Hw4: 26.20, 26.27 26.30

26.20

26.20

$$N_{A_z} = \frac{c\,D_{AB}}{\delta} \ln\left[\frac{1+y_{A0}}{1+y_{As}}\right]$$

If area = $1 m^2$

$$W_A = N_{A_z}(1m^2) = \frac{c\,D_{AB}}{\delta} \ln\left[\frac{1+y_{A0}}{1+y_{As}}\right]$$

and

$$W_A = \frac{\rho_{Si}}{M_{Si}} dV = \frac{\rho_{Si}}{M_{Si}} (1m^2) \frac{d\delta}{dt}$$

$$\frac{\rho_{Si}}{M_{Si}} \frac{d\delta}{dt} = \frac{c\,D_{AB}}{\delta} \ln\left[\frac{1+y_{A0}}{1+y_{As}}\right]$$

$$\frac{d\delta}{dt} = \frac{M_{Si}}{\rho_{Si}} \frac{c\,D_{AB}}{\delta} \ln\left[\frac{1+y_{A0}}{1+y_{As}}\right]$$

$\delta = 5 cm = 0.05 m$

$y_{A_0} = 0.20$

$y_{A_s} = 0.$

$\rho_{Si} = 2.32\ g/cm^3$

$M_{Si} = 28.08\ g/mole$

$$C = \frac{P}{RT} = \frac{70\ Pa}{\left(8.314\ \frac{Pa \cdot m^3}{mole \cdot K}\right)(900K)} =$$

$$= 9.355 \times 10^{-3}\ \frac{mole}{m^3} = 9.355 \times 10^{-9}\ \frac{mole}{cm^3}$$

$$D_{AB} = \frac{0.001858\ T^{3/2}\left[\frac{1}{M_A} + \frac{1}{M_B}\right]^{1/2}}{P\ \sigma_{AB}^2\ \Omega_D}$$

	MW	ϵ_i/K	σ_i
H_2	2.016	33.3	2.968
SiH_4	32.18	207.6	4.08

$$T^{3/2} = (900K)^{3/2} = 27{,}000$$

$$P = \frac{70\ Pa}{1.013 \times 10^5\ Pa/atm} = 6.91 \times 10^{-4}\ atm$$

$$\left[\frac{1}{M_A} + \frac{1}{M_B}\right]^{1/2} = \left[\frac{1}{2.016} + \frac{1}{32.18}\right]^{1/2} = 0.726$$

$$\sigma_{AB} = \frac{2.968 + 4.08}{2} = 3.524$$

$$\sigma_{AB}^2 = 12.418$$

$$\frac{\epsilon_{AB}}{K} = \sqrt{(33.3)(207.6)} = 83.145$$

$$\frac{KT}{\epsilon_{AB}} = \frac{900}{83.145} = 10.82\ ;\ \Omega_D = 0.736$$

$$D_{AB} = \frac{0.001858\ (27{,}000)(0.7260)}{}$$

$$= 5.77 \times 10^3\ cm^2/s = 0.577\ m^2/s$$

$$\frac{d\delta}{dt} = \frac{29.09\ g/mole}{2.32\ g/cm^3}\ \frac{(9.355 \times 10^{-9}\ \frac{mole}{cm^3})(0.577\frac{m^2}{s})}{0.05\ m} \cdot \times \ln\left[\frac{1.2}{1}\right]$$

$$= 2.46 \times 10^{-7}\ \frac{m}{s} \times \frac{60s}{min}$$

$$= 1.476 \times 10^{-5}\ \frac{m}{min} = \underline{\underline{14.76\ \frac{\mu m}{min}}}$$

b) $N_{A\,surface} = k_s C_{As} = c\,k_s\,y_{As}$

$$y_{As} = \frac{N_{Az}}{c\,k_s}$$

$$N_{Az} = \frac{cD_{AB}}{\delta} \ln\left[\frac{1+y_{A0}}{1+\frac{N_{Az}}{ck_s}}\right]$$

$$N_{Az} = \frac{cD_{AB}}{\delta}\left[\ln(1+y_{A0}) - \ln\left(1+\frac{N_{Az}}{ck_s}\right)\right]$$

