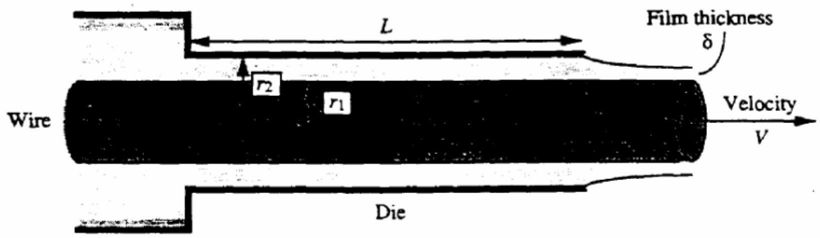
6.2-1

Wire Coating



Z- Momentum Balance (cylindrical coordinates)

$$\rho \frac{Dv_z}{Dt} = \rho\left(\frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z}\right)$$

$$= -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z$$

Assumptions

1. Steady state
2. $v_z \neq 0$; $v_r = v_\theta = 0$; $v_z = v_z(r)$
3. $\rho g_z = 0$ (or is unimportant) 4. p constant

Simplified Momentum Balance

$$\frac{d}{dr}\left(r\frac{dv_z}{dr}\right) = 0$$

Integrate Twice

$$r\frac{dv_z}{dr} = C_1, \qquad v_z = C_1 \ln r + C_2$$

6.2-2

Boundary Conditions

$$\left.\begin{aligned} r = r_1 &: \quad v_z = V \\ r = r_2 &: \quad v_z = 0 \end{aligned}\right\} \text{Hence} \qquad \begin{aligned} c_1 &= -\frac{V}{\ln(r_2/r_1)} \\ c_2 &= \frac{V \ln r_2}{\ln(r_2/r_1)} \end{aligned}$$

Velocity Profile

$$v_z = V\,\frac{\ln(r_2/r)}{\ln(r_2/r_1)} = V\,\frac{\ln(r/r_2)}{\ln(r_1/r_2)}$$

Volumetric Flow Rate

$$Q = \int_{r_1}^{r_2} 2\pi r\,dr\,\frac{V}{\ln(r_1/r_2)}\left(\ln r - \ln r_2\right)$$

Note: $\int r \ln r\,dr = \frac{r^2}{2}\ln r - \frac{r^2}{4}$

$$= \frac{2\pi V}{\ln(r_1/r_2)}\left[\frac{r^2}{2}\ln r - \frac{r^2}{4} - \frac{r^2}{2}\ln r_2\right]_{r_1}^{r_2}$$

$$Q = \frac{2\pi V}{\ln(r_2/r_1)}\left[\frac{1}{4}\left(r_2^2 - r_1^2\right) - \frac{r_1^2}{2}\ln\frac{r_2}{r_1}\right]$$

Coating Thickness

$$Q = V\left[\pi(r_1+\delta)^2 - \pi r_1^2\right] = V\pi\left(\delta^2 + 2r_1\delta\right)$$

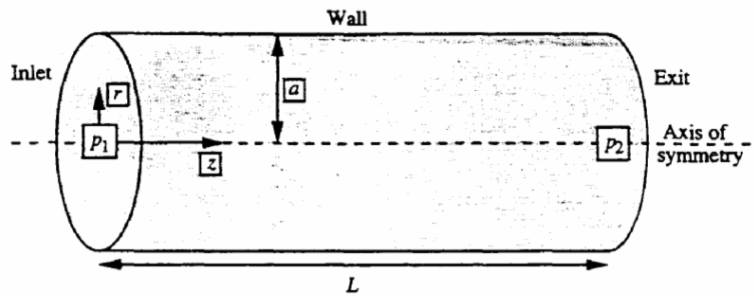
Force to Pull Rod

$$F = 2\pi r_1 L\left(-\tau_{rz}\right)\big|_{r=r_1} \qquad \tau_{rz} = \mu\left(\frac{\partial v_z}{\partial r} + \underbrace{\frac{\partial v_r}{\partial z}}_{\text{zero}}\right) = -\frac{1}{r}\,\frac{\mu V}{\ln(r_2/r_1)}$$

$$F = \frac{2\pi\mu L V}{\ln(r_2/r_1)}$$

6.7-1

Details of Pipe Flow



Assumptions

1. $v_z \neq 0; \quad v_r = v_\theta = 0$
2. Steady, constant ρ and μ.
3. Symmetry about z axis: $\frac{\partial v_z}{\partial \theta} = 0$

