

Homework 4

4.16-1Dimensional Analysis for Ship model

$$F = \psi(\rho, \mu, g, L, u)$$

$$\psi(F, \rho, \mu, g, L, u) = 0$$

Choose ρ, u, L as primary variables. Dimensionless groups for the remaining variables F, μ, g are then:

$$\Pi_1 = \frac{F}{\rho^a u^b L^c} \quad [=] \quad \frac{ML/T^2}{\left(\frac{M}{L^3}\right)^a \left(\frac{L}{T}\right)^b L^c}$$

Equal powers of M, L, T

$$\left. \begin{array}{l} M: 1 = a \\ L: 1 = -3a + b + c \\ T: -2 = -b \end{array} \right\} \text{Hence } a=1, b=2, c=2$$

$$\Pi_1 = \frac{F}{\rho u^2 L^2}$$

Likewise, $\Pi_2 = \frac{\mu}{\rho u L} = \frac{\mu}{\rho u L}$

$$\Pi_3 = \frac{g}{\rho u^2 L^2} = \frac{gL}{u^2}$$

4.16-2

Buckingham "Pi" Theorem

$$\psi(\pi_1, \pi_2, \pi_3) = 0$$

$$\frac{F}{\rho u^2 L^2} = \psi\left(\frac{\rho u L}{\mu}, \frac{u^2}{g L}\right)$$

Re
Froude
No.
No.

To preserve dynamical similarity, we must tow the model with the same Re and Fr as for the full-size ship. Within practical constraints, this is impossible, e.g.

	<u>Full-Size</u>	<u>Model</u>
u	20 ft/sec	? u _{model}
L	500 ft	10 ft (for example)

Since water is the only plausible medium in both cases, ρ and μ are irrelevant.

To maintain same Re

$$20 \times 500 = u_{\text{model}} \times 10 \quad \text{or} \quad u_{\text{model}} = 1000 \frac{\text{ft}}{\text{sec}} (!)$$

Dimensional Analysis for Disk Torque

$$\tau = \Psi(\rho, \mu, \omega, D)$$

$$\Psi(\tau, \rho, \mu, \omega, D)$$

Choose D, ρ, ω as primary variables. Dimensionless groups for τ and μ are:

$$\Pi_1 = \frac{\tau}{D^a \rho^b \omega^c} = \frac{ML^2/T^2}{L^a \left(\frac{M}{L^3}\right)^b \left(\frac{1}{T}\right)^c}$$

Equate powers of L, M, T

$$\left. \begin{array}{l} L: \quad 2 = a - 3b \\ M: \quad 1 = b \\ T: \quad -2 = -c \end{array} \right\}$$

Hence $a = 5, b = 1, c = 2$

$$\Pi_1 = \frac{\tau}{\rho \omega^2 D^5}$$

Likewise,

$$\Pi_2 = \frac{\mu}{D^2 \rho \omega} = \frac{\mu}{D^2 \rho \omega}$$

more conventional
this was found
(Re)

Hence, plot

$$\frac{\tau}{\rho \omega^2 D^5} \quad \text{versus} \quad \frac{\rho D^2 \omega}{\mu}$$

4.21-1

Dimensional Analysis of Centrifugal Pumps

$$\Psi(Q, P, \rho, N, D) = 0$$

$\begin{matrix} \swarrow & \downarrow & \downarrow & \downarrow & \swarrow \\ \frac{L^3}{T} & \frac{ML^2}{T^3} & \frac{M}{L^3} & \frac{1}{T} & L \end{matrix}$

$$\text{Power } \Pi_1 = \frac{P}{\rho^a N^b D^c} \quad [=] \quad \frac{ML^2 T^{-3}}{\left(\frac{M}{L^3}\right)^a \left(\frac{1}{T}\right)^b (L)^c}$$

$$\left. \begin{array}{l} M: 1 = a \\ L: 2 = -3a + c \\ T: -3 = -b \end{array} \right\} \begin{array}{l} a=1 \\ b=3 \\ c=5 \end{array} \quad \Pi_1 = \frac{P}{\rho N^3 D^5}$$

$$\text{Flow rate } \Pi_2 = \frac{Q}{\rho^a N^b D^c} \quad [=] \quad \frac{L^3 T^{-1}}{\left(\frac{M}{L^3}\right)^a \left(\frac{1}{T}\right)^b L^c}$$

$$\left. \begin{array}{l} M: 0 = a \\ L: 3 = -3a + c \\ T: -1 = -b \end{array} \right\} \begin{array}{l} a=0 \\ b=1 \\ c=3 \end{array} \quad \Pi_2 = \frac{Q}{N D^3}$$

Buckingham Pi Theorem

$$\frac{P}{\rho N^3 D^5} = \Psi\left(\frac{Q}{N D^3}\right)$$

4.21-2

Speed for model

$$\frac{Q}{ND^3} = \frac{100}{N_1 (0.5)^3} = \frac{1000}{750 (1.5)^3}$$

Hence $N_1 = 2,025 \text{ rpm}$

Power for full-size pump

$$\frac{P}{\rho N^3 D^5} = \frac{1.2}{62.4 \times 2025^3 \times 0.5^5} = \frac{P_2}{50 \times 750^3 \times 1.5^5}$$

Hence $P_2 = 11.87 \text{ HP}$