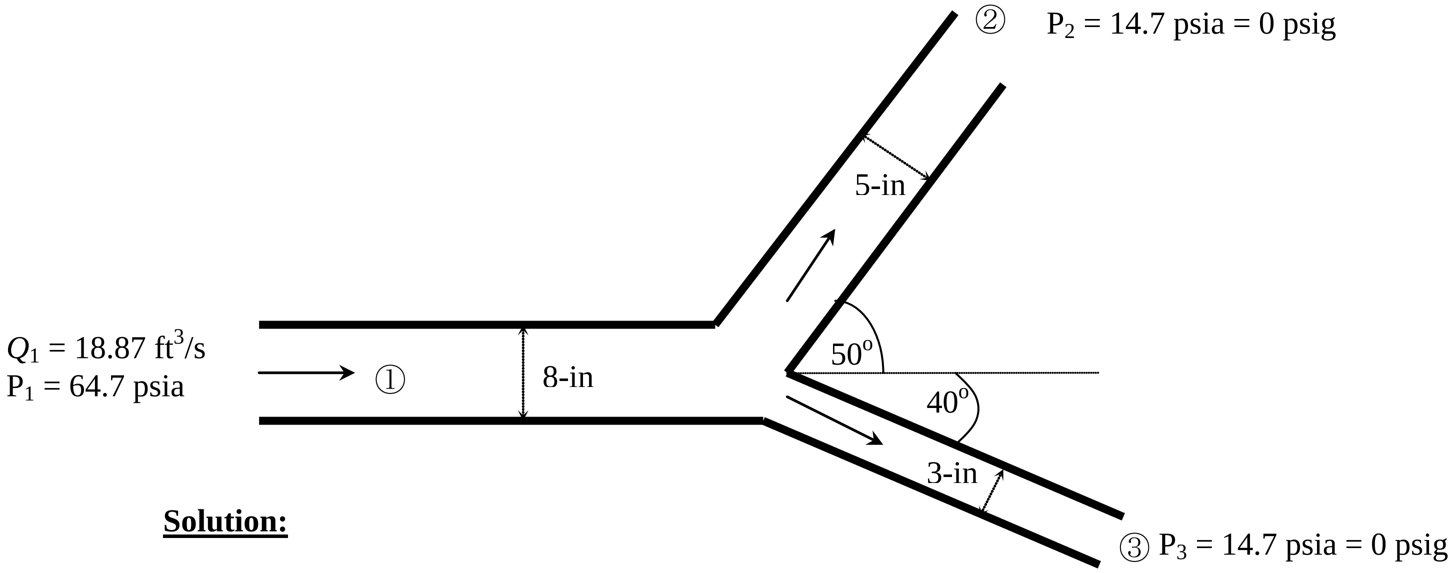
Force on a Horizontal Flow Separator

The figure below shows a horizontal flow separator (i.e., $z_1 = z_2 = z_3$). Water ($\rho_w = 62.4$ lb_m/ft^3) at points ② and ③ exits to the atmosphere. The volumetric flow rate and pressure at point ① are 18.87 ft^3/s and 64.7 psia, respectively. The diameters of the pipes are shown in the figure. **Calculate the forces (magnitude and direction) required to keep the flow separator in place**. <u>Neglect frictional losses in your analysis and assume steady state and incompressible fluid</u>.



<u>Solution:</u>

First we need to calculate the velocities at points 1, 2 and 3.

$$u_1 = \frac{18.87\ \text{ft}^3/\text{s}}{\frac{\pi}{4}\left(\frac{8}{12}\right)^2 \frac{\text{in}^2}{\text{ft}^2}} = 54.06\,\frac{\text{ft}}{\text{s}}$$

u_2 and u_3 are unknowns which can be calculated using a mass and energy balances.

<u>Mass Balance:</u>

$$u_1 A_1 = u_2 A_2 + u_3 A_3 \qquad (\text{unknowns } u_2 \text{ and } u_3) \quad \text{..........} \quad (1)$$

<u>**Energy Balance** (Bernoulli's Eq. from point 1 to point 2)**:**</u>

$$gz_1 + \frac{u_1^2}{2} + \frac{P_1}{\rho} = gz_2 + \frac{u_2^2}{2} + \frac{P_2}{\rho} (\text{unknowns } u_2, \text{and } z_1 = z_2 = 0) \quad \text{..........} \quad (2)$$

Solve for u_2 $\quad u_2 = \sqrt{u_1^2 + 2\left(\frac{P_1 - P_2}{\rho}\right)}$

➔ $$u_2 = \sqrt{(54.06\ \text{ft/s})^2 + 2\left(\frac{(64.7-14.7)\ {}^{\text{lb}_\text{f}}\!/\!_{\text{in}^2}}{62.4\ {}^{\text{lb}_\text{m}}\!/\!_{\text{ft}^3}}\right) 144\frac{\text{in}^2}{\text{ft}^2}32.2\frac{\text{lb}_\text{m}\ \text{ft}}{\text{lb}_\text{f}\,\text{s}^2}} = 101.75\frac{\text{ft}}{\text{s}}$$

