Settling of Particles

Example 1

A stainless steel sphere has a diameter 1 mm and a density of 7870 kg/m^3. Calculate the terminal velocity of the sphere for the following two cases:

(a) The sphere is dropped in polymer of density 1052 kg/m^3 and viscosity 0.1 kg/m s.
(b) The sphere is dropped in kerosene of density 765.52 kg/m^3 and viscosity 0.00818 kg/m s.

Solution:

$$C_D = \frac{4}{3}\frac{g\,D}{u_t^2}\frac{\rho_S - \rho_f}{\rho_f} = \frac{4}{3}\frac{(9.8)(1*10^{-3})}{u_t^2}\;\frac{7870-\rho_f}{\rho_f} = \frac{0.0131}{u_t^2}\;\frac{7870-\rho_f}{\rho_f}$$

(a) ρ_f = 1052 kg/ m^3, μ_f = 0.1 kg / m s

$$C_D = \frac{0.0131}{u_t^2}\;\frac{7870-1052}{1052} = \frac{0.0847}{u_t^2}$$

Assume laminar flow (Re ≤ 1) ➔ $C_D = \frac{24}{\mathrm{Re}}$

$$\frac{24\mu}{\rho_f u_t D} = \frac{0.0847}{u_t^2},$$ Solve for u_t = 0.037 m/s,

Check Re = $\frac{\rho_f u_t D}{\mu} = \frac{(1052)(0.037)(1*10^{-3})}{0.1} = 0.39$

Re < 1 ➔ assumption of laminar flow was correct $\boxed{u_t = 0.037 \text{ m/s}}$

(b) ρ_f = 765.52 kg/ m^3, μ_f = 0.00818 kg/(m.s)

$$C_D = \frac{4}{3}\frac{g\,D}{u_t^2}\frac{\rho_S - \rho_f}{\rho_f} = \frac{0.0131}{u_t^2}\;\frac{7870 - 765.52}{765.52} = \frac{0.1216}{u_t^2}$$

Assume laminar (Re ≤ 1) ➔ $C_D = \frac{24}{\text{Re}}$

$$\frac{0.1216}{u_t^2} = \frac{24\mu}{\rho_f u_t D} = \frac{24(0.1)}{(765.52)u_t\left(1*10^{-3}\right)}$$

Solve for u_t = 0.474 m/s

Check $\text{Re} = \frac{\rho_f u_t D}{\mu} = \frac{(765.52)(0.474)\left(1*10^{-3}\right)}{0.00818} = 44.4$

Re > 1 ➔ laminar assumption was wrong,

Assume transition 1 < Re ≤ 1000

$$C_D = \frac{0.1216}{u_t^2} = \frac{18}{\text{Re}^{0.6}} = \frac{18\mu^{0.6}}{\rho_f^{0.6}u_t^{0.6}D^{0.6}}$$

solve for u_t = 0.1970 m/s

Check Re = $\frac{\rho_f u_t D}{\mu} = \frac{(765.52)(0.1970)\left(1*10^{-3}\right)}{0.00818} = 18.4$

1 < Re < 1000 ➔ transition assumption was correct and $\boxed{u_t = 0.197 \text{ m/s}}$

Example 2

A sphere of 2.2 mm diameter is dropped in water (ρ = 1000 kg/m^3, μ = 1*10 kg m/s). The sphere falls a distance of 10.5 cm in 21 seconds. Assuming steady state conditions:

(a) Calculate the density of the sphere.
(b) If the above sphere is dropped in a fluid of a specific gravity of 0.81 and its terminal velocity was measured as 2.05 cm/s, calculate the viscosity of the fluid.

