Similarly for Residual property

$$d\left(\frac{nG^R}{RT}\right) = \frac{nV^R}{RT}dp - \frac{nH^R}{RT^2}dT + \sum \frac{\bar{G}_i^R}{RT} dn_i$$

but $\bar{G}_i^R = RT \ln \hat{\phi}_i \leftarrow$ sol^n fug. coeff.

$$\therefore \boxed{d\left(\frac{nG^R}{RT}\right) = \frac{nV^R}{RT}dp - \frac{nH^R}{RT^2}dT + \sum \ln \hat{\phi}_i \, dn_i}$$

$\therefore$

$$\frac{V^R}{RT} = \left[\frac{\partial(G^R/RT)}{\partial p}\right]_{T,x_i}$$

$$\frac{H^R}{RT} = -T\left[\frac{\partial(G^R/RT)}{\partial T}\right]_{P,x}$$

and $$\ln \hat{\phi}_i = \left[\frac{\partial(nG^R/RT)}{\partial n_i}\right]_{P,T,n_j}$$

and $$\left(\frac{\partial \ln \hat{\phi}_i}{\partial p}\right)_{T,x} = \frac{\bar{V}_i^R}{RT}$$

$$\left(\frac{\partial \ln \hat{\phi}_i}{\partial T}\right)_{P,x} = -\frac{\bar{H}_i^R}{RT^2}$$

but according to eq. $dM = \left(\frac{\partial M}{\partial p}\right)_{T,x} dp + \left(\frac{\partial M}{\partial T}\right)_{P,x} dT + \sum \bar{M}_i dx_i$

where $M = \sum x_i \bar{M}_i$

so $$\frac{G^R}{RT} = \sum x_i \ln \hat{\phi}_i$$

and according to Gibbs/Duhem equation $\sum x_i d \ln \hat{\phi}_i = 0$