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Drawing Immiscible Liquids from a Tank

From hydrostatics,

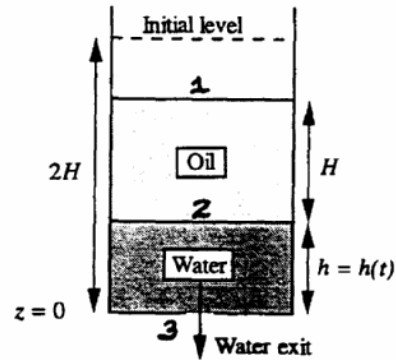
$$p_2 = \rho_o g H$$

Bernoulli (2) → (3)

$$\frac{p_2}{\rho_w} + g h = \frac{u_3^2}{2}$$

Hence

$$u_3 = \sqrt{2g \left( \frac{\rho_o}{\rho_w} H + h \right)}$$



Unsteady-State Mass Balance on Tank (Rate balance)

$$- \underset{\substack{\uparrow \\ \text{leaving through orifice}}}{0.62 a u_3 \rho_w} = - \underset{\substack{\uparrow \\ \text{Rate of accumulation of water in tank (will be negative, but don't enter parenthesis with a minus sign)}}}{0.62 a \rho_w \sqrt{2g \left( \frac{\rho_o}{\rho_w} H + h \right)}} = A \rho_w \frac{dh}{dt}$$

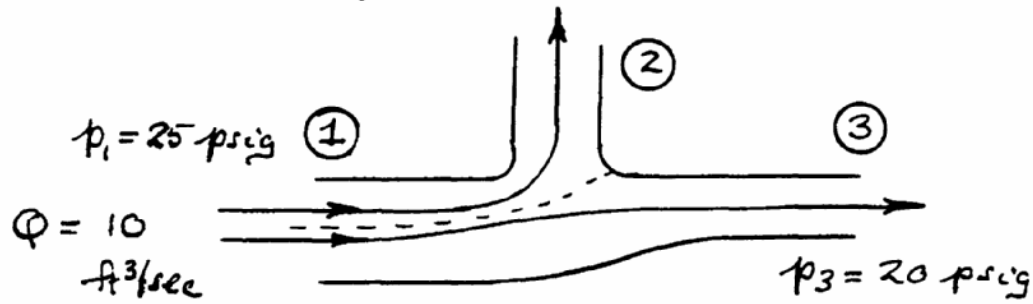
$$\int_0^h dt = - \frac{A}{0.62 a \sqrt{2g}} \int_H^0 \frac{dh}{\sqrt{\frac{\rho_o}{\rho_w} H + h}}$$

$$t = \frac{2A}{\sqrt{2g} \cdot 0.62 a} \left[ \sqrt{\frac{\rho_o}{\rho_w} H + h} \right]_0^H$$

$$t = \frac{2A}{0.62 a \sqrt{2g}} \left[ \sqrt{H \left( \frac{\rho_o}{\rho_w} + 1 \right)} - \sqrt{\frac{\rho_o}{\rho_w} H} \right]$$

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Branch Pipe



Areas  $A_1 = 0.1364$ ,  $A_2 = 0.0873$ ,  $A_3 = 0.0491$  ft<sup>2</sup>

$u_1 = \frac{10}{0.1364} = 73.3$  ft/sec,  $\rho = 0.8 \times 62.4 = 49.9$   $\frac{\text{lbm}}{\text{ft}^3}$

Bernoulli ① → ③  $\frac{u_1^2}{2} + \frac{p_1}{\rho} = \frac{u_3^2}{2} + \frac{p_3}{\rho}$

$$\frac{73.3^2}{2} + \frac{25 \times 32.2 \times 144}{49.9} = \frac{u_3^2}{2} + \frac{20 \times 32.2 \times 144}{49.9}$$

Hence  $u_3 = 79.4$  ft/sec

Continuity (mass Balance) ( $\rho$  constant)

$$10 = 0.0873 u_2 + 0.0491 \times 79.4$$

$u_2 = 69.9$  ft/sec

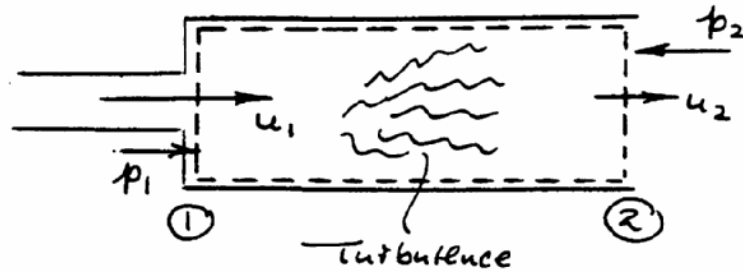
Bernoulli ① → ②

$$\frac{73.3^2}{2} + \frac{25 \times 32.2 \times 144}{49.9} = \frac{69.9^2}{2} + \frac{p_2 \times 32.2 \times 144}{49.9}$$

Hence  $p_2 = 27.6$  psig.

