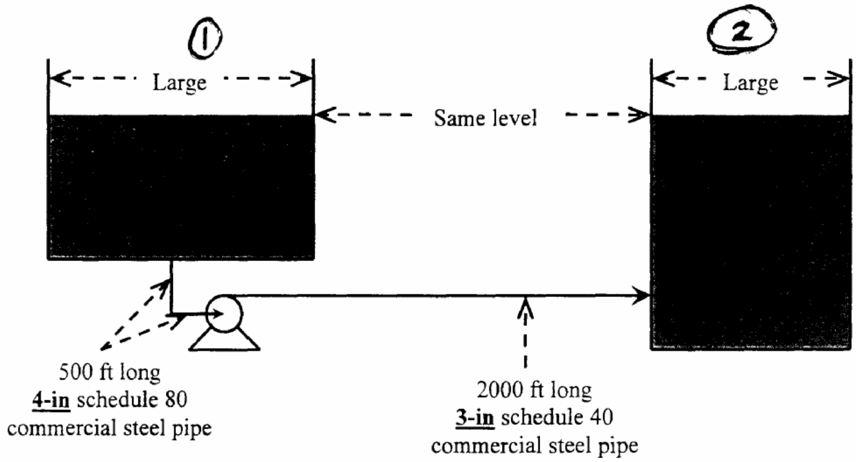
2nd Major Examination
Wednesday, December 15, 2004

Q1. (25 points)

Water ($\rho = 62.4\ lb_m/ft^3$, $\mu = 0.000672\ lb_m/ft\,s$) is pumped from one reservoir to another at a rate 200 gpm$ using a 70% efficient pump. The levels in the reservoirs are the same and both are open to atmosphere, see the figure below. Calculate the power requirement of the pump. Do not neglect the frictional losses in the pipes before and after the pump. $(gpm = 1 US gallon per minute)



MEB 1 & 2

$$\cancel{\frac{\Delta P}{\rho}}^{0} + \cancel{\Delta \frac{U^2}{2}}^{0} + \cancel{g \Delta z}^{0} + w + f = 0$$

$$-w = +f$$

2 sections, after and before the pump.

Section 1: from Table 3.3 $D = 3.826$ in

Comercial steel pipe Table 3.2

$\epsilon = 0.00015$ ft

$$\epsilon/D = \frac{.00015}{3.826/12} = 0.00047$$

$$U = \frac{Q}{\pi/4\, D^2} = \frac{200\ gal}{min} * \frac{4}{\pi} * \left(\frac{12\ in/ft}{3.826\ in}\right)^2 * \frac{ft^3}{7.48\ gal} * \frac{min}{60 s}$$

$U = 5.5816 \ ft/s$

$$Re = \frac{\rho U D}{\mu} = \frac{62.4 \frac{lbm}{ft^3} * 5.5816 \frac{ft}{s} * \frac{3.826 \ in}{12 \ in/ft}}{.000672 \ lbm/ft \cdot s}$$

$$Re = 165{,}249. = 1.7 * 10^5$$

Using EQ. 3.41 or Fig. 3.10

$$f_F \cong 0.0048$$

Using EQ. 3.37 $\quad f = 2 f_F U^2 \frac{L}{D}$

$$f = 2 * 0.0048 * (5.5816)^2 \left(\frac{ft}{s}\right)^2 * \frac{500 \ ft}{\frac{3.826 \ in}{12 \ in/ft}}$$

$$\simeq 469 \ ft^2/s^2$$

Section 2

$D = 3.068 \qquad \epsilon = 0.00015$

$\epsilon/D = 0.0005867$

$U = 8.680 \qquad Re = 206077 \simeq 2. * 10^5$

$$f_F \cong 4.81 * 10^{-3} \Rightarrow f = 5670$$

$\therefore \ W = 5670 + 469 = 6140 \ ft^2/s^2$

$$Power = \frac{\dot{m} w}{\eta} = \frac{Q \rho w}{\eta}$$

$$P = \frac{200 \frac{gal}{min} * \frac{ft^3}{7.4805\ gal} * \frac{min}{60 s} * 62.4 \frac{lbm}{ft^3} * 6140 \frac{ft^2}{s^2}}{0.7 * 32.2 \frac{lbm \cdot ft}{lb_f \cdot s^2}}$$

$$P = 7574 \frac{lb_f \cdot ft}{s} * \frac{1.341 * 10^{-3}\ hp}{0.7376 \frac{ft \cdot lb_f}{s}}$$

$$P = 13.8\ hp$$

Q2. (25 points)

Calculate the drag coefficient for a dust particle of diameter 1 micron[$] and SG = 2 settling through air of density 1.2 kg/m^3 and viscosity $1.8*10^{-5}$ Pa/s.

