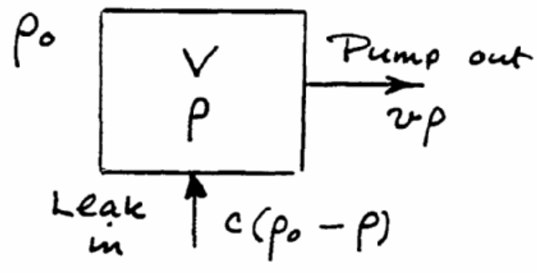
2.1

Evacuation of Leaking Tank



Transient Mass Balance

$$-v\rho + c(\rho_0 - \rho) = \frac{d}{dt} V\rho = V\frac{d\rho}{dt}$$

Separate variables and integrate:

$$\int_0^t dt = V\int_{\rho_0}^{\rho} \frac{d\rho}{c\rho_0 - (c+v)\rho} \quad \text{or} \quad t = \frac{-V}{c+v} \ln[c\rho_0 - (c+v)\rho]\Big|_{\rho_0}^{\rho}$$

$$t = \frac{V}{c+v} \ln \frac{-v\rho_0}{c\rho_0 - (c+v)\rho} = \frac{V}{c+v} \ln \frac{v\rho_0}{(c+v)\rho - c\rho_0}$$

$$= \frac{V}{c+v} \ln \frac{1}{\left(1+\frac{c}{v}\right)\frac{p}{p_0} - \frac{c}{v}} \quad \text{since } \rho = \frac{Mp}{RT}$$

and T is constant

Lowest attainable pressure

$$\text{As } t \to \infty \quad \underbrace{\left(1+\frac{c}{v}\right)}_{1.01}\frac{p^*}{p_0} = \underbrace{\frac{c}{v}}_{0.01}$$

$c = 10^{-5}\ m^3/s$

$v = 10^{-3}\ m^3/s$

$$p^* = \frac{0.01}{1.01} \times 1 = \underline{\underline{0.00990\ bar}}$$

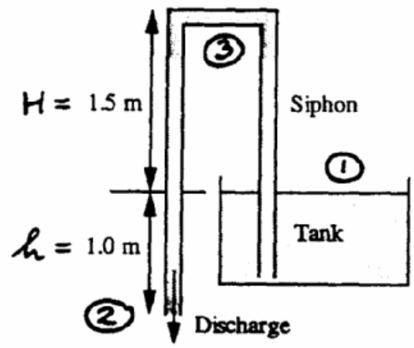
Time to fall half way to p^*, to $p = \frac{1+0.00990}{2} = 0.50495\ bar$

$$t = \frac{1}{0.00101} \ln \frac{1}{1.01\times 0.50495 - 0.01}$$

$$= 686.3\ s$$

2.8-1

Performance of a Siphon



Bernoulli ① → ② (ignoring friction)

$$\frac{u_1^2}{2} + \frac{p_1}{\rho} + g z_1 = \frac{u_2^2}{2} + \frac{p_2}{\rho} + g z_2$$

zero zero zero zero $-h$

$$u_2 = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 1.00} = \underline{\underline{4.43 \text{ m/s}}}$$

Lowest pressure occurs at point 3 at the top

Bernoulli ① → ③

$$\frac{u_1^2}{2} + \frac{p_1}{\rho} + g z_1 = 0 = \frac{u_3^2}{2} + \frac{p_3}{\rho} + g H$$

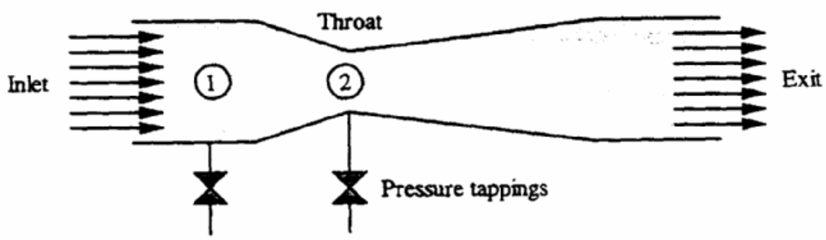
Continuity ③ → ②

$$A u_3 = A u_2$$

$$u_3 = u_2$$

2.12

Venturi "Meter"