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### Sudden Expansion in a Pipe



Bernoulli cannot be applied between 1 and 2 because of the turbulence and energy losses that are characteristic of an expanding jet.

Continuity ① → ②       $A_1 u_1 = A_2 u_2$       (1)

Momentum ① → ②

$p_1 A_2 - p_2 A_2 + \rho A_1 u_1^2 - \rho A_2 u_2^2 = 0$       (2)

Substitute  $u_1 = u_2 \frac{A_2}{A_1}$  from (1) into (2)

$p_2 - p_1 = \rho u_2^2 \left( \frac{A_2}{A_1} - 1 \right)$ ,      (3)

which is always positive, so pressure increases. Increase in pressure energy is at the expense of kinetic energy

Overall Energy Balance

$\Delta \left( \frac{u^2}{2} \right) + \frac{\Delta p}{\rho} + g \Delta z + \cancel{w} + \cancel{M} = 0$

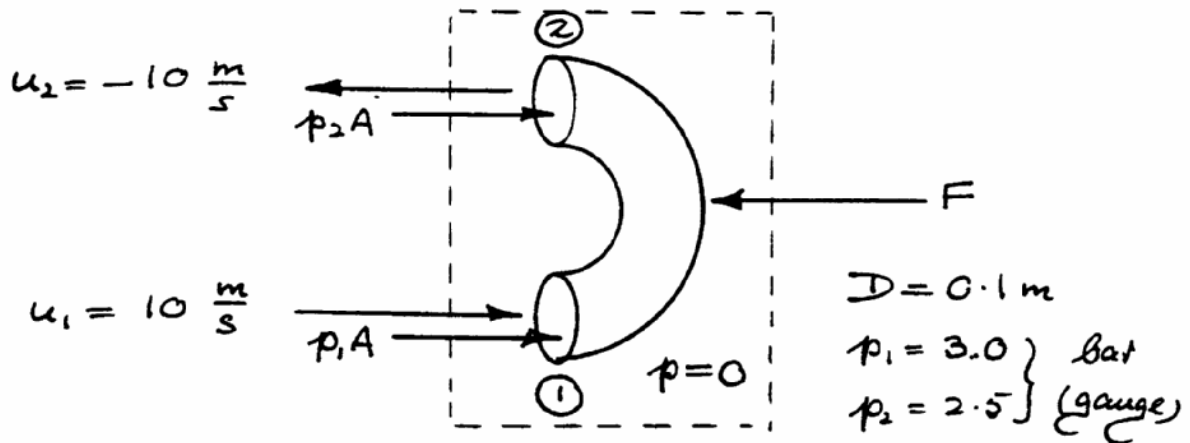
$M = \frac{u_1^2}{2} - \frac{u_2^2}{2} + \frac{p_1 - p_2}{\rho} = \frac{u_1^2}{2} - \frac{u_2^2}{2} + \rho u_2^2 \left( 1 - \frac{A_2}{A_1} \right)$

$= \frac{u_2^2}{2} \left( \frac{A_2^2}{A_1^2} - 1 + 2 - 2 \frac{A_2}{A_1} \right) = \frac{u_2^2}{2} \left( \frac{A_2}{A_1} - 1 \right)^2$

which is always positive. 67

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Force on Return Elbow



$$A = \frac{\pi D^2}{4} = \frac{\pi (0.1)^2}{4} = 0.00785 \text{ m}^2$$

$$m = \rho A u_1 = 1,000 \frac{\text{kg}}{\text{m}^3} \times 0.00785 \text{ m}^2 \times 10 \frac{\text{m}}{\text{s}} = 78.54 \frac{\text{kg}}{\text{s}}$$

Momentum Balance on Return Elbow

$$+ m u_1 - m u_2 + p_1 A + p_2 A - F = 0$$

Addition
Loss
Both  $p_1$  and  $p_2$  fluid
Elbow stays put.  
By flow
to push elbow to right

$$\begin{aligned}
 F &= (p_1 + p_2) A + 2 m u_1 \\
 &= (3.0 + 2.5) \times 10^5 \frac{\text{N}}{\text{m}^2} \times 0.00785 + 2 \times 78.54 \times 10 \\
 &= 4,317 + 1,571 = \underline{\underline{5,888 \text{ N}}}
 \end{aligned}$$