Substitute result for u_2 in mass balance equation (1) and solve for u_3:

➔ $$u_3 = \frac{u_1 A_1 - u_2 A_2}{A_3} = \frac{u_1 \frac{\pi}{4} D_1^2 - u_2 \frac{\pi}{4} D_2^2}{\frac{\pi}{4} D_3^2} = \frac{u_1 D_1^2 - u_2 D_2^2}{D_3^2} = 101.75\frac{\text{ft}}{\text{s}}$$

Momentum Balance x-direction:

$$(\text{Rate Momentum})_{in}\big|_x - (\text{Rate Momentum})_{out}\big|_x + \sum F\big|_x = 0$$

$$(m_1 u_1) - (m_2 u_2 \text{Cos}(50) + m_3 u_3 \text{Cos}(40)) + (P_1 A_1 - P_2 A_2 \text{Cos}(50) - P_3 A_3 \text{Cos}(40)) + B_x = 0$$

Where B_x is the x-force required to hold the separator in the x-direction. Utilizing $m = \rho u A$ and solving for B_x:

$$B_x = -\{(\rho A_1 u_1^2) - (\rho A_2 u_2^2 \text{Cos}(50) + \rho A_3 u_3^2 \text{Cos}(40)) + (P_1 A_1 - P_2 A_2 \text{Cos}(50) - P_3 A_3 \text{Cos}(40))\}$$

Note: in the momentum balance always use **gauge** pressure

$P_1 = 50$ psig, $P_2 = P_3 = 0$ psig

$B_x = -1976.6\ \text{lb}_\text{f}$ (do it yourself)

Momentum Balance y-direction:

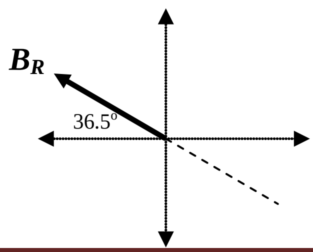
$$(\text{Rate Momentum})_{in}\big|_y - (\text{Rate Momentum})_{out}\big|_y + \sum F\big|_y = 0$$

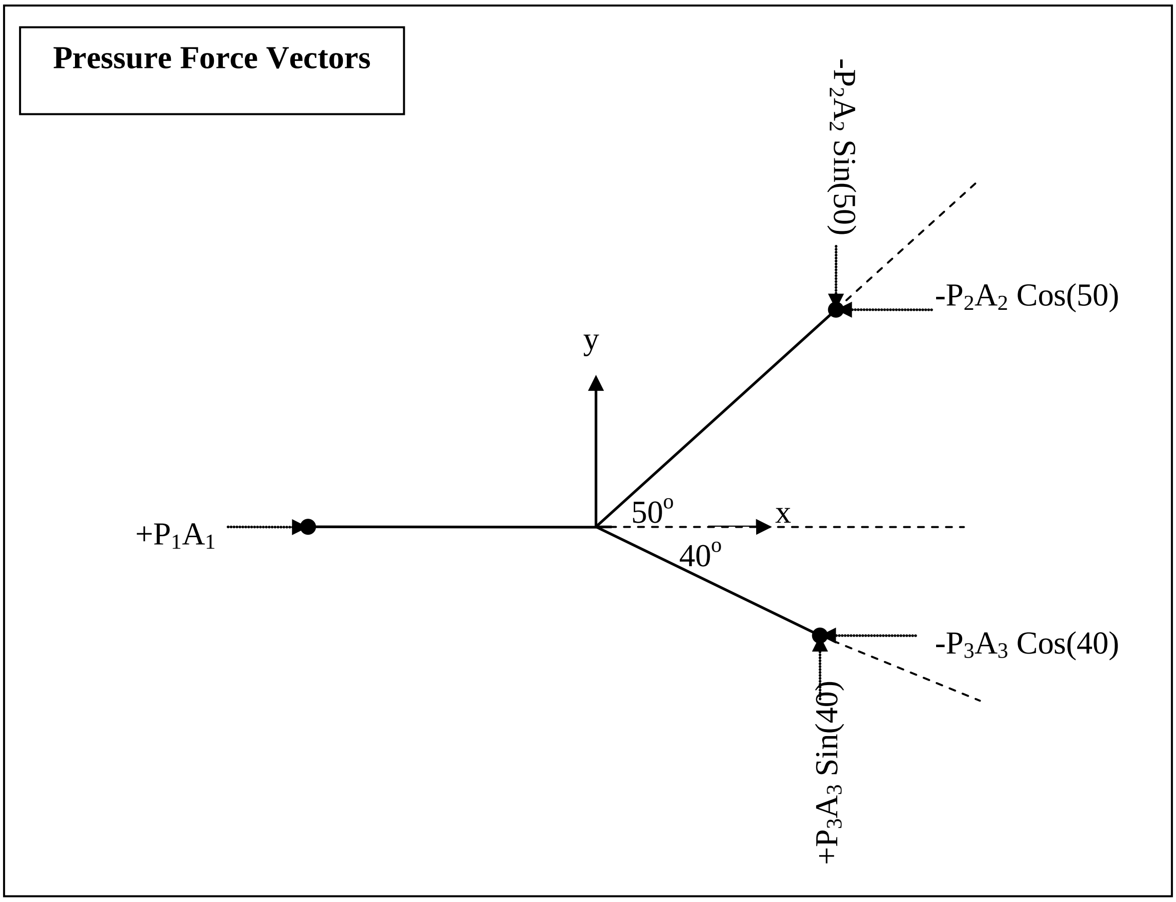
$$(0) - (m_2 u_2 \text{Sin}(50) - m_3 u_3 \text{Sin}(40)) + (-P_2 A_2 \text{Sin}(50) - P_3 A_3 \text{Sin}(40)) + B_y = 0$$