[$](1 micron = $1*10^{-6}$ meter)

$$C_D = \frac{4}{3}\,\frac{gD}{u_t^2}\,\frac{\rho_s-\rho_f}{\rho_f}$$

$$= \frac{4}{3}\,\frac{(9.8)\,(1*10^{-6})}{u_t^2}\left(\frac{2000-1.2}{1.2}\right)$$

$$= \frac{0.02176}{u_t^2} \quad \text{----} \quad \text{(5 points)}$$

<u>Find u_t</u> assume laminar flow $\Rightarrow C_D = \frac{24}{Re}$
(Re < 1)

$$\Rightarrow \frac{24}{\frac{\rho_f\, u_t\, D}{\mu}} = \frac{0.02176}{u_t^2} \quad \text{----} \quad \text{(5 point}$$

$$\frac{24}{\frac{(1.2)\,(1*10^{-6})}{1.8*10^{-5}}} = \frac{0.02176}{u_t}$$

$$\Rightarrow u_t = 6.0458*10^{-5}\left(\frac{m}{s}\right) \quad \text{---} \quad \text{(3 po}$$

<u>check Re</u> (5 points)

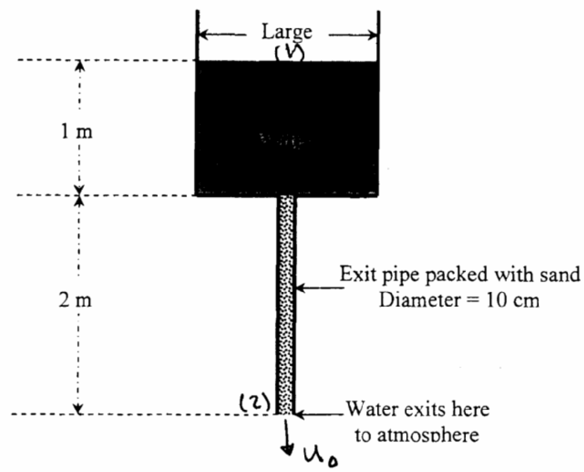
$$Re = \frac{(1.2)\,(6.0458*10^{-3})\,(1*10^{-6})}{1.8*10^{-5}}$$

$$= 4.0305*10^{-6} < 1 \Rightarrow$$ laminar assumpt was corre

$$\Rightarrow C_D = \frac{0.02176}{(6.0458*10^{-5})^2} = 5953211.9 \quad \text{(}$$

Q3. (25 points)

Water drains from a tank by gravity through a pipe packed with sand. The system is sketched in the figure below. The sand particles are uniform with an average diameter of 0.2 mm and a void fraction of 0.3. Calculate the rate of water drainage in ml/min.



$$Q = \frac{\pi}{4} D^2 u_o$$

M.E.B. (1) and (2):

$$\underbrace{\cancel{\frac{\Delta P}{\rho}}}_{=0\ (P_1 = P_2 = P_{atm})} + \underbrace{\cancel{\frac{\Delta u^2}{2}}}_{= \frac{u_o^2}{2}\ (u_1 = 0)} + g\Delta z + \underbrace{\cancel{W}}_{=0\ (\text{No shaft work})} + F = 0$$

$$\Rightarrow \quad \frac{u_o^2}{2} + g\Delta z + F = 0 \qquad (1)$$

+10

+3 $$F = \frac{150 u_o \mu L (1-\varepsilon)^2}{\rho D_p^2 \varepsilon^3} + 1.75 \frac{u_o^2 L (1-\varepsilon)}{D_p \varepsilon^3} \quad \leftarrow \quad (4.25) \text{ in the text}$$

or $F = a u_o + b u_o^2$

$$a = \frac{150 \mu L (1-\varepsilon)^2}{\rho D_p^2 \varepsilon^3} \quad , \quad b = \frac{1.75 L (1-\varepsilon)}{D_p \varepsilon^3}$$

