Chapter 4

Unsteady State Conduction

4-1 Introduction
Transient Conduction

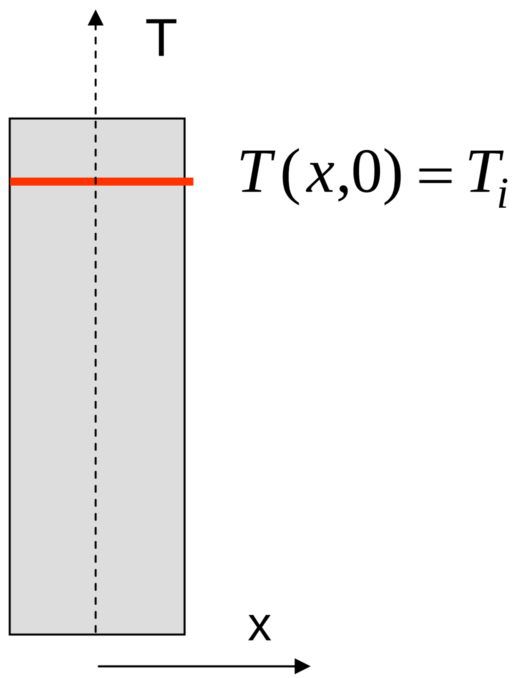
- Many heat transfer problems are time dependent
- Changes in operating conditions in a system cause temperature variation with time, as well as location within a solid, until a new steady state (thermal equilibrium) is obtained.

- In this chapter we will develop procedures for determining the time dependence of the temperature distribution
- Real problems may include finite and semi-infinite solids, or complex geometries, as well as two and three dimensional conduction
- Solution techniques involve the lumped capacitance method, exact and approximate solutions, and finite difference methods.

➢ We will focus on the Lumped Capacitance Method, which can be used for solids within which *temperature gradients are negligible.*

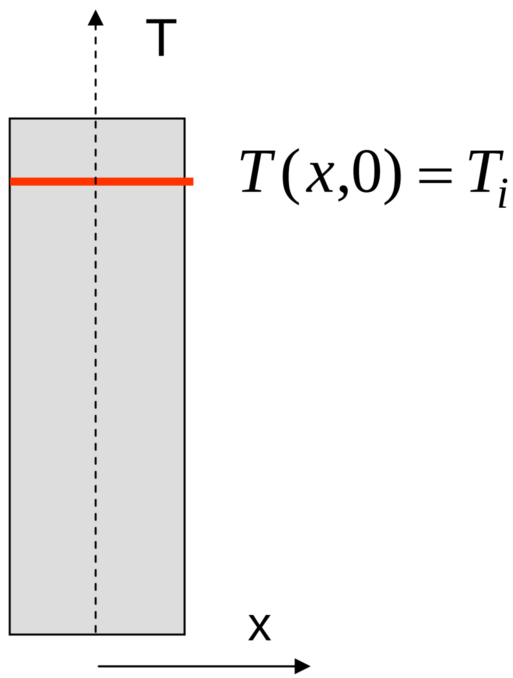
4-2 Lumped Capacitance Method

- Consider a hot metal that is initially at a uniform temperature, T_i , and at t=0 is quenched by immersion in a cool liquid, of lower temperature T_∞
- The temperature of the solid will decrease for time t>0, due to convection heat transfer at the solid-liquid interface, until it reaches T_∞



Lumped Capacitance Method

- If the thermal conductivity of the solid is very high, resistance to conduction within the solid will be small compared to resistance to heat transfer between solid and surroundings.
- Temperature gradients within the solid will be negligible, i.e.. ***the temperature of the solid is spatially uniform at any instant.***



Lumped Capacitance Method

Starting from an overall energy balance on the solid: $-\dot{E}_{out} = \dot{E}_{st}$

$$-hA_s(T - T_\infty) = \rho Vc \frac{dT}{dt}$$

The time required for the solid to reach a temperature T is:

$$t = \frac{\rho Vc}{hA_s} \ln \frac{\theta_i}{\theta}$$

where $\theta = T - T_\infty$

$\theta_i = T_i - T_\infty$

The temperature of the solid at a specified time t is:

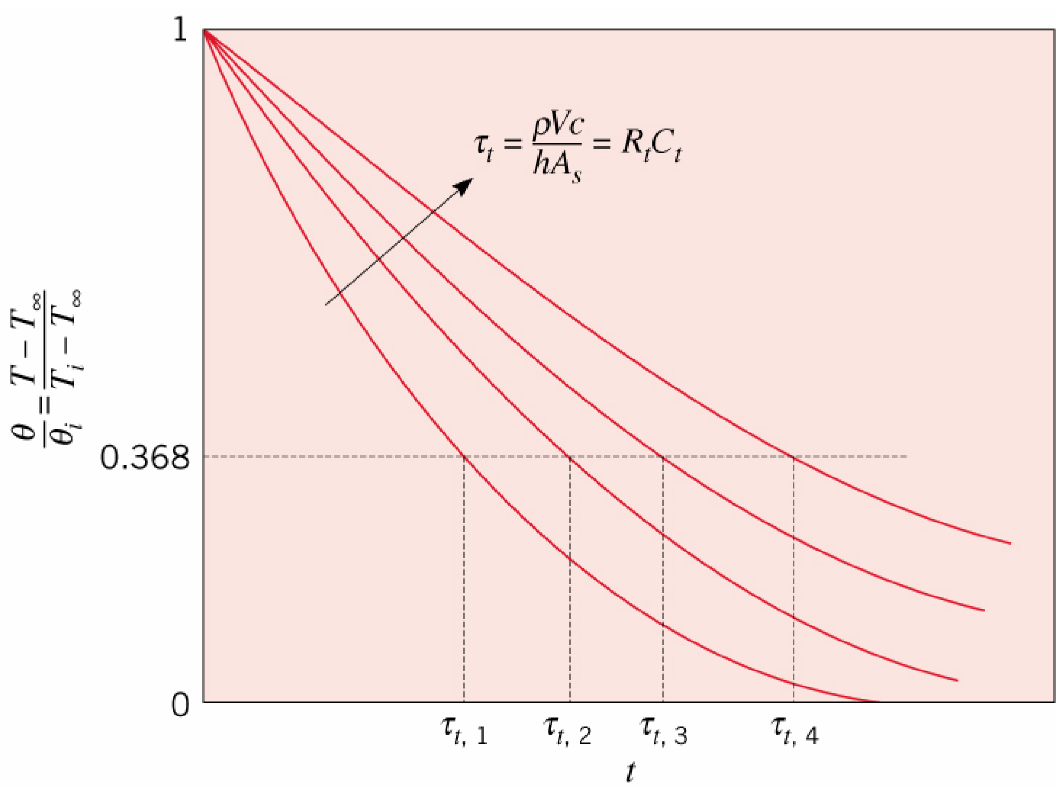
$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = \exp\left[-\left(\frac{hA_s}{\rho Vc}\right)t\right] \qquad (4.5)$$

The total energy transfer, Q, occurring up to some time t is:

$$Q = \int_0^t q\, dt = hA_S \int_0^t \theta\, dt = (\rho Vc)\theta_i \left[1 - \exp(-t/\tau_t)\right]$$

Transient Temperature Response

Based on eq. (4.5), the temperature difference between solid and fluid decays exponentially.



- Let's define a thermal time constant

$$\tau_t = \left(\frac{1}{hA_s}\right)(\rho V c) = R_t C_t$$

R_t is the resistance to convection heat transfer,

C_t is the lumped thermal capacitance of the solid

- Increase in R_t or C_t causes solid to respond more slowly and more time will be required to reach thermal equilibrium.

Validity of Lumped Capacitance Method

- Need a suitable criterion to determine validity of method. Must relate relative magnitudes of temperature drop in the solid to the temperature difference between surface and fluid.

$$\frac{\Delta T_{solid\,(\text{due to conduction})}}{\Delta T_{solid/liquid\,(\text{due to convection})}} = \frac{(L/kA)}{(1/hA)} = \frac{R_{cond}}{R_{conv}} = \frac{hL}{k} \equiv Bi$$

? What should be the relative magnitude of ΔT solid versus ΔT solid/liquid for the lumped capacitance method to be valid?

Example 4-1

- A steel ball (c=0.46 kj/kg. C, k= 35 W/m.C) 5 cm in diameter and initially at a uniform temp. of 450 C is suddenly placed in a controlled environment in which the temperature is maintained at 100 C. The convection heat transfer coefficient is 10 W/m^2 . C.
- Calculate the time required for the ball to attain a temperature of 150 C.

Example 4-1

We anticipate that the lumped-capacity method will apply because of the low value of *h* and high value of *k*. We can check by using Equation (4-6):

$$\frac{h(V/A)}{k} = \frac{(10)[(4/3)\pi(0.025)^3]}{4\pi(0.025)^2(35)} = 0.0023 < 0.1$$

so we may use Equation (4-5). We have

$$T = 150^o C \qquad p = 7800 kg/m^3 \qquad [486 lb_m / ft^3]$$

$$T_\infty = 100^o C \qquad h = 10 W/m^2.^o C \qquad [1.76 Btu/h.ft^2.oF]$$

$$T_0 = 450^o C \qquad c = 460 J/kg.^o C \qquad [0.11 Btu/lb_m.^o F]$$

$$\frac{hA}{pcV} = \frac{(10)4\pi(0.025)2}{(7800)(460)(4\pi/3)(0.025)^3} = 3.44\times10^{-4} s^{-1}$$

$$\frac{T - T_\infty}{T_o - T_\infty} = e^{-[hA/pcV]\tau}$$

$$\frac{150-100}{450-100} = e^{-3.344\times10-4\tau}$$

$$\tau = 5819 s = 1.62 h$$

Example 4-2

- **Semi-Infinite Solid with Sudden Change in Surface Conditions**

- A large block of steel is initially at a uniform temperature of 35oC. The surface is exposed to a heat flux (a) by suddenly raising the surface temperature to 250oC and (b) through a constant surface heat flux of W/m2.
- Calculate the temperature at a depth of 2.5 cm after a time of 0.5 min for both these cases.