Solution:

(a) D = 2.2 mm = 2.2 *10^{-3} m, ρ_f = 1000 kg/m^3 , μ = 1* 10 $^{-3}$ kg/m s, u_t =10.5/21 cm/s = 0.005 m/s

$$\mathrm{Re} = \frac{(1000)(5*10^{-3})(2.2*10^{-3})}{1*10^{3}} = 11$$ ➔ 1 < Re ≤ 1000 (transition)

For Transition: $$C_D = \frac{18}{\mathrm{Re}^{0.6}} = \frac{18}{(11)^{0.6}} = 4.27$$

$$C_D = \frac{4}{3}\frac{g\,D}{u_t^2}\frac{\rho_S - \rho_f}{\rho_f} = 4.27$$

$$4.27 = \frac{4}{3}\frac{(9.8)\left(2.2*10^{-3}\right)}{(0.005)^2}\frac{\rho_S - 1000}{1000}$$ ➔ ρ_S = 1003.7 kg/m^3

(b) ρ_f = (0.81) (1000) = 810 kg/m^3

$$C_D = \frac{4}{3}\frac{g\,D}{u_t^2}\frac{\rho_S - \rho_f}{\rho_f} = \frac{4}{3}\frac{(9.8)(2.2*10^{-3})}{(2.05*10^{-2})^2}\frac{1003.7 - 810}{810} = 16.4$$

We know C_D but don't know Re:

Assume laminar (Re < 1) ➔ $C_D = \frac{24}{\mathrm{Re}} = 16.4 \quad \Rightarrow \quad \mathrm{Re} = 1.46$ Assumption was wrong

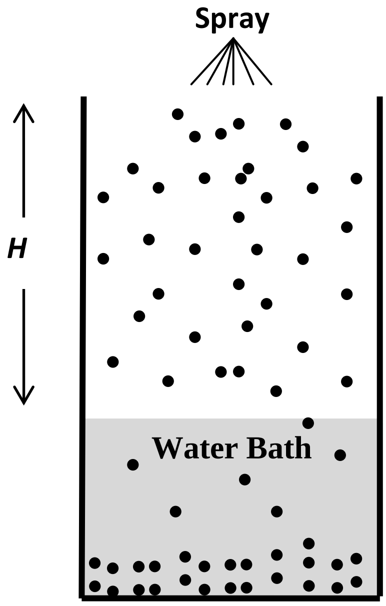
Assume Transition (1 < Re ≤ 1000) ➔ $C_D = \frac{18}{\mathrm{Re}^{0.6}} = 16.4$

$\Rightarrow \quad \mathrm{Re} = 1.17$ Transition assumption was correct

$$\mathrm{Re} = \frac{(810)(2.05*10^{-3})(2.2*10^{-3})}{\mu} = 1.17 \quad \Rightarrow \quad \mu = 0.031\ \frac{\mathrm{kg}}{\mathrm{m\,s}}$$

Example 3

Lead particles of diameter 3 mm are manufactured by spraying hot molten lead of density 11400 kg/m^3 from the top of a spray tower in which the hot lead droplets are cooled by the surrounding stagnant air of density 1.21 kg/m^3 and viscosity 18.3 x 10^{-6} kg/m s. By the time they fall a distant ***H*** to the base of the tower they solidify and quench in a cold water bath as shown in the figure below.



You are designing the spray tower and you are told by a heat transfer expert that molten lead particles need about 10 seconds to solidify if they fall freely in air. Perform the following:

(a) Assuming steady state, calculate the particle terminal velocity (steady state velocity) and based on that calculate the height of the tower ***H***. In this case you assume that the particles reach terminal velocity in a very short time << 10 s.

(b) Do not assume steady state but assume that the drag coefficient is constant and it is equal to that calculated by steady state approximation. Calculate the particle velocity as a function of time and calculate the height of the tower ***H***

Solution:

(a) Recall for steady state:

$$C_D = \frac{4}{3}\frac{g\,D}{u_t^2}\frac{\rho_S-\rho_f}{\rho_f} = \frac{4}{3}\frac{(9.8)(3*10^{-3})}{u_t^2}\;\frac{11400-1.21}{1.21} = \frac{369.28}{u_t^2}$$

Because we have high density particles falling in low density and low viscosity air, they are expected to have high velocities and hence we can assume turbulent flow. However, we have to validate our assumption after calculating $\boldsymbol{u_t}$ to ensure that Re > 1000.

