Flow through Packed Beds

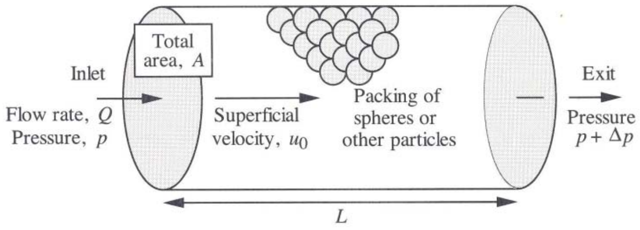

Fig. 4.10 Flow through a packed bed.

Table 4.2 Notation for Flow Through Packed Beds

Symbol	*Meaning*
A	Cross-sectional area of bed
a_v	Surface area of a particle divided by its volume
D_p	Effective particle diameter, $6/a_v$
L	Bed length
Q	Volumetric flow rate
u_0	Superficial fluid velocity, Q/A
ε	Fraction void (not occupied by particles)
ρ, μ	Fluid density and viscosity

The mechanical energy balance can be applied for flow across a packed bed:

$$\frac{\Delta P}{\rho} + \Delta\left(\frac{u_0^2}{2}\right) + g\,\Delta z + \Im + w_S = 0 \qquad (1)$$

If the cross-sectional area of the bed ***A*** is constant and there is no shaft work between the bed's inlet and outlet, the mechanical energy balance can be simplified:

$$-\frac{\Delta P}{\rho} = g\,\Delta z + \Im \qquad (2)$$

The pressure drop across the bed is due gravity and friction. For packed beds the frictional losses is given by the following equation:

$$\Im = 3 f_F \,\frac{1-\varepsilon}{\varepsilon^3} u_0^2 \,\frac{L}{D_P} \qquad (3)$$

where $\boldsymbol{\varepsilon}$ is the fraction of void not occupied by particles (void fraction), ***L*** is the total length of the bed and $\boldsymbol{D_P}$ is the effective particle diameter defined as follows:

$$D_P = \frac{6}{a_v} \qquad (4)$$

and a_v is the total external surface area of the particle divided by particle volume. For spherical particles, D_P is simply the particle diameter. f_F appearing in equation (3) is the friction factor for packed beds which accounts for both laminar and turbulent flow regimes:

$$f_F = \frac{1}{3}\left[\overbrace{\frac{150}{\text{Re}}}^{\text{Laminar Contribution}} + \overbrace{1.75}^{\text{Turbulent Contribution}}\right] \tag{5}$$

and **Re** is the Reynolds number for packed beds defined as:

$$\text{Re} = \frac{\rho\, u_0\, D_P}{(1-\varepsilon)\mu} \tag{6}$$

where ρ and μ are the fluid density and viscosity, respectively. u_0 is the fluid superficial velocity defined as the ratio of fluid volumetric flow rate by the cross-sectional area of the bed:

$$u_0 = \frac{Q}{A} \tag{7}$$

Substituting equations (3-5) into the mechanical energy balance equation (2) and rearranging, the following equation can be derived for flow through packed beds known as Ergun equation:

$$-\frac{\Delta P}{\rho\, u_0^2}\frac{D_P}{L}\frac{\varepsilon^3}{1-\varepsilon} = \left[\frac{150}{\text{Re}} + 1.75\right] + \frac{1}{u_0^2}\frac{D_P}{L}\frac{\varepsilon^3}{1-\varepsilon} g\,\Delta z \tag{8}$$

The above equation is similar to equation [4.26] of the textbook, however, it is more general to account for non-horizontal beds. Hence, equation [4.26] is for the special case when Δz = 0.

Rearranging equation (8), the following equation can be written for the pressure drop across packed beds:

$$-\frac{\Delta P}{\rho} = \overbrace{\left[\frac{150}{\text{Re}} + 1.75\right] u_0^2 \frac{L}{D_P}\frac{1-\varepsilon}{\varepsilon^3}}^{\text{Pressure drop due to friction}} + \overbrace{g\,\Delta z}^{\text{Pressure drop due to gravity}} \tag{9}$$

Once again the above equation is similar to equation [4.29] of the textbook, however, it is more general to account for non-horizontal beds. Hence, equation [4.29] is for the special case when Δz = 0.