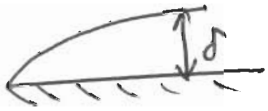
Hydrodynamic Boundary Layer:

$$U_x = a + by + cy^2 + dy^3 \quad -(1)$$



Boundary Conditions

$$U_x = 0, \ @ \ y = 0$$
$$U_x = U_\infty \ @ \ y = \delta$$
$$\frac{\partial U_x}{\partial y} = 0, @ \ y = \delta$$
$$\frac{\partial^2 U_x}{\partial y^2} = 0, \ @ \ y = 0$$

$$\therefore \quad a = 0, \quad b = \frac{3}{2\delta U_\infty}, \quad c = 0, \quad d = -\frac{U_\infty}{2\delta^3}$$

$$\therefore \quad \frac{U_x}{U_\infty} = \frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\frac{y^3}{\delta^3} \quad -(2)$$

von-Karman Equation

$$\frac{d}{dx}\int_0^\delta U_x(U_\infty - U_x)\,dy = \frac{\tau_0}{\rho} \quad -(3)$$

$$\frac{\tau_0}{\rho} = +\frac{\mu}{\rho}\left.\frac{\partial U_x}{\partial y}\right|_{y=0} = +\nu\cdot\frac{3}{2}\frac{U_\infty}{\delta}.$$

$$\frac{d}{dx}\int_0^\delta \left(\frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\frac{y^3}{\delta^3}\right)U_\infty\left[1 - \frac{y}{\delta} + \frac{1}{2}\frac{y^3}{\delta^3}\right]\cancel{U_\infty}\,dy = \nu\frac{3}{2}\frac{\cancel{U_\infty}}{\delta}.$$

$$\Rightarrow \frac{d}{dx}\left[U_\infty^2\,\delta\,\frac{39}{280}\right] = \frac{3}{2}\nu\frac{U_\infty}{\delta}.$$

$$\Rightarrow \quad \delta\frac{d\delta}{dx} = \frac{140}{13}\frac{\nu}{U_\infty}.$$

$$\Rightarrow \quad \int_0^\delta \delta\, d\delta = \frac{140}{130}\frac{\nu}{U_\infty}\int_0^x x$$

$$\Rightarrow \quad \frac{\delta^2}{2} = 10.77\,\frac{\nu}{U_\infty}.x$$

$$\delta^2 = 21.54\,\frac{\nu x}{U_\infty}$$

$$\frac{\delta^2}{x^2} = 21.54\,\frac{\nu}{U_\infty x} \qquad \frac{\delta}{x} = \frac{4.64}{\sqrt{Re_x}}.$$

$$C_f = \frac{\tau_0}{\frac{1}{2}\rho U_\infty^2} = \frac{0.646}{\sqrt{Re_x}}$$

Concentration B.L

$$C_A = a + by + cy^2 + dy^3 \quad \text{— (1)}$$

B.C. $y=0,\ C_A = C_{AS},\ \frac{d^2 C_A}{dy^2} = 0$

$y = \delta,\ C_A = C_{Ab},\ \frac{dC_A}{dy} = 0$

$$\therefore \frac{C_A - C_{AS}}{C_{Ab} - C_{AS}} = \frac{3}{2}\left(\frac{y}{\delta_c}\right) - \frac{1}{2}\left(\frac{y}{\delta_c}\right)^3 \quad \text{— (2)}$$

But $k(C_{AS} - C_{Ab}) = -D_{AB} \left.\frac{dC_A}{dy}\right|_{y=0}$

$$\therefore k(C_{AS} - C_{Ab}) = -D_{AB} * \frac{1}{(C_{Ab} - C_{AS})} \cdot \frac{3}{2}\frac{1}{\delta_c}$$

$$k = \frac{3 D_{AB}}{2\,\delta_c} \quad \text{— (3)}$$

Von-Karman Eqn

$$\frac{d}{dx}\int_0^{\delta_c} (C_A - C_{Ab})\, U_x\, dy = k_c (C_{AS} - C_{Ab}) \quad \text{— (3)}$$