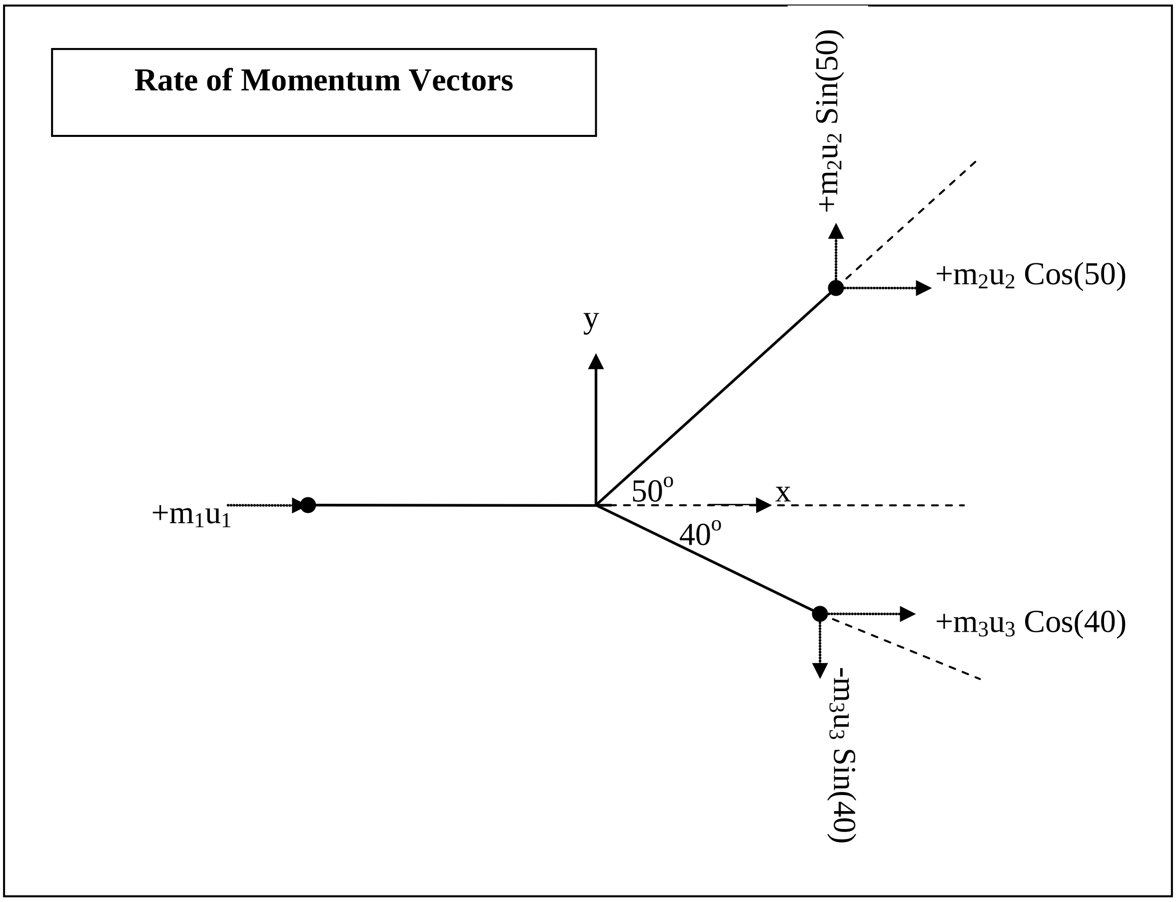
$P_2 = P_3 = 0$ psig ➔ $B_y = +1462.6\ \text{lb}_\text{f}$ (do it yourself)

Magnitude of Resultant Force $B_R = \sqrt{(B_x^2 + B_y^2)} = 2459\ \text{lb}_\text{f}$

Direction of Resultant Force $\theta = \tan^{-1}\left(\frac{B_y}{B_x}\right) = -36.5^\circ$




Pressure Force Vectors
-P2A2 Sin(50)
-P2A2 Cos(50)
y
+P1A1
50o
x
40o
-P3A3 Cos(40)
+P3A3 Sin(40)


Rate of Momentum Vectors
+m2u2 Sin(50)
+m2u2 Cos(50)
y
+m1u1
50o
x
40o
+m3u3 Cos(40)
-m3u3 Sin(40)