Chapter 3

Multiple-Dimensions Steady-State Conduction

Introduction

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}=\rho c_p\frac{\partial T}{\partial t}$$

Heat Equation

Net conduction of heat into the CV

rate of energy generation per unit volume

time rate of change of thermal energy per unit volume

❖At any point in the medium the rate of energy transfer by conduction into a unit volume plus the volumetric rate of thermal energy generation must equal the rate of change of thermal energy stored within the volume

Introduction

- For Steady state with no heat generation the 2D equation is:
  - Mathematical analysis of 2D heat conduction (3-2 read only)
  - Graphical analysis (3-3 read only)
  - Numerical method analysis (3-5 read only)

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

3-4 The conduction shape factor

- In 2D system, we may define shape factor S such as
-
- $q = k S \Delta T$

- S have been worked out for several shapes (Table 3-1)

- For 3D wall (as in a furnace) (Figure 3-4)

$$S_{wall} = \frac{A}{L}$$

$$S_{edge} = 0.54\ D$$

$$S_{corner} = 0.15\ L$$

A = area of wall

L = wall thickness

D = length edge

Example 3-1 (Buried pipe)

- A horizontal pipe 15 cm in diameter and 4m long is buried in the earth at a depth of 20 cm. The pipe-wall temp. is 75 C and the earth surface temp. is 5 C. Assuming that the thermal conductivity of the earth is 0.8 W/m. C, calculate the heat lost by the pipe.

- Table 3-1, since D<3r

$$S = \frac{2\pi L}{\cosh^{-1}(D/r)} = \frac{2\pi(4)}{\cosh^{-1}(20/7.5)} = 15.35m$$

$\therefore$

The heat flow is

$$q = kS\Delta T = (0.8)(15.35)(75-5) = 859.6W$$

Example 3-2 (Cubical furnace)

- A small cubical furnace 50 by 50 cm on the inside is constructed of fireclay brick (k= 1.04 W/m. C) with a wall thickness of 10 cm. The inside of the furnace is maintained at 500 C, and the outside is maintained at 50 C. Calculate the heat lost through the walls

$$S_{wall} = \frac{A}{L} = \frac{(0.5)(0.5)}{0.1} = 2.5m$$

$$S_{edge} = 0.54D = (0.54)(0.5) = 0.27m$$

$$S_{corner} = 0.15L = (0.15)(0.1) = 0.015m$$

$$\mathrm{A} = \text{area of wall}$$

$$\mathrm{L} = \text{wall thickness}$$

$$\mathrm{D} = \text{length edge}$$

$$\mathrm{S} = (6 \text{ wall sections})(2.5) + (12 \text{ edges})(0.27) + (8 \text{ corners})(0.015) = 18.36 \text{ m}$$

$$\mathrm{q} = \mathrm{kS}\Delta\mathrm{T} = (1.04)(18.36)(500 - 50) = 8.59 \text{ kw}$$