Chapter 7
Lecture # 1-3

- **Overview of Chapter 7.**
- **Introduction.**
- **Interest Rate.**

Overview of Chapter 7

Title: **Engineering Economic Analysis**

Topics:

- Interest Rates
- Cash Flow Diagram
- Depreciation
- Inflation
- Taxation.

Introduction

Purpose of Chapter

- To discuss the principles of economic analysis

Importance

- This chapter covers all of the major topics required for completion of the Fundamentals of Engineering (FE) examination.

Introduction

The goal of any manufacturing company is to make money

Low-Value Raw Materials → **Chemical Processing Company** → **High-Value Chemicals**

Introduction

Money when invested earns money.

$1 today is worth more than $1 in the future.

Introduction

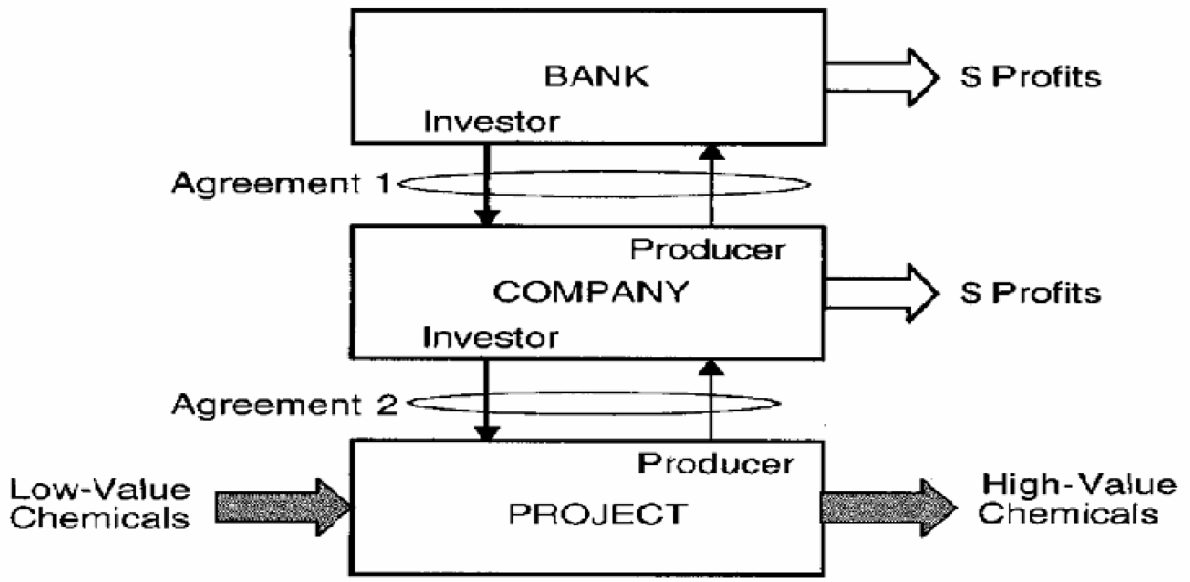
An *investment* is an agreement between two parties, whereby, one party, the *investor*, provides money, P, to a second party, the *producer*, with the expectation that the *producer* will return money, F, to the *investor* at some future specified date, where $F > P$. The terms used in describing the investment are:

P – Principal or Present Value

F – Future Value

n – Years between F and P

Introduction



A Typical Financing Scheme for a Chemical Plant

Interest Rate

The amount of money earned from the investment

$$E = F - P \qquad (7.1)$$

The yearly earnings rate is

$$i_s = \frac{E}{Pn} = \frac{(F - P)}{Pn} \qquad (7.2)$$

where i_s is termed the simple interest rate.

From Equation 7.2, we have:

$$\frac{F}{P} = (1 + ni_s) \text{ or, in general, } \frac{F}{P} = f(n,i) \qquad (7.3)$$

Interest Rate

Example 7.2

You decide to put $1000 into a bank that offers a special rate if left in for two years. After two years you will be able to withdraw $1150.

a. Who is the producer?

b. Who is the investor?

c. What are the values of P, F, i_s, and n?

