Consider a vertical bed of particles for the case when the fluid flows upwards through the bed as shown in the figure below. When the fluid velocity through the bed exceeds a critical value the particles in the bed will start moving like a fluid and this action is called “fluidization”. The objective of this analysis is to calculate the minimum fluidization velocity.

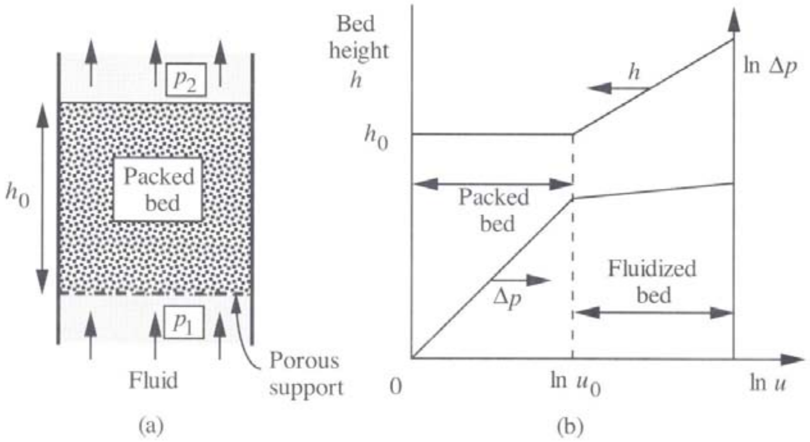


Fig. 4.17 Fluidization: (a) upwards flow through a packed bed, and (b) variation of bed height with superficial velocity.

Before fluidization the problem is similar to flow through a packed bed. Apply Mechanical Energy Balance for upwards flow through the bed:

$$\frac{\Delta P}{\rho_f} + \Delta\left(\frac{u_0^2}{2}\right) + g\,\Delta z + \Im + w_S = 0 \tag{1}$$

If the cross-sectional area of the bed ***A*** is constant and there is no shaft work between the bed’s inlet and outlet, the mechanical energy balance can be simplified:

$$-\Delta P = \left(P_1 - P_2\right) = \rho_f\, g\, h_0 + \rho_f\, \Im \tag{2}$$

For packed beds the frictional losses is given by the following equation:

$$\Im = 3 f_F \frac{1-\varepsilon_0}{\varepsilon_0^3} u_0^2 \frac{L}{D_P} \tag{3}$$

where ε_0 is the void fraction before fluidization. $\boldsymbol{f_F}$ appearing in equation (3) is the friction factor for packed beds which accounts for both laminar and turbulent flow regimes:

$$f_F = \frac{1}{3}\left[\overbrace{\frac{150}{\text{Re}}}^{\text{Laminar Contribution}} + \overbrace{1.75}^{\text{Turbulent Contribution}}\right] \tag{4}$$

and **Re** is the Reynolds number for packed beds defined as:

$$\mathrm{Re} = \frac{\rho_f \, u_0 \, D_P}{(1-\varepsilon_0)\mu_f} \tag{5}$$

Substituting equations (3-5) into the mechanical energy balance equation (2) and rearranging, the following equation can be derived for flow through packed beds: known as Ergun equation:

$$(P_1 - P_2) = \overbrace{\left[\frac{150(1-\varepsilon_0)\mu_f}{\rho_f \, u_0 \, D_P} + 1.75\right]\rho_f \, u_0^2 \, \frac{L}{D_P}\frac{1-\varepsilon_0}{\varepsilon^3}}^{\text{Pressure drop due to friction}} + \overbrace{\rho_f \, g \, h_0}^{\text{Pressure drop due to gravity}} \tag{6}$$

To derive an equation for the minimum fluidization velocity, we have to apply the following physical concept:

Fluidization of the packed bed starts when the pressure drop across the packed bed is equal the weight of the bed/unit area.

$$\text{Pressure drop across bed} = \left[\frac{150(1-\varepsilon_0)\mu_f}{\rho_f \, u_0 \, D_P} + 1.75\right]\rho_f \, u_0^2 \, \frac{L}{D_P}\frac{1-\varepsilon_0}{\varepsilon_0^3} + \rho_f \, g \, (h_0)$$

$$\frac{\text{Weight of the bed}}{\text{Unit Area}} = \overbrace{(1-\varepsilon_0)\rho_s h_0 g}^{\text{Weight of Particles/Unit Area}} + \overbrace{\varepsilon_0 \rho_f h_0 g}^{\text{Weight of fluid between particles/Unit Area}}$$