but $\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 + \cdots$

if $x = \frac{N_{Az}}{ck_s}$, x^2, x^3... will be very small

$$\therefore \ln\left(1+\frac{N_{Az}}{ck_s}\right) \approx \frac{N_{Az}}{ck_s}$$

$$N_{Az} = \frac{cD_{AB}}{\delta}\ln(1+y_{A0}) - \frac{cD_{AB}}{\delta}\frac{N_{Az}}{ck_s}$$

$$N_{Az}\left(1+\frac{D_{AB}}{\delta k_s}\right) = \frac{cD_{AB}}{\delta}\ln(1+y_{A0})$$

$$N_{Az} = \frac{\frac{cD_{AB}}{\delta}\ln[1+y_{A0}]}{1+\frac{D_{AB}}{\delta k_s}}$$

$$1+\frac{D_{AB}}{\delta k_s} = 1 + \frac{5.77\times 10^{3}\,cm^2/s}{(5\,cm)(1.25\,\frac{cm}{s})} = 924$$

$$N_{Az} = \frac{(9.355\times 10^{3}\,\frac{mole}{m^3})(0.577\,\frac{m^2}{s})\ln(1.2)}{(0.05\,m)(924)}$$

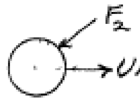
$$= \underline{\underline{2.13\times 10^{-5}\,mole/m^2\cdot s}}$$

a) $N_A = \frac{cD_{AB}}{\delta}\ln(1.2) = (2.13\times 10^{-5})(924)$

$$= \underline{\underline{1.97\times 10^{-2}\,mole/m^2\cdot s}}$$

26.27

<u>26.27</u>



$$U + 3F_2(g) \rightarrow UF_6(g)$$

$$\nabla \cdot N_A + \cancel{\frac{\partial c_A}{\partial t}}^{\,0} = \cancel{R_A}^{\,0}$$

$$\frac{d}{dr}(r^2 N_{A_r}) = 0 \;,\; r^2 N_{A_r} \text{ is constant}$$

$$N_{UF_6} = -cD_{AB}\frac{dy_{UF_6}}{dr} + y_{UF_6}(N_{UF_6} + N_{F_2})$$

$$N_{F_2} = -3\,N_{UF_6}$$

$$N_{UF_6} = \frac{-c\,D_{AB}}{1+2y_{UF_6}}\frac{dy_{UF_6}}{dr}$$

$$\underbrace{4\pi r^2 N_{UF_6}}_{W_{UF_6}}\int_{R_s}^{\infty}\frac{dr}{r^2} = -\frac{4\pi c}{2}D_{AB}\int_{1.0}^{0}\frac{2dy_{UF_6}}{1+2y_{UF_6}}$$

$$W_{UF_6}\left(\frac{1}{R_s}\right) = \frac{4\pi c\,D_{AB}}{2}\ln\left[\frac{3.0}{1.0}\right]$$

$$W_{UF_6} = 2\pi c\,D_{AB}\ln(3.0)$$

$$c = \frac{P}{RT} = \frac{1.013\times 10^5\,\text{Pa}}{\left(8.314\,\frac{\text{Pa}\cdot\text{m}^3}{\text{mole}\cdot\text{K}}\right)(1000\,\text{K})}$$

$$= 12.18\,\frac{\text{mole}}{\text{m}^3} = 1.218\times 10^{-5}\,\frac{\text{mole}}{\text{cm}^3}$$

$$W_{UF_6} = 2\pi\left(1.218\times 10^{-5}\,\frac{\text{mole}}{\text{cm}^3}\right)\left(0.233\,\frac{\text{cm}^2}{\text{s}}\right)\times$$

$$\times(0.2\,\text{cm})\ln(3.0$$

$$= \underline{\underline{4.59\times 10^{-6}\,\text{mole/s}}}$$

26.30

26.30

For sphere:

$$\frac{1}{r^2}\frac{d}{dr}(r^2 N_{A_r}) = -k_1 c_A$$

$$N_{A_r} = -D_{AB}\frac{dc_A}{dr}$$

$$-\frac{D_{AB}}{r^2}\frac{d}{dr}\left(r^2\frac{dc_A}{dr}\right) = -k_1 c_A$$

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dc_A}{dr}\right) = \frac{k_1}{D_{AB}}c_A \qquad (a)$$

let $y = c_A r$.