Interest Rate

a. Producer: The bank has to produce $150.00 after two years.

b. Investor: You invest $1000 in an account at the beginning of the two-year period.

c. P = $1000 (given)

F = $1150 (given)

n = 2 years (given)

From Equation 7.2,

i_s = ($1150 – $1000)/($1000)/(2) = 0.075 or 7.5% per year

Interest Rate

- Simple Interest – Annual Basis
  - Interest paid in any year = Pi_s
    - Pi_s – Fraction of investment paid as interest per year
  - After n years total interest paid = $Pi_s n$
  - Total investment is worth = $P + Pi_s n$
  - Could earn interest on earned interest

Interest Rate

- Compound Interest

At time 0 we have P

At the end of Year 1, we have $F_1 = P(1 + i)$

At the end of Year 2, we have $F_2 = P(1 + i)^2$

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At the end of Year n, we have $F_n = P(1 + i)^n$

or $P = F_n / (1 + i)^n$

Interest Rate

SIMPLE INTEREST

$$F_n = P(1 + i_s n) \quad (7.4)$$

COMPOUND INTEREST

$$F_n = P(1 + i)^n \quad (7.5)$$

Interest Rate

Example

How much would I need to invest at 8 % p.a. to yield $5000 in 10 years ?

$$i = 0.08$$

$$n = 10$$

$$F_{10} = 5000$$

$$P = \frac{5000}{(1 + 0.08)^{10}} = \$2315.97$$

Interest Rate

Example 7.6

I need to borrow a sum of money (*P*) and have two loan alternatives:

- **a.** I borrow from my local bank who will lend me money at an interest rate of 7% p.a. and pay compound interest.
- **b.** I borrow from "Honest Sam" who offers to loan me money at 7.3% p.a. using simple interest.

In both cases, I need the money for three years. How much money would I need in three years to pay off this loan? Consider each option separately.

Interest Rate

Bank: From Equation 7.5 for $n = 3$ and $i = 0.07$ we get

$$F_3 = (P)(1 + 0.07)^3 = 1.225\,P$$

Sam: From Equation 7.4 for $n = 3$ and $i = 7.3$ we get

$$F_3 = (P)(1 + (3)(0.073)) = 1.219\,P$$

Even though Sam stated a higher interest rate to be paid, I would borrow the money from Sam because $1.219P < 1.225P$. This was because Sam used simple interest, and the bank used compound interest.

Interest Rate

If we have an investment over a period of years, and the interest rate changes each year, then the appropriate calculation for compound interest is given by

$$F_n = P\prod_{j=1}^{n}(1 + i_j) = P(1 + i_1)(1 + i_2)\cdots(1 + i_n) \qquad (7.7)$$

Interest Rate

Different Time Basis for Interest Calculations

- Relates to statement "Your loan is 6 % p.a. compounded monthly"
- Define actual interest rate per compounding period as *r*
  - i_{nom} = Nominal annual interest rate
  - m = Number of compounding periods per year (12)

Interest Rate

Different Time Basis for Interest Calculations

- i_{eff} = Effective annual interest rate

$$r = \frac{i_{nom}}{m}$$

- Look at condition after 1 year

$$F_1 = P\left(1 + i_{eff}\right)$$

$$i_{eff} = \left(1 + \frac{i_{nom}}{m}\right)^m - 1$$

Interest Rate

Example

Invest $1000 at 10 % p.a. compounded monthly. How much do I have in 1 year, 10 years?

$$F_1 = P\left(1+\frac{i_{nom}}{m}\right)^m = 1000\left(1+\frac{0.10}{12}\right)^{12} = \$1104.71$$

$$i_{eff} = \left(1+\frac{0.10}{12}\right)^{12} - 1 = 0.1047$$

$$F_{10} = P\left(1+i_{eff}\right)^{10} = \$2707.04$$

Interest Rate

- As m increases, i_{eff} increases
- Is there a limit as *m* goes to infinity?
  - Yes – continuously compounded interest
  - i_{eff} (continuous) = $e^{i_{nom}} - 1$

Interest Rate

IN COMPARING ALTERNATIVES, THE EFFECTIVE ANNUAL RATE AND NOT THE NOMINAL ANNUAL RATE OF INTEREST MUST BE USED