Frictional losses in an Orifice plate

Derive an expression for the frictional losses in an orifice plate problem as function of u_1, A_1 (area of pipe) and A_2 (area of jet leaving orifice).

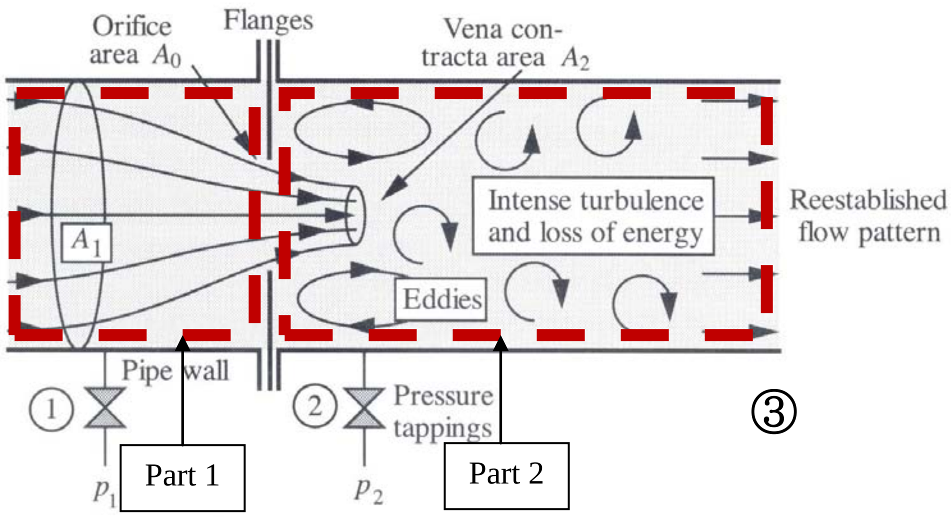

Fig. 2.9 Flow through an orifice plate.

Solution:

In this case the problem can be divided into two parts:

Part1. From point 1 to point 2, there is no mixing and hence no frictional losses. Bernoulli's Equation ban be applied in this part.

Part 2. From point 2 to point 3, there is mixing and hence there is frictional losses. Bernoulli's Equation ban be applied in this part.

Apply mechanical energy balance (MEB) from point 1 to point 3:

$$\Delta\left(\frac{u^2}{2}\right) + g\Delta z + \frac{\Delta P}{\rho} + w_s + \Im = 0 \qquad (A_1 = A_3 \Rightarrow u_1 = u_3, z_1 = z_3, w_s = 0)$$

$$\boxed{\Rightarrow \Im = -\frac{\Delta P}{\rho} = \frac{P_1 - P_3}{\rho} \qquad (1)}$$

Apply Bernoulli's equation (BE) from point 1 to point 2:

$$\frac{u_1^2}{2} + g\,z_1 + \frac{P_1}{\rho} = \frac{u_2^2}{2} + g\,z_2 + \frac{P_2}{\rho} \qquad \left(z_1 = z_2,\quad \text{also} \quad u_2 = u_1\frac{A_1}{A_2}\right)$$

$$\boxed{\Rightarrow \frac{u_1^2}{2}\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right] = \frac{P_1 - P_2}{\rho} \qquad (2)}$$

Apply momentum balance (MB) from point 2 to point 3:

$$\left(\dot{M}_2 - \dot{M}_3 + \sum F\right)_x = 0$$

$$m_2 u_2 - m_3 u_3 + \underbrace{P_2 A_3 - P_3 A_3}_{\text{Pressure Forces}} = 0 \qquad \left(m_2 = m_3 = m, \quad \text{also} \quad A_1 = A_3 \Rightarrow u_3 = u_1, u_2 = u_1 \frac{A_1}{A_2} \right)$$

Note: The fluid at point 2 exerts pressure on the whole area of the pipe A_1, not only the area of the jet A_2.

Simplify MB:

$$m\, u_1 \left(\frac{A_1}{A_2} - 1 \right) + (P_2 - P_3) A_1 = 0 \qquad (m = \rho\, u_1 A_1)$$

$$\boxed{\Rightarrow u_1^2 \left[1 - \frac{A_1}{A_2} \right] = \frac{P_2 - P_3}{\rho} \qquad (3)}$$

Add (2) + (3)

$$\frac{u_1^2}{2} \left[\left(\frac{A_1}{A_2} \right)^2 - 1 \right] + u_1^2 \left[1 - \frac{A_1}{A_2} \right] = \frac{P_1 - P_2}{\rho} + \frac{P_2 - P_3}{\rho}$$

Simplify:

$$\frac{u_1^2}{2} \left(\frac{A_1}{A_2} - 1 \right)^2 = \frac{P_1 - P_3}{\rho}$$

Substitute this result in Eq. (1)

$$\boxed{\Rightarrow \Im = \frac{u_1^2}{2} \left(\frac{A_1}{A_2} - 1 \right)^2}$$