King Fahd University of Petroleum & Minerals
Chemical Engineering Department
CHE 204 – Transport Phenomena I
Homework # 9

Problem 1.

In the analysis of laminar boundary layer flow over a flat plate, a velocity profile is assumed as followes:

$$\frac{v_x}{v_{x\infty}} = f(\zeta),$$

where $\zeta \equiv \dfrac{y}{\delta(x)}$ and then substituted in the momentum balance equation eqn. (8.4).

Assuming the following form: $f(\zeta) = a + b\zeta + c\zeta^2 + d\zeta^3$, where a, b, c and d are constants. Answer the following questions:

(a) Use the properties of the function $f(\zeta)$ to evaluate the constants a, b, c and d.

(b) Obtain an experssion for the thichness of the boundary layer, δ, along the plate, as function of x and Re_x where x is the distance from the edge of the plate.

(c) Show that the drag coefficient c_f, is given by: $c_f\sqrt{\mathrm{Re}_x} = 0.646$.

Problem 2.

Carbon dioxide gas (ρ = 1.8 kg/m^3 and μ = 1.5 x 10^{-5} Pa s) flows parallel to a flat plate at a velocity 1 km/hr. Use the simplified analysis of the boundary layer flow discussed in sections 8.2 and 8.5 of your textbook to answer the following questions:

(a) Calculate the distance L_t at which the boundary layer undergoes a transition from laminar to turbulent flow.

(b) Calculate the thickness of the boundary layer at a distance L_t -1 and L_t +1 meters.

(c) Calculate the drag force over the flat plate at a distance L_t +1 meters.

a) $f = a + b\xi + c\xi^2 + d\xi^3$

$f = \frac{v_x}{v_\infty}$, $\xi = \frac{y}{\delta}$

$y = 0$ $\quad v_x = 0 \quad \Rightarrow \quad \xi = 0 \quad f = 0$

$y = \delta$ $\quad v_x = v_\infty \quad \Rightarrow \quad \xi = 1 \quad f = 1$

$y = \delta$ $\quad \frac{dv_x}{dy} = 0 \quad \Rightarrow \quad \xi = 1 \quad \frac{df}{d\xi} = 0$

$y = 0$ $\quad \frac{d^2v_x}{dy^2} = 0 \quad \Rightarrow \quad \xi = 0 \quad \frac{d^2f}{d\xi^2} = 0$

Therefore:

$0 = a$ (1)

$1 = a + b + c + d$ (2)

$0 = b + 2c + 3d$ (3)

$0 = 2c$ (4)

$\therefore$ $\boxed{a = 0}$, $\boxed{c = 0}$

$b + d = 1 \quad \Rightarrow \quad b = 1 - d$

$b + 3d = 0 \quad \Rightarrow \quad 1 - d + 3d = 0 \quad \Rightarrow \quad \boxed{d = -\frac{1}{2}}$

and $\boxed{b = \frac{3}{2}}$

$\therefore$ $\boxed{f = \frac{3}{2}\xi - \frac{1}{2}\xi^3}$ Ans.

b) $v_\infty \frac{d}{dx}\int_0^\delta v_x\,dy = \frac{\tau_w}{\rho} + \frac{d}{dx}\int_0^\delta v_x^2\,dy$ (1)

$v_x = v_\infty * \left\{ \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3 \right\}$

$$v_x^2 = v_\infty^2 * \left\{ \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3 \right\}^2$$

$$\Rightarrow \quad v_x^2 = v_\infty^2 * \left\{ \frac{9}{4}\left(\frac{y}{\delta}\right)^2 - \frac{3}{2}\left(\frac{y}{\delta}\right)^4 + \frac{1}{4}\left(\frac{y}{\delta}\right)^6 \right\}$$

$$\int_0^\delta v_x \, dy = v_\infty * \left\{ \frac{3}{2\delta}\frac{y^2}{2} - \frac{1}{2\delta^3}\frac{y^4}{4} \right\}\Bigg|_0^\delta$$

$$= v_\infty \left\{ \frac{3}{4}\delta - \frac{1}{8}\delta \right\} = \frac{5}{8} v_\infty \delta .$$

$$\int_0^\delta v_x^2 \, dy = v_\infty^2 * \left\{ \frac{9}{4\delta^2}\frac{y^3}{3} - \frac{3}{2\delta^4}\frac{y^5}{5} + \frac{1}{4\delta^6}\frac{y^7}{7} \right\}\Bigg|_0^\delta$$

$$= v_\infty^2 * \left\{ \frac{9}{12}\delta - \frac{3}{10}\delta + \frac{1}{28}\delta \right\} = v_\infty^2 \delta \left(\frac{3}{4} - \frac{3}{10} + \frac{1}{28} \right)$$

$$= \frac{17}{35} v_\infty^2 \delta$$

$$\tau_w = \mu \left(\frac{\partial v_x}{\partial y} \right)\Bigg|_{y=0} = v_\infty \mu \left(\frac{3}{2\delta} - \frac{3}{2\delta} y^2 \right)\Bigg|_{y=0} = \frac{3 v_\infty \mu}{2\delta}$$

$$\Rightarrow \quad \boxed{\frac{\tau_w}{\rho} = \frac{3 v_\infty \mu}{2\rho\delta}} \qquad (2)$$

$$v_\infty \frac{d}{dx} \int_0^\delta v_x \, dy = v_\infty \frac{d}{dx} \left(\frac{5}{8} v_\infty \delta \right)$$

$$\Rightarrow \quad \boxed{v_\infty \frac{d}{dx} \int_0^\delta v_x \, dy = \frac{5}{8} v_\infty^2 \frac{d\delta}{dx}} \qquad (3)$$

$$\frac{d}{dx} \int_0^\delta v_x^2 \, dy = \frac{17}{35} \frac{d}{dx} \left(v_\infty^2 \delta \right)$$

$$\Rightarrow \quad \boxed{\frac{d}{dx} \int_0^\delta v_x^2 \, dy = \frac{17}{35} v_\infty^2 \frac{d\delta}{dx}} \qquad (4)$$

