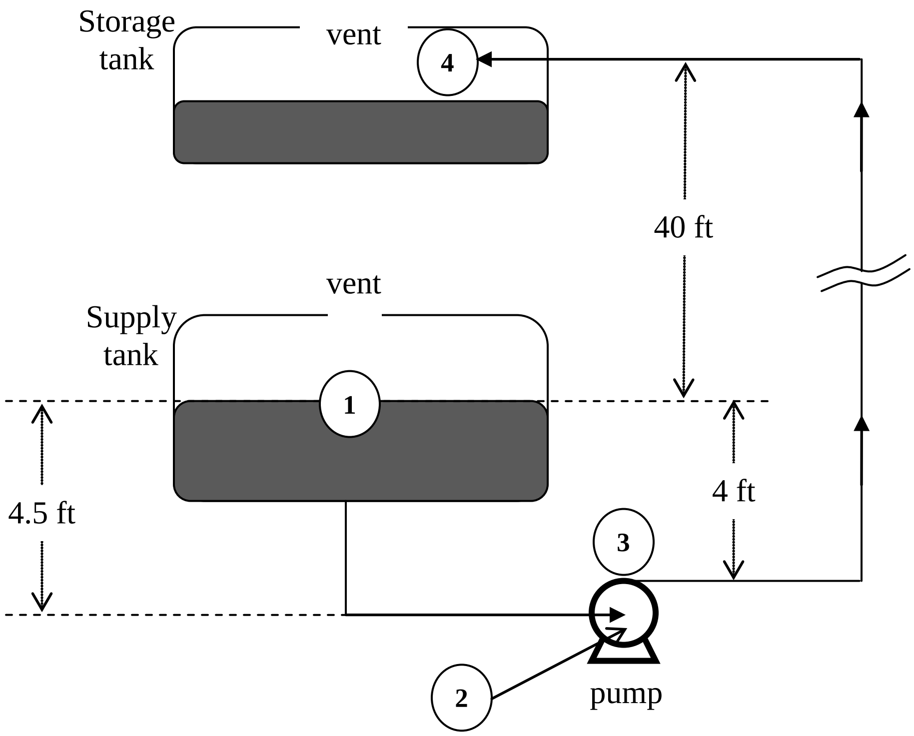
Calculation of Pumping Power

(see example 2.2 of the textbook)



The figure above shows an arrangement for pumping n-pentane ($\rho_{\text{n-pentane}}$ = 39.3 lbm/ft^3) at 25^{o}C from one tank to another, through a vertical distance of 40 ft. All piping is 3-in. I.D. Assume that the overall frictional losses in the pipes are given by (methods to be described later in Chapter 3):

$$F = 2.5u_m^2 \frac{\text{ft}^2}{\text{s}^2} = \frac{2.5u_m^2}{g_c} \frac{\text{ft lb}_\text{f}}{\text{lb}_\text{m}}$$

For simplicity, you may ignore friction in the short length of pipe leading to the pump inlet. Also, the pump and its motor have a combined efficiency of 75%. If the mean velocity u_m is 25 ft/s, determine the following:

(a) The power required to drive the pump.
(b) The pressure at the inlet of the pump, and compare it with 10.3 psia, which is the vapor pressure of n-pentane at 25^{o}C.
(c) The pressure at the pump exit.

Solution:

(a) Power, $\underline{P} = m * w_s$

$$m = \rho u_m A_{\text{pipe}} = 39.3 * 25 * \frac{\pi}{4}\left(\frac{3}{12}\right)^2 = 48.2\,\frac{\text{lb}_\text{m}}{\text{s}}$$

Mechanical Energy Balance (MEB) to find w_s:

$$\Delta\left(\frac{u^2}{2}\right) + g\Delta z + \frac{\Delta P}{\rho} + w_s + F = 0$$

Apply MEB between points (1) and (4)

$$\Rightarrow \left(\frac{u_4^2}{2} - \frac{u_1^2}{2}\right) + g(z_4 - z_1) + \frac{P_4 - P_1}{\rho} + w_s + 2.5u_m^2 = 0 \qquad \text{Note that } (P_1 = P_4)$$

$$\Rightarrow w_s = -\left\{\left(\frac{25^2}{2} - \frac{0}{2}\right) + 32.2 * 40 + 0 + 2.5 * 25^2\right\} = -3163\,\frac{\text{ft}^2}{\text{s}^2} * \frac{1}{32.2\,\frac{\text{lb}_\text{m}\,\text{ft}}{\text{lb}_\text{f}\,\text{s}^2}} = -98.3\,\frac{\text{ft lb}_\text{f}}{\text{lb}_\text{m}}$$

Note: -ve sign is due to the fact that the work is done on the system.

$$\text{Real work required} = \text{W}_{\text{real}} = \frac{\text{- work}_{\text{ideal}}}{\text{efficiency}} = \frac{-w_s}{\eta} = \frac{98.3}{0.75} = 131.0\,\frac{\text{lb}_\text{f}\text{ ft}}{\text{lb}_\text{m}}$$

$$\text{Power} = m\,\text{W}_{\text{real}} = 48.2\,\frac{\text{lb}_\text{m}}{\text{s}} * 131.0\,\frac{\text{lb}_\text{f}\text{ ft}}{\text{lb}_\text{m}} = 6313.0\,\frac{\text{lb}_\text{f}\text{ ft}}{\text{s}} * \frac{1}{737.6\,\frac{\text{lb}_\text{f}\text{ ft/s}}{\text{kw}}} = 8.56\text{ kw}$$

(b) To find P_2 at the inlet of the pump apply MEB between (1) and (2)

$$\left(\frac{u_2^2}{2} - \frac{u_1^2}{2}\right) + g(z_2 - z_1) + \frac{P_2 - P_1}{\rho} + w + F = 0$$

- No work between (1) and (2)
- Short pipe assume negligible frictional losses

$$\left(\frac{25^2}{2}-\frac{0}{2}\right)+32.2*(-4.5)+\frac{P_2-0}{39.3}+0+0=0$$

$$P_2=-6586.7\ \frac{\text{ft}^2}{\text{s}^2}\frac{\text{lb}_\text{m}}{\text{ft}^3}*\frac{1}{32.2\frac{\text{ft}\ \ \text{lb}_\text{m}}{\text{lb}_\text{f}\ \ \text{s}^2}}=-204.6\frac{\text{lb}_\text{f}}{\text{ft}^2}\quad \text{(gauge pressure)}$$

$$P_2=-204.6\frac{\text{lb}_\text{f}}{\text{ft}^2}*\frac{\text{ft}^2}{144\,\text{in}^2}=-1.42\,\text{psig}=-1.42+14.7=13.3\,\text{psia}$$

(c) To find P_3 MEB between (2) and (3)

$$\frac{u_3^2-u_2^2}{2}+g(z_3-z_2)+\frac{P_3-P_2}{\rho}+w+F=0$$

P_2 = -1.42 psig, $u_2 = u_3$, z_3-z_2 = 0.5 ft, $w_s=-98.3\frac{\text{ft}\,\text{lb}_\text{f}}{\text{lb}_\text{m}}$ (see part a)

solving $\Rightarrow P_3=25.2\,\text{psig}$