Here. $U_x = u_\infty\left[\frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\frac{y^3}{\delta^3}\right]$

and. $\frac{C_A - C_{AS}}{C_{Ab} - C_{AS}} - 1 = \frac{3}{2}\frac{y}{\delta_c} - \frac{1}{2}\frac{y^3}{\delta^3} - 1$

$$\Rightarrow \frac{C_A - \cancel{C_{AS}} - C_{Ab} + \cancel{C_{AS}}}{C_{Ab} - C_{AS}} = \frac{3}{2}\frac{y}{\delta_c} - \frac{1}{2}\frac{y^3}{\delta^3} - 1$$

$$\therefore (C_A - C_{Ab}) = (C_{Ab} - C_{AS})\left[\frac{3}{2}\frac{y}{\delta_c} - \frac{1}{2}\frac{y^3}{\delta^3} - 1\right]$$

$$\frac{d}{dx}\int_0^{\delta_c} \cancel{(C_{Ab} - C_{AS})}\cdot\left[\frac{3}{2}\frac{y}{\delta_c} - \frac{1}{2}\frac{y^3}{\delta_c^3} - 1\right]\left[\frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\frac{y^3}{\delta^3}\right] u_\infty\, dy = k_c\,\cancel{(C_{AS} - C_{Ab})}$$

$$\Rightarrow \frac{d}{dx}\int_0^{\delta_c}\left[\frac{3}{2}\frac{y}{\delta_c} - \frac{1}{2}\frac{y^3}{\delta_c^3} - 1\right]\left[\frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\frac{y^3}{\delta^3}\right] dy = -\frac{k_c}{u_\infty} = -\frac{3}{2}\frac{D_{AB}}{\delta_c u_\infty}$$

$$\frac{d}{dx}\int_0^{\delta_c}\left(\frac{3}{2}\frac{y}{\delta_c} - \frac{1}{2}\frac{y^3}{\delta_c^3} - 1\right)\left(\frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\frac{y^3}{\delta^3}\right) dy = -\frac{3}{2}\frac{D_{AB}}{\nu}\cdot\frac{\nu}{\delta_c u_\infty}$$

(3)

$$\frac{d}{dx}\int_0^{\delta_c}\left(\frac{3}{2}\frac{y}{\delta_c}-\frac{1}{2}\frac{y^3}{\delta_c^3}-1\right)\left(\frac{3}{2}\frac{y}{\delta}-\frac{1}{2}\frac{y^3}{\delta^3}\right)dy = -\frac{k_c}{u_\infty}$$

$$\Rightarrow \frac{d}{dx}\int_0^{\delta_c}\left[\frac{9}{4}\frac{y^2}{\delta_c\delta}-\frac{3}{4}\frac{y^4}{\delta_c\delta^3}-\frac{3}{4}\frac{y^4}{\delta_c^3\delta}+\frac{1}{4}\frac{y^6}{\delta_c^3\delta^3}-\frac{3}{2}\frac{y}{\delta}+\frac{1}{2}\frac{y^3}{\delta^3}\right]dy = -\frac{3}{2}\frac{D_{AB}}{\delta_c u_\infty}$$

$$\Rightarrow \frac{d}{dx}\left[\frac{9}{4}\cdot\frac{y^3}{3}\cdot\frac{1}{\delta_c\delta}-\frac{3}{4}\cdot\frac{y^5}{5}\cdot\frac{1}{\delta_c\delta^3}-\frac{3}{4}\cdot\frac{y^5}{5}\cdot\frac{1}{\delta_c^3\delta}+\frac{1}{4}\cdot\frac{y^7}{7}\frac{1}{\delta_c^3\delta^3}-\frac{3}{2}\frac{y^2}{2}\cdot\frac{1}{\delta}+\frac{1}{2}\frac{y^4}{4}\cdot\frac{1}{\delta^3}\right]_0^{\delta_c} = -\frac{3}{2}\frac{D_{AB}}{\delta_c u_\infty}$$

