# Flow Measurement by a Rotameter

The figure below shows another flow measuring device known as a rotameter. It consists of a vertical graduated glass tube with a "*float*" inside it. As the fluid passes upwards through the tube the float will be suspended until it reaches a equilibrium position, from which the fluid flow rate can be read from the adjacent scale in the tube wall.

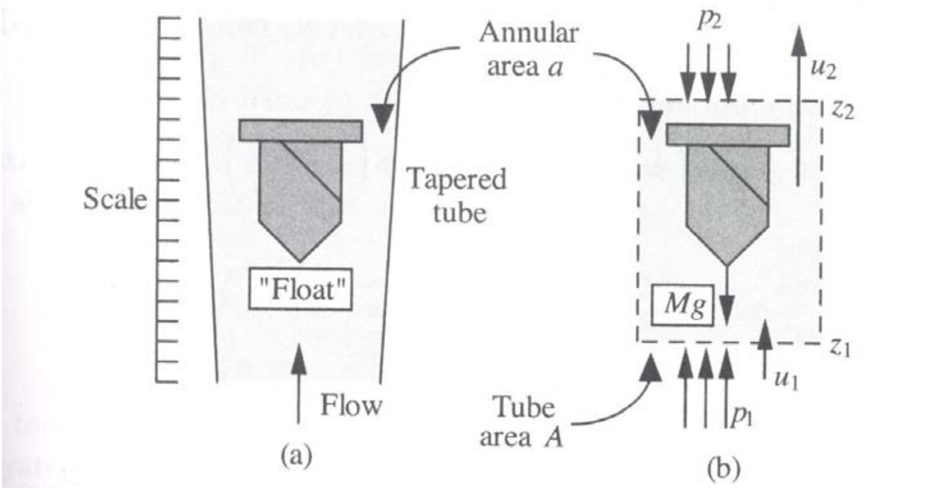

*Fig. E2.6 (a) Section of rotameter, and (b) control volume.*

Derive an expression for the fluid flow rate as a function of the mass of the float, ***M***, density of the fluid, $\boldsymbol{\rho}$, density of the float, $\boldsymbol{\rho_f}$, and area of the pipe, ***A***, and annular area around the float, ***a***.

**<u>Solution:</u>**

Consider the control volume show in figure (b):

**Apply continuity equation (CE) from point 1 to point 2:**

$$\rho u_1 A = \rho u_2 a \qquad \Rightarrow u_2 = \frac{A}{a} u_1$$

**Apply Bernoulli's equation (BE) from point 1 to point 2:**

$$\frac{u_1^2}{2} + g z_1 + \frac{P_1}{\rho} = \frac{u_2^2}{2} + g z_2 + \frac{P_2}{\rho} \qquad \Rightarrow P_1 - P_2 = -\rho \frac{u_1^2}{2}\left(1 - \left(\frac{A}{a}\right)^2\right) + \rho g \left(z_2 - z_1\right)$$

**Apply momentum balance (MB) from point 1 to point 2:**

$$\left(\dot{M}_1 - \dot{M}_2 + \sum F\right)_y = 0$$

$$m_1u_1 - m_2u_2 + \underbrace{P_1A - P_2A}_{\text{Pressure Forces}} - \underbrace{\left[\overbrace{(z_2 - z_1)A - \frac{M}{\rho_f}}^{\text{Volume of fluid around float}}\right]\rho\, g - \overbrace{M}^{\text{mass of float}}\, g}_{\text{Gravity Forces}} = 0 \qquad \left(m_1 = m_2 = m = \rho\, u_1 A, u_2 = \frac{A}{a}\, u_1\right)$$

**Note:** The fluid at point 2 exerts pressure on the whole area of the pipe *A*, not only the annular area *a*.

Simplify MB:

$$\rho\, u_1^2 A\left(1 - \frac{A}{a}\right) + (P_1 - P_2)A - (z_2 - z_1)A\rho\, g - \left(1 - \frac{\rho}{\rho_f}\right)Mg = 0$$

Substitute BE in Simplified MB:

$$\rho\, u_1^2 A\left(1 - \frac{A}{a}\right) - \rho\frac{u_1^2}{2}\left(1 - \left(\frac{A}{a}\right)^2\right)A + \rho g\,(z_2 - z_1)A - (z_2 - z_1)A\rho\, g - \left(1 - \frac{\rho}{\rho_f}\right)Mg = 0$$

Simplify:

$$\rho\frac{u_1^2}{2}A\left(2 - 2\frac{A}{a}\right) - \rho\frac{u_1^2}{2}A\left(1 - \left(\frac{A}{a}\right)^2\right) - \left(1 - \frac{\rho}{\rho_f}\right)Mg = 0$$

$$\ldots\ \rho\, A\frac{u_1^2}{2}\left[\left(\frac{A}{a}\right)^2 - 2\frac{A}{a} + 1\right] = \left(1 - \frac{\rho}{\rho_f}\right)Mg$$

$$\ldots\ u_1 = \sqrt{\frac{2\left(1 - \frac{\rho}{\rho_f}\right)Mg}{\rho\, A\left[\left(\frac{A}{a}\right) - 1\right]^2}} \qquad \boxed{\Rightarrow Q = u_1 A = A\sqrt{\frac{2\left(1 - \frac{\rho}{\rho_f}\right)Mg}{\rho\, A\left[\left(\frac{A}{a}\right) - 1\right]^2}}}$$