Equating:

$$\left[\frac{150(1-\varepsilon_0)\mu_f}{\rho_f \, u_0 \, D_P} + 1.75\right]\rho_f \, u_0^2 \, \frac{L}{D_P}\frac{1-\varepsilon_0}{\varepsilon_0^3} + \rho_f \, g \, h_0 = (1-\varepsilon_0)\rho_s h_0 g + \varepsilon_0 \rho_f h_0 g \tag{7}$$

And rearranging:

$$\overbrace{\left(1.75\frac{\rho_f}{D_P \, \varepsilon_0^3}\right)}^{a} u_0^2 + \overbrace{\left(\frac{150(1-\varepsilon_0)\mu_f}{D_P^2 \, \varepsilon_0^3}\right)}^{b} u_0 + \overbrace{\left(-g(\rho_S - \rho_f)\right)}^{c} = 0 \tag{8}$$

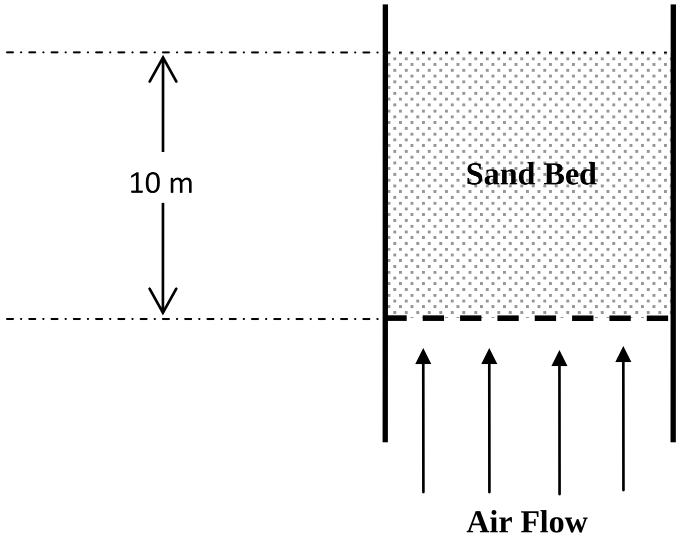
The above equation is similar to equation 4.43 of the textbook. Solving for u_0 gives the minimum fluidization velocity:

$$\Rightarrow u_0 = \frac{-\mathrm{b} \pm \sqrt{\mathrm{b}^2 - 4\,\mathrm{a}\,\mathrm{c}}}{2\,\mathrm{a}} \tag{9}$$

Hint: Which answer do you take the +ve of –ve ?

Example

Air of density 1.21 kg/m^3 and viscosity 18.3 x 10^{-6} kg/m s flows upwards through a sand bed, as shown in figure below. The sand particles are uniform, with an average diameter of 0.2 mm and void fraction of 0.3 (before fluidization) and density 4500 kg/m^3. If the height of the bed before fluidization is 10 m, calculate the minimum fluidization velocity.



Recall the minimum fluidization velocity equation:

$$\overbrace{\left(1.75\frac{\rho_f}{D_P\ \varepsilon_0^3}\right)}^{a}u_0^2+\overbrace{\left(\frac{150(1-\varepsilon_0)\mu}{D_P^2\ \varepsilon_0^3}\right)}^{b}u_0+\overbrace{\left(-g(\rho_S-\rho_f)\right)}^{c}=0$$

$$a=1.75\frac{\rho_f}{D_P\ \varepsilon^3}=1.75\frac{(1.21)}{(0.2\times10^{-3})(0.3)^3}=392129.6$$

$$b=\frac{150(1-\varepsilon_0)\mu}{D_P^2\ \varepsilon^3}=\frac{150(1-0.3)(18.3\times10^{-6})}{(0.2\times10^{-3})^2(0.3)^3}=1779166.7$$

$$c=-g(\rho_S-\rho_f)=-9.8(4500-1.21)=-44088.1$$

$$\Rightarrow u_0=\frac{-\mathrm{b}+\sqrt{\mathrm{b}^2-4\,\mathrm{a}\,\mathrm{c}}}{2\,\mathrm{a}}=\frac{-1779166.7+1798495.8}{784259.2}=0.0246\ \mathrm{m/s}=88.7\ \mathrm{m/hr}$$