Bernoulli ① → ② $\quad \frac{u_1^2}{2} + \frac{p_1}{\rho} = \frac{u_2^2}{2} + \frac{p_2}{\rho}$

Continuity ① → ② $\quad A u_1 = a u_2$

Eliminate u_2 gives $\quad \frac{p_1 - p_2}{\rho} = \frac{u_1^2}{2}\left(\frac{A^2}{a^2} - 1\right)$

Flow rate $\quad Q = A u_1 = C_D A \sqrt{\frac{2(p_1 - p_2)}{\rho\left(\frac{A^2}{a^2} - 1\right)}} = C_D A \sqrt{\frac{2(\rho_M - \rho_I) g \Delta h}{\rho_I\left(\frac{A^2}{a^2} - 1\right)}}$

M = mercury $\quad$ I = isopentane

Substituting numbers, $\quad A = \frac{\pi}{4}\left(\frac{6}{12}\right)^2 = 0.196 \text{ ft}^2, \quad C_D = 0.98$

$$Q = 0.98 \times 0.196 \sqrt{\frac{2\,\overbrace{(13.57 \times 62.4 - 38.75) \times 32.2 \times \left(\frac{20}{12}\right)}^{\alpha}}{38.75\left(\frac{6^4}{3^4} - 1\right)}}$$

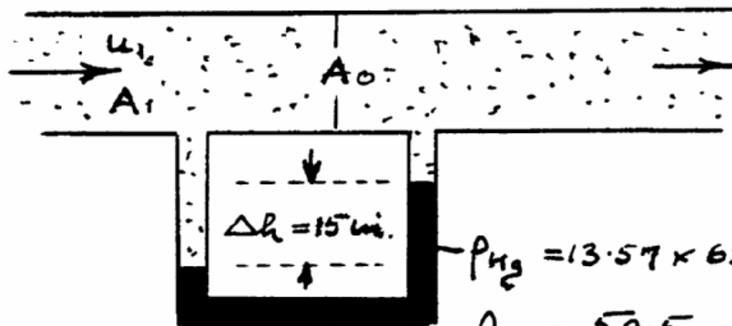
$= 0.98 \times 0.196 \times 12.22 = \underline{\underline{2.35 \text{ ft}^3/\text{sec}}}$

$= \underline{\underline{1053 \text{ gpm}}}$

$p_1 - p_2 = \frac{\alpha}{32.2 \times 144} = 9.35$ psi

2.13
Orifice Plate

Use the orifice-plate equation $\quad u_1 = C_D\sqrt{\dfrac{2(p_1-p_2)}{\rho\left(\dfrac{A_1^2}{A_0^2}-1\right)}}$



$\rho_{Hg} = 13.57 \times 62.4 = 847$

$\rho_K = 50.5$

$A_1 = \dfrac{\pi}{4}\left(\dfrac{2}{12}\right)^2 = 0.0218 \text{ ft}^2 \qquad u_1 = \dfrac{560}{60 \times 50.5 \times 0.0218} = 8.48 \dfrac{\text{ft}}{\text{sec}}$

For most orifice plates, $C_D \doteq 0.62$, which can be used as first trial.

$$8.48 = 0.62\sqrt{\dfrac{2 \times \frac{15}{12}(847-50.5) \times \overset{g}{32.2}}{50.5\left(\dfrac{A_1^2}{A_0^2}-1\right)}}$$

whence

$$\frac{A_1}{A_0} = \frac{D_1^2}{D_0^2} = 2.79 \qquad \frac{D_1}{D_0} = 1.67 \qquad \underline{\underline{D_0 = 1.2 \text{ in dia}}}$$

Check on value of C_D

$$Re_0 = \frac{D_1}{D_0} Re_1 = \frac{\underset{D_1/D_0}{1.67} \times \overset{u_1}{8.48} \times \overset{\rho}{50.5} \times \overset{D_1}{\frac{2}{12}}}{\underset{\mu}{3.18 \times \frac{1}{3600}}} = 1.35 \times 10^5$$

From chart, for $\dfrac{D_0}{D_1} = 0.60, \quad Re = 1.35 \times 10^5$

$C_D = 0.62 \quad$ Assumption was OK