is compressed from 37.6 ^{o}C and 9.3 bar to 18.66 bar using 80% adiabatic compressor. Find the temperature at the exit (T_2) and the work required (Ws).



For propylene: $\frac{C_p^{ig}}{R} = 1.64 + 22.71 \text{ x } 10^{-3} T$ (simplified by ignoring T^2 term), ω = 0.14, T_C = 365.6 K and P_C = 46.65 bar.

Tr_1 = 0.85, Pr_1= 0.2 $\Rightarrow$ $\frac{S_1^R}{R} = -0.255,\quad \frac{H_1^R}{RT_C} = -0.365$

Solution:

Step 1: Assume isentropic compressor: $\Rightarrow$ $\Delta S = 0 = \int_{T_1}^{T_2^{\backslash}} \frac{C_p^{ig}}{T} dT - R\ln\left(\frac{P_2}{P_1}\right) + \left(S_2^R - S_1^R\right)$

$$R\left(1.63\ln\left(\frac{T_2^{\backslash}}{T_1}\right) + 22.71 \text{ x } 10^{-3}\left(T_2^{\backslash} - T_1\right)\right) - R\ln\left(\frac{P_2}{P_1}\right) + \left(S_2^R - S_1^R\right) = 0$$

Tr_1 = 0.85, Pr_1= 0.2 $\Rightarrow \frac{S_1^R}{R} = -0.255,\ \frac{S_2^R}{R} =$ unknown since $T_2^{\backslash}$ unknown

$$\Rightarrow F\left(T_2^{\backslash}\right) = \left(1.63\ln\left(\frac{T_2^{\backslash}}{310.75}\right) + 22.71 \text{ x } 10^{-3}\left(T_2^{\backslash} - 310.75\right)\right) - \ln\left(\frac{18.66}{9.3}\right) + \left(\frac{S_2^R}{R} - (-0.255)\right) = 0$$

Since $S_2^R =$ unknown a trail and error procedure is needed.

$T_2^{\backslash}$ (K)	$\dfrac{S_2^R}{R}$	$F(T_2^{\backslash})$
350	-0.449	0.1948
340	-0.491	-0.1214
343.8 (by interpolation)	-0.47	0.0050
343.7 (by interpolation)	-0.47	0.0008

Answer $T_2^{\backslash} = 343.7\ K$

$$\Delta H^{\backslash} = \int_{T_1}^{T_2^{\backslash}} C_p^{ig}\,dT + \left(H_2^R - H_1^R\right) = R\left(1.63\left(T_2^{\backslash} - T_1\right) + \frac{22.71\,\mathrm{x}\,10^{-3}}{2}\left(\left(T_2^{\backslash}\right)^2 - \left(T_1\right)^2\right)\right) + \left(H_2^R - H_1^R\right)$$

$$\frac{\Delta H^{\backslash}}{R} = \left(1.63\left(T_2^{\backslash} - T_1\right) + \frac{22.71\,\mathrm{x}\,10^{-3}}{2}\left(\left(T_2^{\backslash}\right)^2 - \left(T_1\right)^2\right)\right) + \left(\frac{H_2^R}{R} - \frac{H_1^R}{R}\right)$$

$Tr_1 = 0.85$, $Pr_1 = 0.2$ $\Rightarrow$ $\dfrac{H_1^R}{R} = -133.4$

$Tr_2^{\backslash} = 0.94$, $Pr_1 = 0.4$ $\Rightarrow$ $\dfrac{H_2^R}{R} = -223.4$

$$\frac{\Delta H^{\backslash}}{R} = \left(1.63(343.7 - 310.75) + \frac{22.71\,\mathrm{x}\,10^{-3}}{2}\left(343.7^2 - 310.7^2\right)\right) + (-223.4 + 133.4) = 208.6$$

Step 2: 80% Compressor $\Rightarrow$ $\Delta H = \dfrac{\Delta H^{\backslash}}{\eta} \Rightarrow \boxed{\dfrac{\Delta H}{R} = 260.75 = \dfrac{Ws}{R}}$

$$\frac{\Delta H}{R} = 260.75 = \int_{T_1}^{T_2} \frac{C_p^{ig}}{R}dT + \left(\frac{H_2^R}{R} - \frac{H_1^R}{R}\right)$$

$$\Rightarrow F(T_2) = \left(1.63\left(T_2 - T_1\right) + \frac{22.71\,\mathrm{x}\,10^{-3}}{2}\left(\left(T_2\right)^2 - \left(T_1\right)^2\right)\right) + \left(\frac{H_2^R}{R} - \frac{H_1^R}{R}\right) - 260.75 = 0$$

$$F(T_2) = \left(1.63\left(T_2 - 310.75\right) + \frac{22.71\,\mathrm{x}\,10^{-3}}{2}\left(\left(T_2\right)^2 - (310.75)^2\right)\right) + \left(\frac{H_2^R}{R} - (-133.4)\right) - 260.75 = 0$$

Since T_2 is unknown $\Rightarrow H_2^R$ is unknown

T_2 (K)	$\frac{H_2^R}{R}$	$F(T_2^\backslash)$
345	223	39.49558719
350	210	21.11303781
348.3	210	4.464114692
347.8	210	0.011713145

Answer $T_2 = 347.8\ K$