Example 4-2

We can make use of the solutions for the semi-infinite solid given as Equations (4-8) and (4-13a). For case a.

$$\frac{\times}{2\sqrt{\varepsilon\tau}} = \frac{0.025}{(2)\left(1.4\times10^{-5}\right)(30)^{1/2}} = 0.61$$

The error function id determined from Appendix A as

$$erf\,\frac{\times}{2\sqrt{\alpha\tau}} = erf\,0.61 = 0.61164$$

We have $T_i = 35^oC$ and $T_o = 250^oC$, so the temperature at x=2.5cm is determined from Equation (4-8)as

$$T(\times,\tau) = T_o + (T_i - T_o)erf\,\frac{\times}{2\sqrt{\alpha\tau}}$$

$$= 250 + (35-250)(0.61164) = 118.5^oC$$

Example 4-2

For the constant-heat-flux case b, we make use of Equation (4-13a). Since q_o / A is given as $3.2 \times 10^5 W / m^2$, we can insert the numerical Values to give

$$T(\times, \tau) = 35 + \frac{(2)\left(3.2 \times 10^5\right)\left[\left(1.4 \times 10^{-5}\right)(30) l \pi\right]^{1/2}}{45} e^{-(0.61)}$$

$$- \frac{(0.025)\left(3.2 \times 10^5\right)}{45}(1 - 0.61164)$$

$$= 79.3^{o}\mathrm{C} \quad \times = 2.5 cm, \tau = 30 s$$

For the constant-heat-flux case the surface temperature after 30 s would be evaluated with x=0 in Equation (4-13a). Thus,

$$T(\times = 0) = 35 + \frac{(2)\left(3.2 \times 10^5\right)\left[\left(1.4 \times 10^{-5}\right)(30) / \pi\right]^{1/2}}{45} = 199.4^o C$$

Biot and Fourier Numbers

- The lumped capacitance method is valid when

$$Bi = \frac{hL_c}{k} < 0.1$$

where the characteristic length: L_c=V/A_s=Volume of solid/surface area

We can also define a “dimensionless time”, the Fourier number:

$$Fo = \frac{\alpha t}{L_c^2}$$

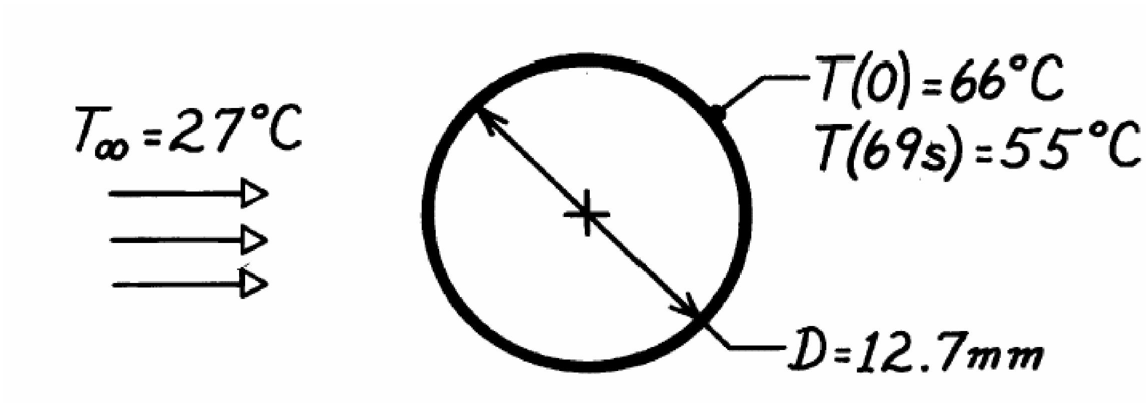
Eq. (5.2) becomes:

$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = \exp[-Bi \cdot Fo] \qquad (5.4)$$

Example (Problem 5.6 Textbook)

The heat transfer coefficient for air flowing over a sphere is to be determined by observing the temperature-time history of a sphere fabricated from pure copper. The sphere, which is 12.7 mm in diameter, is at 66°C before it is inserted into an air stream having a temperature of 27°C. A thermocouple on the outer surface of the sphere indicates 55°C, 69 s after the sphere is inserted in the air stream.

- Calculate the heat transfer coefficient, assuming that the sphere behaves as a spacewise isothermal object. Is your assumption reasonable?



Other transient problems

- When the lumped capacitance analysis is not valid, we must solve the partial differential equations analytically or numerically
- Exact and approximate solutions may be used
- Tabulated values of coefficients used in the solutions of these equations are available
- Transient temperature distributions for commonly encountered problems involving semi-infinite solids can be found in the literature