(2), (3), and (4) in (1) gives:

$$\frac{5}{\delta} v_\infty^2 \frac{d\delta}{dx} = \frac{3 v_\infty \mu}{2 \rho \delta} + \frac{17}{35} v_\infty^2 \frac{d\delta}{dx}$$

$$\Rightarrow v_\infty^2 \left(\frac{5}{8} - \frac{17}{35}\right) \frac{d\delta}{dx} = \frac{3 v_\infty \mu}{2\rho} \frac{1}{\delta}$$

$$\int_0^\delta \delta \, d\delta = \frac{3/2}{(5/8 - 17/35)} * \frac{\mu}{\rho v_\infty} * \int_0^x dx$$

$$\frac{\delta^2}{2} = \frac{3/2}{(5/8 - 17/35)} * \frac{\mu x}{\rho v_\infty}$$

$$\Rightarrow \delta = \sqrt{\frac{3}{(5/8 - 17/35)}} * \sqrt{\frac{\mu x}{\rho v_\infty}} = 4.641 \sqrt{\frac{\mu}{\rho v_\infty x}} \, x$$

or $\boxed{\delta = \frac{4.641 \, x}{\sqrt{Re_x}}}$ Ans.

$\underline{\underline{c}}$ $$C_f = \frac{\tau_w}{1/2 \rho v_\infty^2} = \frac{\tau_w / \rho}{1/2 \, v_\infty^2}$$

From equation (2): $$\frac{\tau_w}{\rho} = \frac{3 v_\infty \mu}{2 \rho \delta} = \frac{3 v_\infty \mu}{2 \rho} \frac{\sqrt{Re_x}}{4.641 x}$$

$$\Rightarrow C_f = \frac{\sqrt{Re_x} / (4.641 x)}{\cancel{1/2} v_\infty^{\cancel{2}}} * \frac{3 \cancel{v_\infty} \mu}{\cancel{2} \rho}$$

$$= \left(\frac{3}{4.641}\right) * \sqrt{Re_x} * \left(\frac{\mu}{\rho v_\infty x}\right)$$

$1/Re_x$

$$\Rightarrow C_f = \frac{\left(\frac{3}{4.641}\right) \cancel{\sqrt{Re_x}}}{\sqrt{Re_x} * \cancel{\sqrt{Re_x}}} = \frac{0.646}{\sqrt{Re_x}}$$

HW #9

Problem #2 $V_{x\infty} = 1 \frac{Km}{hr} * \frac{hr}{3600s} * \frac{1000\ m}{Km} = 0.278 \frac{m}{s}$

[a] Transition from Laminar to turbulent B.L. @

$$Re_x = 3.2 * 10^5 \Rightarrow \frac{\rho V_{x\infty} L_t}{\mu} = 3.2 * 10^5$$

$$L_t = 3.2 * 10^5 \frac{1.5 * 10^{-5}}{(1.8)(0.278)} = 9.59\ m$$

[b] $L_t - 1 \Rightarrow$ laminar $\frac{\delta}{x} = \frac{4.79}{\sqrt{Re_x}}$.

$$x = L_t - 1 = 8.59 \Rightarrow Re_x = 286562.4$$

$$\Rightarrow \delta = \frac{4.79}{\sqrt{286562.4}} 8.59 = 0.0769\ m$$

$$= 7.69\ cm$$

$L + 1 \Rightarrow$ Turbulent $\frac{\delta}{x} = \frac{0.376}{(Re_x)^{1/5}}$

$$\cancel{X} = L_t + 1 = 10.59 \Rightarrow Re_x = 353282.4$$

$$\Rightarrow \delta = \frac{0.376}{(353282.4)^{1/5}} 10.59 = 0.31\ m$$

$$= 31\ cm$$

$\boxed{c}$ $F_D = \int_0^{L_t} \tau_{w\,Laminar}\, dx + \int_{L_t}^{L_t+1} \tau_{w\,Turbulent}\, dx$

$$= \int_0^{L_t} \left(\tfrac{1}{2}\rho V_{x\infty}^2 (0.656)\right) \frac{1}{\sqrt{\frac{\rho V_{x\infty}}{\mu}}} x^{-\frac{1}{2}}\, dx$$

0.046

$$+ \int_{L_t}^{L_t+1} \frac{\tfrac{1}{2}\rho V_{x\infty}^2 \; 0.0576}{\left(\frac{\rho V_{x\infty}}{\mu}\right)^{0.2}} x^{-0.2}\, dx$$

4.01×10^{-3}

$$F_D = \frac{0.046}{\sqrt{\frac{\rho V_{x\infty}}{\mu}}} \cdot 2\left[(L_t)^{1/2} - (0)^{1/2}\right]$$

$$+ \frac{4.01 \times 10^{-3}}{\left(\frac{\rho V_{x\infty}}{\mu}\right)^{0.2}} \frac{1}{0.8} \left[(L_t + 1)^{0.8} - (L_t)^{0.8}\right].$$

$$= 1.56 \times 10^{-3} + 3.15 \times 10^{-4}$$

$$= 1.87 \times 10^{-3} \; \frac{N}{m}$$

per unit width.