For turbulent flow (Re > 1000), $C_D = 0.44$

$$C_D = 0.44 = \frac{369.28}{u_t^2}$$ solve for u_t = 29.0 m/s

Check Re = $\dfrac{\rho_f u_t D}{\mu} = \dfrac{(1.21)(29)(3*10^{-3})}{18.3*10^{-6}} = 5752.5 > 1000$

Indeed the turbulent assumption was valid and ***u_t* = 29.0 m/s**

For steady state $H = u_t \times t = 29 \times 10 = 290$ m

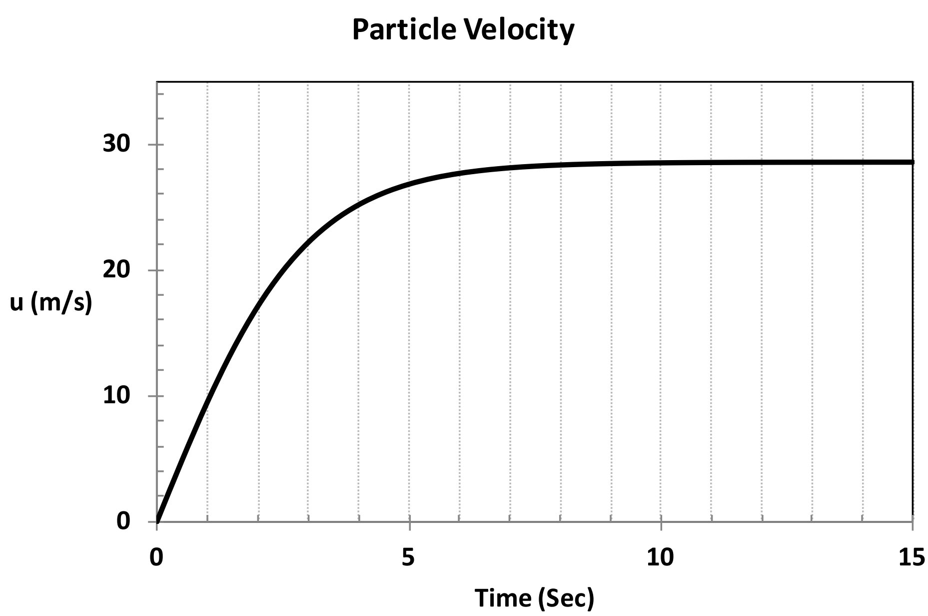
(b) Recall equation (E4.2.5) of the textbook for unsteady state particle velocity when the fluid density is very small compared to the particle density $(1 - \rho_f / \rho_s)g \approx g$:

$$u = \sqrt{\frac{g}{c}}\,\frac{e^{(2\sqrt{gc})t} - 1}{e^{(2\sqrt{gc})t} + 1}$$

$$c = \frac{3\rho_f C_D}{4D\,\rho_s} = \frac{3(1.21)(0.44)}{4(3\times10^{-3})(11400)} = 0.012$$

$$u = \sqrt{\frac{9.8}{0.012}}\,\frac{e^{(2\sqrt{9.8\times0.012})t} - 1}{e^{(2\sqrt{9.8\times0.012})t} + 1} = 28.6\frac{e^{0.69\,t} - 1}{e^{0.69\,t} + 1}$$

The figure below shows *u* as a function of time calculated by using the above equation. It is clear that the particle reaches the terminal velocity (steady state) in 7 seconds.



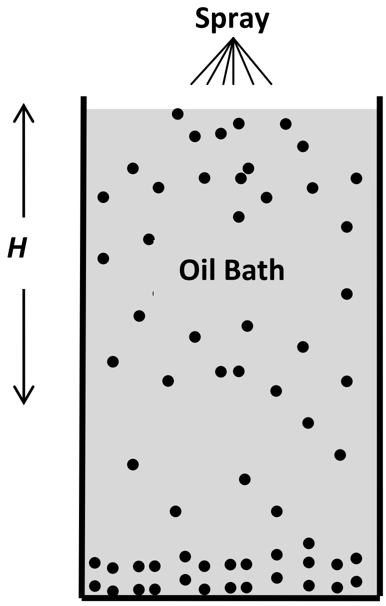
The distant travelled by particle as function of time is given in equation (E4.2.7) of the textbook

$$x = \frac{1}{2c}\sqrt{\frac{g}{c}}\left[2\ln\left(\frac{e^{\left(2\sqrt{gc}\right)t}+1}{2}\right)-\left(2\sqrt{gc}\right)t\right]$$

$$H = \frac{1}{2(0.0117)}\left[2\ln\left(\frac{e^{\left(2\sqrt{9.8\times 0.0117}\right)10}+1}{2}\right)-\left(2\sqrt{9.8\times 0.0117}\right)10\right] = 230\ \text{m}$$

Conclusion: By assuming steady state for turbulent flow there is 26% error in calculating the height of the tower.