$$\Rightarrow \frac{d}{dx}\left[\frac{3}{4}\cdot\frac{\delta_c^2}{\delta}-\frac{3}{20}\cdot\frac{\delta_c^4}{\delta^3}-\frac{3}{20}\cdot\frac{\delta_c^2}{\delta}+\frac{1}{28}\cdot\frac{\delta_c^4}{\delta^3}-\frac{3}{4}\cdot\frac{\delta_c^2}{\delta}+\frac{1}{8}\frac{\delta_c^4}{\delta^3}\right] = -\frac{3}{2}\frac{D_{AB}}{\delta_c u_\infty}$$

$$\Rightarrow \frac{d}{dx}\left[-\frac{3}{20}\frac{\delta_c^2}{\delta}+\frac{3}{280}\frac{\delta_c^4}{\delta^3}\right] = -\frac{3}{2}\frac{D_{AB}}{\delta_c u_\infty}$$

$$\phi = \frac{\delta_c}{\delta} \; : \; \delta_c = \phi\delta$$

$$\Rightarrow \frac{d}{dx}\left[-\frac{3}{20}\cdot\frac{\phi^2\delta^2}{\delta}+\frac{3}{280}\cdot\frac{\phi^4\delta^4}{\delta^3}\right] = \text{''}$$

$$\Rightarrow \frac{d}{dx}\left[-\frac{3}{20}\delta\phi^2+\frac{3}{280}\cdot\delta\phi^4\right] = -\frac{3}{2}\frac{D_{AB}}{\delta_c u_\infty}$$

$$\phi^4 \simeq 0$$

$$\Rightarrow \phi\delta\frac{d}{dx}\left[\delta\phi^2\right] = \frac{10 D_{AB}}{u_\infty}$$

$$\Rightarrow \phi\delta\left[\delta\, 2\phi\frac{d\phi}{dx}+\phi^2\frac{d\delta}{dx}\right] = \frac{10 D_{AB}}{u_\infty}$$

$$\Rightarrow 2\delta^2\phi^2\frac{d\phi}{dx}+\phi^3\delta\frac{d\delta}{dx} = \frac{10 D_{AB}}{u_\infty}$$

$$\delta^2 = 21.54\frac{\nu x}{u_\infty}$$

$$2\delta\frac{d\delta}{dx} = 21.54\cdot\frac{\nu}{u_\infty}$$

$$\delta\frac{d\delta}{dx} = 10.77\frac{\nu}{u_\infty}$$

$$\Rightarrow 2 * 21.54\frac{\nu x}{u_\infty}\phi^2\frac{d\phi}{dx}+\phi^3\, 10.77\frac{\nu}{\cancel{u_\infty}} = \frac{10 D_{AB}}{\cancel{u_\infty}}$$

$$\Rightarrow 4x\phi^2\frac{d\phi}{dx}+\phi^3 = \frac{10}{10.77}\frac{D_{AB}}{\nu}$$

$$\frac{d}{dx}(\phi^3) = 3\phi^2\frac{d\phi}{dx}$$

$$\Rightarrow 4x\frac{1}{3}\frac{d\phi^3}{dx}+\phi^3 = \frac{0.9285}{Sc}$$

$$\Rightarrow \frac{4}{3}x\cdot\frac{dP}{dx}+P = \frac{0.9285}{Sc}$$

$$x=0, \; \phi=0.$$

$$\therefore \phi^3 = \frac{0.9285}{Sc}$$

$\therefore \quad \phi = 0.9756\ Sc^{1/3}$

$\therefore \delta_c = 0.9756\ Sc^{-1/3}\ \delta$

$= 0.9756 * Sc^{-1/3} * 4.64 * x\ Re_x^{-1/2}$

$= 4.5267\ x\ Re_x^{-1/2}\ Sc^{1/3}$

Now

$$k = \frac{3}{2} \frac{D_{AB}}{\delta_c}$$

$$= \frac{3}{2} \frac{D_{AB}}{4.5267\ x\ Re_x^{-1/2}\ Sc^{1/3}}$$

$$\therefore \frac{k\ x}{D_{AB}} = 0.331\ Re_x^{1/2}\ Sc^{1/3}$$

$$Sh_x = 0.331\ Re_x^{1/2}\ Sc^{1/3}.$$