Substituting in (1) gives:

$$\frac{u_o^2}{2} + a u_o + b u_o^2 + c = 0 \qquad \left\{ c = g\Delta z \right.$$

$$\Rightarrow \quad (1/2 + b) u_o^2 + a u_o + c = 0$$

$$\text{or} \quad u_o^2 + a' u_o + c' = 0 \qquad \left\{ \begin{array}{l} a' = \dfrac{a}{(1/2 + b)} \\ c' = \dfrac{c}{1/2 + b} \end{array} \right.$$

one equation in one unknown (u_o)

$$\Rightarrow \quad u_o = -\frac{a'}{2} + \frac{1}{2}\sqrt{a'^2 - 4c'} \qquad +5$$

$$a = \frac{150 \mu L}{\rho D_p^2} \frac{(1-\varepsilon)^2}{\varepsilon^3}$$

$$\rho = 1000 \frac{kg}{m^3} \quad , \quad \mu = 0.001 \frac{kg}{m.s}$$

$$L = 2m \quad , \quad D_p = 0.2 * 10^{-3} \, m \quad , \quad \varepsilon = 0.3$$

$$\Rightarrow \quad a = 136,111 \quad m/s$$

$$b = \frac{1.75 L}{D_p \varepsilon^3} (1-\varepsilon) \qquad \Rightarrow \qquad b = 453,704$$

$$c = g \Delta z = -3 * 9.81 = -29.43 \; m^2/s^2$$

$$a' = \frac{a}{1/2 + b} = 0.30$$

$$c' = \frac{c}{1/2 + b} = -6.49 * 10^{-5}$$

$$u_o = \frac{1}{2}\sqrt{a'^2 - 4c'} - \frac{a'}{2} = 2.16 * 10^{-4} \; m/s$$

$$Q = \frac{\pi}{4} D^2 u_o = \frac{\pi}{4} * (0.1)^2 * 2.16 * 10^{-4} = 1.70 * 10^{-6} \; \frac{m^3}{s}$$

or $$Q = 1.70 * 10^{-6} \frac{\cancel{m^3}}{\cancel{s}} * \frac{1000 \cancel{lit}}{1 \cancel{m^3}} * \frac{1000 \, mL}{1 \cancel{lit}} * \frac{60 \cancel{s}}{1 \, min}$$

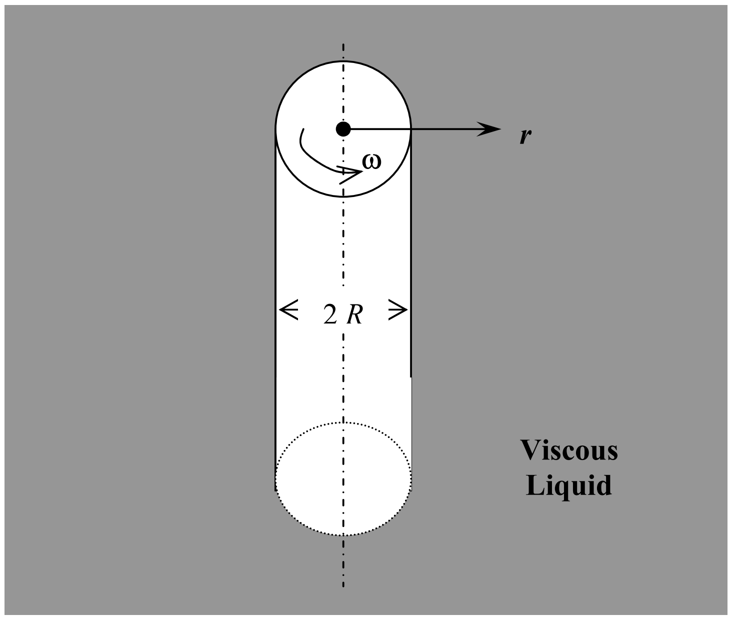
$$\Rightarrow \boxed{Q = 102 \; \frac{mL}{min}}$$

+7

Q4. (25 points)

A long vertical cylindrical rod of radius $\boldsymbol{R}$ is steadily rotated counterclockwise with angular velocity ω in a very large volume of an incompressible Newtonian liquid. The liquid extends from the surface of the rod effectively to infinity, where it may be considered at rest.