Example 4
Refer to the previous example but now the Lead particles are dropped immediately in an oil bath of density 911 kg/m^3 and viscosity 1 kg/m s.



You are designing a spray tower for this case and you are told by the heat transfer expert that molten lead particles need about 1 second to solidify if they fall freely in oil. Perform the following:

(a) Assuming steady state, calculate the particle terminal velocity (steady state velocity) and based on that calculate the height of the tower ***H***. In this case you assume that the particles reach terminal velocity in a very short time << 10 s.
(b) Do not assume steady state but assume that the drag coefficient is constant and it is equal to that calculated by steady state approximation. Calculate the particle velocity as a function of time and calculate the height of the tower ***H***

Solution:

(a) $$C_D = \frac{4}{3}\frac{g\,D}{u_t^2}\frac{\rho_S - \rho_f}{\rho_f} = \frac{4}{3}\frac{(9.8)(3*10^{-3})}{u_t^2}\;\frac{11400-911}{911} = \frac{0.45}{u_t^2}$$

Because the droplets are falling now in high density and high viscosity oil, they are expected to have low velocities and hence we can assume laminar flow. However, we have to validate our assumption after calculating $\boldsymbol{u_t}$ to ensure that Re ≤ 1.

Assume laminar flow (Re ≤ 1) ➔ $C_D = \dfrac{24}{\mathrm{Re}}$

$$\frac{24\mu}{\rho_f u_t D} = \frac{0.45}{u_t^2} \quad \Rightarrow \quad \frac{24(1)}{(911)u_t\left(3\times10^{-3}\right)} = \frac{0.45}{u_t^2},$$ Solve for u_t = 0.051 m/s,

Check Re = $\frac{\rho_f u_t D}{\mu} = \frac{(911)(0.051)\left(3\times10^{-3}\right)}{1} = 0.14 \le 1$

The laminar assumption was valid $\quad u_t$ = 0.051 m/s

For steady state $H = u_t \times t = 0.051\times1 = 0.051\ \text{m} = 5.1\ \text{cm}$

(b) Recall equation (E4.2.5) of the textbook for unsteady state particle velocity when the fluid density is very small compared to the particle density $(1-\rho_f/\rho_s)g \approx g$:

$$u = \sqrt{\frac{g}{c}}\,\frac{e^{\left(2\sqrt{gc}\right)t}-1}{e^{\left(2\sqrt{gc}\right)t}+1} \qquad \text{where} \qquad c = \frac{3\rho_f C_D}{4D\,\rho_s}$$

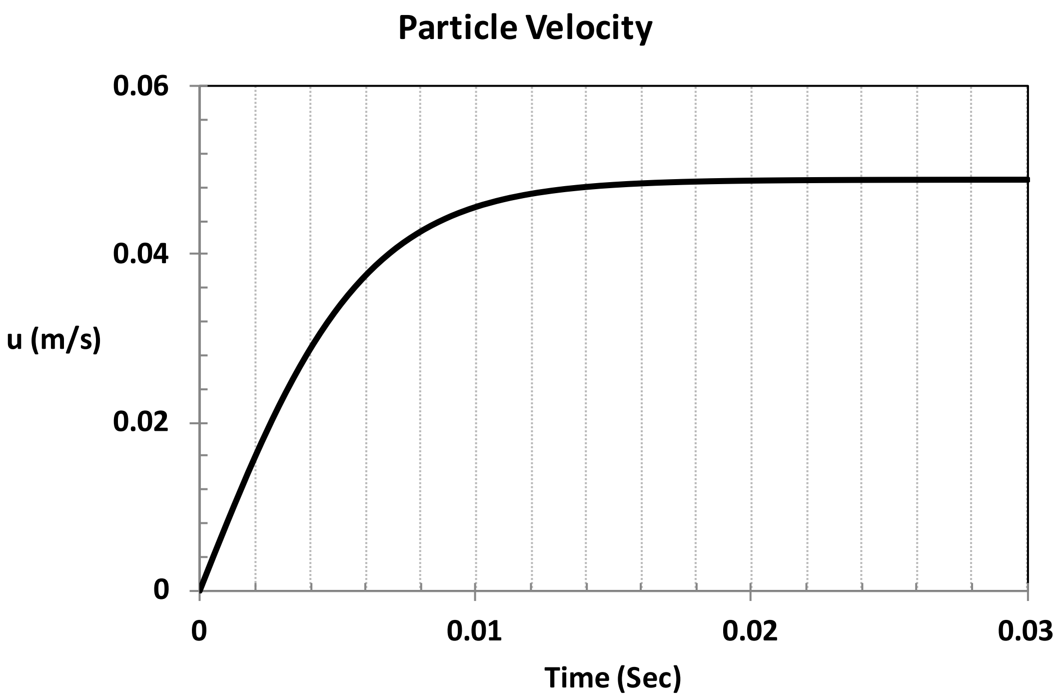
Since the oil density is quite high $(1-\rho_f/\rho_s)g \ne g$ the above unsteady state velocity can be modified as follows:

$$u = \sqrt{\frac{(1-\rho_f/\rho_s)g}{c}}\,\frac{e^{\left(2\sqrt{(1-\rho_f/\rho_s)gc}\right)t}-1}{e^{\left(2\sqrt{(1-\rho_f/\rho_s)gc}\right)t}+1}$$

$$c = \frac{3\rho_f C_D}{4D\,\rho_s} = \frac{3(911)\left(\frac{24}{0.14}\right)}{4\left(3\times10^{-3}\right)(11400)} = 3424.8$$

$$u = \sqrt{\frac{\left(1-\frac{911}{11400}\right)9.8}{3424.8}}\,\frac{e^{\left(2\sqrt{\left(1-\frac{911}{11400}\right)9.8\times3424.8}\right)t}-1}{e^{\left(2\sqrt{\left(1-\frac{911}{11400}\right)9.8\times3424.8}\right)t}+1} = 0.049\,\frac{e^{337.1\,t}-1}{e^{337.1\,t}+1}$$

The figure below shows *u* as a function of time calculated by the above equation. It is clear from the figure that the particle reaches a terminal velocity (constant velocity) in a very short time 0.01 sec.



The distant travelled by particle as function of time is given by equation (E4.2.7) of the textbook when the fluid density is very small compared to the particle density $(1-\rho_f/\rho_s)g \approx g$:

$$x = \frac{1}{2c}\left[2\ln\left(\frac{e^{(2\sqrt{gc})t}+1}{2}\right)-\left(2\sqrt{gc}\right)t\right]$$

Since the oil density is quite high $(1-\rho_f/\rho_s)g \neq g$ the above equation can be modified as follows:

$$x = \frac{1}{2c}\left[2\ln\left(\frac{e^{\left(2\sqrt{(1-\rho_f/\rho_s)gc}\right)t}+1}{2}\right)-\left(2\sqrt{(1-\rho_f/\rho_s)gc}\right)t\right]$$

$$H = \frac{1}{2(3424.8)}\left[2\ln\left(\frac{e^{337.1}+1}{2}\right)-337.1\right] = 0.05\,\mathrm{m} = 5\ \mathrm{cm}$$

Note: The calculation of the above formula is not straight forward because $e^{337.1} = \infty$! See details below for simplification.

$$H = \frac{1}{2(3424.8)}\left[2\ln\left(\frac{e^{337.1}+1}{2}\right) - 337.1\right] = \frac{1}{2(3424.8)}\left[2\left\{\ln\left(e^{337.1}+1\right) - \ln(2)\right\} - 337.1\right]$$

$$\approx \frac{1}{2(3424.8)}\left[2\left\{\ln\left(e^{337.1}\right) - \ln(2)\right\} - 337.1\right] \approx \frac{1}{2(3424.8)}\left[2\left\{\ln\left(e^{337.1}\right)\right\} - 337.1\right]$$

$$\approx \frac{1}{2(3424.8)}\left[2\{337.1\} - 337.1\right] \approx \frac{1}{2(3424.8)}\left[337.1\right] = 0.05 \text{ m}$$

Conclusion: by assuming steady state for laminar flow there is small error in calculating the height of the tower. The problem reaches steady state in a very short time for laminar flow and hence it is safe to assume steady state for laminar flow.