Continuity Equation

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial(\rho r v_r)}{\partial r} + \frac{1}{r}\frac{\partial(\rho v_\theta)}{\partial \theta} + \frac{\partial(\rho v_z)}{\partial z} = 0$$

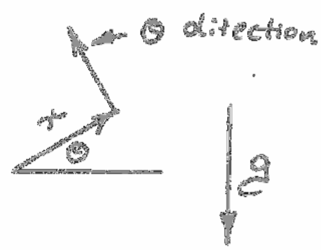
Simplifies to $\frac{\partial v_z}{\partial z} = 0$ Hence $v_z = v_z(r)$

Three Momentum Balances (after simplification)

$$\frac{\partial p}{\partial r} = \rho g_r = \rho(-g \sin\theta)$$

$$\frac{1}{r}\frac{\partial p}{\partial \theta} = \rho g_\theta = \rho(-g \cos\theta)$$

$$\frac{\partial p}{\partial z} = \mu \frac{1}{r}\frac{d}{dr}\left(r \frac{dv_z}{dr}\right)$$



$\underline{6.7-2}$

Integrate r and θ Momentum Balances

$$\left.\begin{aligned} p &= -\rho g r \sin\theta + f_1(z,\theta) \\ p &= -\rho g r \sin\theta + f_2(z,r) \end{aligned}\right\} \text{Hence } f_1(z,\theta) = f_2(z,r) = f(z) \text{ only}$$

Pressure Variation in z direction

Assume (to be verified) a linear decline from p_1 to p_2

Hence

$$p = p_1 - \frac{z}{L}(p_1 - p_2) - \rho g r \sin\theta$$

Integrate z - Momentum Balance Once

$$r\frac{dv_z}{dr} = -\frac{1}{2\mu}\left(-\frac{\partial p}{\partial z}\right) r^2 + c_1$$

Integrate Again

$$v_z = -\frac{1}{4\mu}\left(-\frac{\partial p}{\partial z}\right) r^2 + c_1 \ln r + c_2$$

Determination of Constants of Integration

c_1 must be zero, to avoid physically impossible infinite velocity at center from $c_1 \ln r$ term

or Use Symmetry that $\frac{dv_z}{dr} = 0$ at $r = 0$

will give the same result

At the wall, $r = a$:

$$0 = -\frac{1}{4\mu}\left(-\frac{\partial p}{\partial z}\right) a^2 + c_2 \quad \text{or} \quad c_2 = \frac{1}{4\mu}\left(-\frac{\partial p}{\partial z}\right) a^2$$

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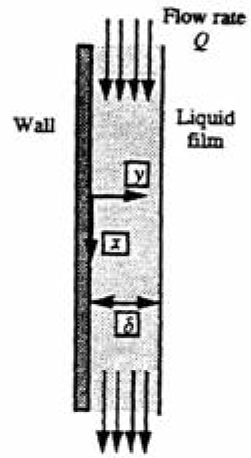
6.14-1
Wetted-Wall Column



Assume

1. Steady, constant ρ, μ.
2. $v_x \neq 0$; $v_y = v_z = 0$
3. $\frac{\partial v_x}{\partial z} = 0$ 4. $g_x = g$

$g_y = 0$

Continuity (ρ constant)

$$\nabla \cdot \bar{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0$$

Hence $\frac{\partial v_x}{\partial x} = 0$ and so $v_x = v_x(y)$

Simplified Momentum Balances

$$x: \quad \frac{\partial p}{\partial x} = \mu \frac{d^2 v_x}{dy^2} + \rho \underbrace{g}_{g_x}$$

$$y: \quad \frac{\partial p}{\partial y} = 0$$

That is, there is no variation of pressure horizontally across the film. Since the pressure outside the film is everywhere atmospheric ($p = 0$, say), the pressure everywhere inside the film must be $p = 0$ also. Hence $\partial p / \partial x = 0$ and the x-momentum balance is

$$\frac{d^2 v_x}{dy^2} = -\frac{\rho g}{\mu}$$

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