(a) Using the proper simplifying assumptions, derive an equation for the fluid velocity, v_θ, around the rod as a function of the radial coordinate.
(b) Derive an expression for the shear stress, $\tau_{r\theta}$, at the surface of the rod.
(c) If the length of the rode is $\boldsymbol{L}$, derive an expression for the torque required to rotate the rod.



a)

Assumptions: 1) steady state, 2) Newtonian, incompressible fluid
3) Axisymmetry, 4) $v_r = v_z = 0$, 5) $v_\theta \neq f(z \text{ or } \theta)$

$\Rightarrow v_\theta = f(r)$.

Momentum Balance: {Equation 5.75 in the text}

only θ-direction is needed:

Axisymmetry

$v_\theta \neq f(\theta)$ $\quad$ $v_r = 0$ $\quad$ $v_z = 0$ $\quad$ $p \neq f(\theta)$ Axisymmetry

$$\rho\left(\cancelto{0\,(SS)}{\frac{\partial v_\theta}{\partial t}} + \cancelto{0\;v_r=0}{v_r\frac{\partial v_\theta}{\partial r}} + \frac{v_\theta}{r}\cancelto{0}{\frac{\partial v_\theta}{\partial \theta}} + \cancelto{0}{\frac{v_r v_\theta}{r}} + \cancelto{0}{v_z\frac{\partial v_\theta}{\partial z}}\right) = \cancelto{0}{-\frac{1}{r}\frac{\partial p}{\partial \theta}}$$

$$+ \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial (r v_\theta)}{\partial r}\right) + \cancelto{0}{\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2}} + \cancelto{0}{\frac{2}{r^2}\frac{\partial v_r}{\partial \theta}} + \cancelto{0}{\frac{\partial^2 v_\theta}{\partial z^2}}\right] + \rho\, \cancelto{0}{g_\theta}$$

$v_\theta \neq f(\theta)$ Axisymmetry $\quad$ $v_r = 0$ $\quad$ $v_\theta \neq f(z)$ $\quad$ $g_\theta = 0$

gravity has no effect in the θ direction.

$$\Rightarrow \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial (r v_\theta)}{\partial r}\right)\right] = 0$$

or

$$\frac{d}{dr}\left(\frac{1}{r}\frac{d}{dr}(r v_\theta)\right) = 0$$

+5

$$\Rightarrow \frac{1}{r}\frac{d}{dr}(r v_\theta) = C_1$$

$$\Rightarrow \frac{d}{dr}(r v_\theta) = C_1 r$$

$$\Rightarrow r v_\theta = C_1 \frac{r^2}{2} + C_2$$

$$\Rightarrow v_\theta = C_1 \frac{r}{2} + \frac{C_2}{r}$$

+6

Boundary conditions: at $r = R$ $\quad v_\theta = \omega R$

$r \longrightarrow \infty \quad v_\theta = 0$

$\therefore C_1 = 0$ (for finite solution)

$\therefore \omega R = \frac{C_2}{R} \Rightarrow C_2 = \omega R^2$

$$\therefore \boxed{v_\theta = \omega R^2 / r}$$

+7

b) $\tau_{r\theta} = \mu \left\{ r \frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \underbrace{\cancel{\frac{1}{r}\frac{\partial v_r}{\partial \theta}}}_{=0\ (v_r = 0)} \right\}$ {eq. 5.63 in the text}

$$\therefore \tau_{r\theta} = \mu \left\{ r \frac{d}{dr}\left(\frac{v_\theta}{r}\right)\right\}$$

$$\frac{v_\theta}{r} = \frac{\omega R^2}{r^2} \Rightarrow \frac{d}{dr}\left(\frac{v_\theta}{r}\right) = -2\omega R^2 / r^3$$

$$\Rightarrow \tau_{r\theta} = -2\omega \mu R^2 / r^2 \Rightarrow \tau_{r\theta}\Big|_{r=R} = -2\omega\mu$$

5

c) $\mathcal{T} = -\tau_{r\theta}\Big|_{r=R} * (2\pi R L) * R$

$$= -2\pi R^2 L * \tau_{r\theta}\Big|_{r=R}$$

$$\Rightarrow \boxed{\mathcal{T} = 4\pi R^2 L \omega \mu}$$