$$\frac{dy}{dr} = r\frac{dc_A}{dr} + c_A$$

and $r^2\frac{dc_A}{dr} = r\frac{dy}{dr} - y$

substituting into (a) yields

$$\frac{d^2y}{dr^2} - \frac{k_1}{D_{AB}}y = 0.$$

$$\therefore y = C_1 \cosh\left(r\sqrt{k_1/D_{AB}}\right) + C_2 \sinh\left(r\sqrt{k_1/D_{AB}}\right) = c_A r$$

@ $r = 0$, $y = 0$ and $C_1 = 0$

@ $r = R$, $c_A = c_{Ao}$

$$c_{Ao}R = C_2 \sinh\left(R\sqrt{k_1/D_{AB}}\right)$$

$$C_2 = \frac{c_{Ao}R}{\sinh\left(R\sqrt{k_1/D_{AB}}\right)}$$

$$c_A = c_{Ao}\frac{R}{r}\frac{\sinh\left(r\sqrt{k_1/D_{AB}}\right)}{\sinh\left(R\sqrt{k_1/D_{AB}}\right)}$$

$$c_A = \frac{c_{Ao}R}{\sinh\left(R\sqrt{k_1/D_{AB}}\right)}\cdot\frac{1}{r}\sinh\left(r\sqrt{k_1/D_{AB}}\right)$$

$$\frac{dc_A}{dr} = \frac{c_{Ao}R}{\sinh\left(R\sqrt{k_1/D_{AB}}\right)}\left[-\frac{1}{r^2}\sinh\left(r\sqrt{k_1/D_{AB}}\right) + \frac{\sqrt{k_1/D_{AB}}}{r}\cosh\left(r\sqrt{k_1/D_{AB}}\right)\right]$$

$$\left.\frac{dc_A}{dr}\right|_R = \frac{c_{Ao}R}{\sinh\left(R\sqrt{k_1/D_{AB}}\right)}\left[-\frac{1}{R^2}\sinh\left(R\sqrt{k_1/D_{AB}}\right) + \frac{\sqrt{k_1/D_{AB}}}{R}\cosh\left(R\sqrt{k_1/D_{AB}}\right)\right]$$

$$= -\frac{c_{Ao}}{R} + c_{Ao}\sqrt{k_1/D_{AB}}\coth\left(R\sqrt{k_1/D_{AB}}\right)$$

$$-D_{AB}\frac{dc_A}{dr}\bigg|_R = \frac{D_{AB}c_{Ao}}{R}\left[1 - R\sqrt{\frac{k_1}{D_{AB}}}\,\text{Coth}\left(R\sqrt{\frac{k_1}{D_{AB}}}\right)\right]$$

$$N_{Ar}\Big|_R = -D_{AB}\frac{dc_A}{dr}\bigg|_R$$

$$= \frac{D_{AB}c_{Ao}}{R}\left[1 - R\sqrt{\frac{k_1}{D_{AB}}}\,\text{Coth}\left(R\sqrt{\frac{k_1}{D_{AB}}}\right)\right]$$

From example 26.4

$D_{AB} = 2\times 10^{-10}\ m^2/s$

$R = 0.002\ m$

$c_{Ao} = 0.02\ mol/m^3$

$k_1 = 1.9\times 10^{-2}/s$

$$R\sqrt{\frac{k_1}{D_{AB}}} = 0.002\,m\sqrt{\frac{1.9\times 10^{-2}/s}{2\times 10^{-10}m^2/s}} = 19.49$$

$$\frac{D_{AB}c_{Ao}}{R} = \frac{(2\times 10^{-10}m^2/s)(0.02\frac{mol}{m^3})}{0.002\,m} = 2\times 10^{-9}\frac{mol}{m^2\cdot s}$$

$$N_{Ar}\Big|_R = \left(2\times 10^{-9}\frac{mol}{m^2\cdot s}\right)\left[1 - 19.49\,\text{Coth}(19.49)\right]$$

$$= \underline{1.02\times 10^{-12}\ mol/m^2 s}$$

major differences, the areas.