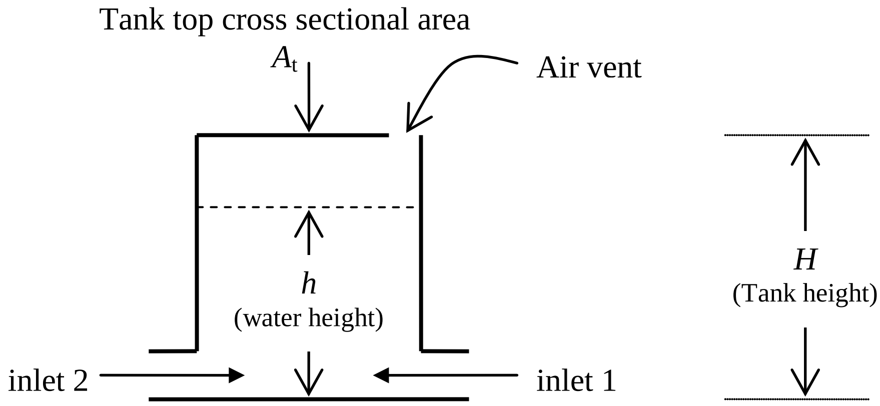
Mass Balance

CHE 204, Prepared by: Dr. Usamah Al-Mubaiyedh

A tank is filled with water by two inlets, as shown in the figure below:



(a) Derive an expression for the rate of water height increase as a function of time, dh/dt .

(b) Calculate dh/dt if A_t = 2 ft^2, D_1 = 1-in, D_2 = 3-in, u_1 = 3 ft/s, u_2 = 2 ft/s.

Solution:

(a) Mass Balance: $$m_{\text{in}} - m_{\text{out}} = \frac{d}{dt}\left(\mathrm{M}_{\text{syst.}}\right), \qquad \left(m_{\text{out}} = 0\right)$$

$$m_{\text{in}} = \rho u_1 A_1 + \rho u_2 A_2 \quad , \quad M_{\text{syst.}} = \rho A_t h$$

$$\rho u_1 A_1 + \rho u_2 A_2 = \frac{d}{dt}\left(\rho A_t h\right), \qquad \left(m_{\text{out}} = 0\right)$$

$$\frac{dh}{dt} = \frac{u_1 A_1 + u_2 A_2}{A_t}$$

(b)

$$\frac{dh}{dt} = \frac{3\,\frac{\pi}{4}\left(\frac{1}{12}\right)^2 + 2\,\frac{\pi}{4}\left(\frac{3}{12}\right)^2}{2} = 0.057 \quad \text{ft/s}$$

Mass Balance

(see example 2.1 in the textbook)

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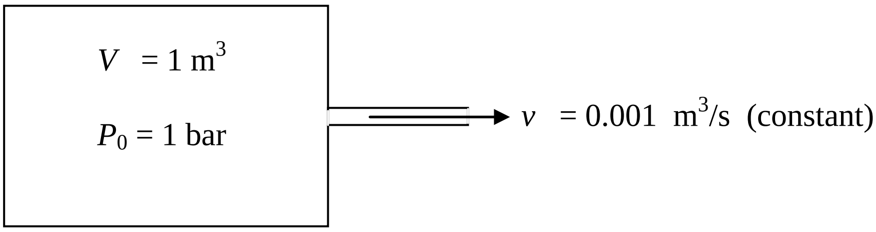
The tank shown in the figure below has a volume $V = 1\ \mathrm{m}^3$ and contains air that is maintained at a constant temperature by being in thermal equilibrium with its surroundings.



If the initial absolute pressure is $P_0 = 1$ bar, how long will it take for the pressure to fall to a final pressure of 0.0001 bar if the air is evacuated at a constant rate of $v = 0.001\ \mathrm{m}^3/\mathrm{s}$.

<u>Solution:</u>

(a) Mass Balance:

$$m_{\mathrm{in}} - m_{\mathrm{out}} = \frac{d}{dt}\left(\mathrm{M}_{\mathrm{system}}\right), \qquad \left(m_{\mathrm{in}} = 0\right)$$

$$0 - \rho_{\mathrm{out}}\, v = \frac{d}{dt}\left(\rho_{\mathrm{syst.}} V\right), \qquad \left(V = \mathrm{constant}\right)$$

$$\rho_{\mathrm{out}} = \rho_{\mathrm{syst.}} = \rho = \frac{P\, M_{\mathrm{w}}}{RT} \qquad \left(\text{for I.G.}\right)$$

$$\Rightarrow \quad -v\, \frac{P\, M_{\mathrm{w}}}{RT} = V \frac{d}{dt}\left(\frac{P\, M_{\mathrm{w}}}{RT}\right) \qquad \left(\frac{M_{\mathrm{w}}}{RT} = \mathrm{constant}\right)$$

Simplify

$$\frac{dP}{dt} = -\frac{v}{V} P$$

$$\int_{P_0}^{P} \frac{dP}{P} = \int_0^t -\frac{v}{V}\, dt \qquad \left(\frac{v}{V} = \mathrm{constant}\right)$$

$$P = P_0 \quad e^{-\frac{v}{V}t}$$

Or

$$t = -\frac{V}{v} \ln\left(\frac{P}{P_0}\right) = -\frac{1}{0.001} \ln\left(\frac{0.0001}{1}\right) = 9210\ \mathrm{s} = 2.56\ \mathrm{hr}$$