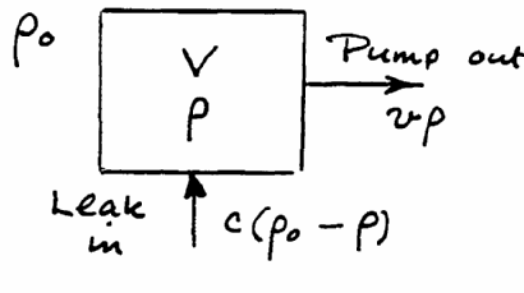


2.1

Evacuation of Leaking Tank



Transient Mass Balance

$$-vp + c(p_0 - p) = \frac{d}{dt} Vp = V \frac{dp}{dt}$$

Separate variables and integrate:

$$\int_0^t dt = V \int_{p_0}^p \frac{dp}{c p_0 - (c+v)p} \quad \text{or} \quad t = \frac{-V}{c+v} \ln [c p_0 - (c+v)p] \Big|_{p_0}^p$$

$$t = \frac{V}{c+v} \ln \frac{-v p_0}{c p_0 - (c+v)p} = \frac{V}{c+v} \ln \frac{v p_0}{(c+v)p - c p_0}$$

$$= \frac{V}{c+v} \ln \frac{1}{\left(1 + \frac{c}{v}\right) \frac{p}{p_0} - \frac{c}{v}}$$

since $p = \frac{M \bar{p}}{RT}$
and \bar{v} is constant

Lowest attainable pressure

$$\text{As } t \rightarrow \infty \quad \underbrace{\left(1 + \frac{c}{v}\right)}_{1.01} \frac{p^*}{p_0} = \frac{c}{v} \quad 0.01$$

$$c = 10^{-5} \text{ m}^3/\text{s}$$

$$v = 10^{-3} \text{ m}^3/\text{s}$$

$$p^* = \frac{0.01}{1.01} \times 1 = \underline{\underline{0.00990 \text{ bar}}}$$

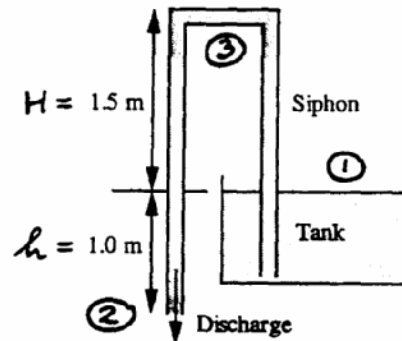
Time to fall half way to p^* , to $p = \frac{1 + 0.00990}{2}$

$$t = \frac{1}{0.00101} \ln \frac{1}{1.01 \times 0.50495 - 0.01} = 0.50495 \text{ bar}$$

$$= 686.3 \text{ s}$$

2.8-1

Performance of a Siphon



Bernoulli ① → ② (ignoring friction)

$$\underbrace{\frac{u_1^2}{2}}_{\text{Zero}} + \underbrace{\frac{p_1}{\rho}}_{\text{Zero}} + \underbrace{gz_1}_{\text{Zero}} = \underbrace{\frac{u_2^2}{2}}_{\text{Zero}} + \underbrace{\frac{p_2}{\rho}}_{\text{Zero}} + \underbrace{gz_2}_{-h}$$

$$u_2 = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 1.00} = \underline{\underline{4.43 \text{ m/s}}}$$

Lowest pressure occurs at point 3 at the top

Bernoulli ① → ③

$$\frac{u_1^2}{2} + \frac{p_1}{\rho} + gz_1 = 0 = \frac{u_3^2}{2} + \frac{p_3}{\rho} + gH$$

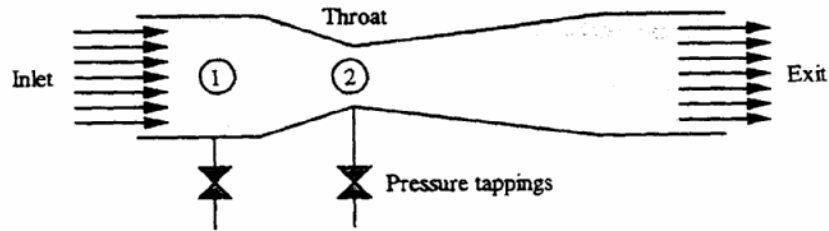
Continuity ③ → ②

$$A u_3 = A u_2$$

$$u_3 = u_2$$

2.12

Venturi "meter"



Bernoulli ① → ② $\frac{u_1^2}{2} + \frac{p_1}{\rho} = \frac{u_2^2}{2} + \frac{p_2}{\rho}$

Continuity ① → ② $A u_1 = a u_2$

Eliminate u_2 gives $\frac{p_1 - p_2}{\rho} = \frac{u_1^2}{2} \left(\frac{A^2}{a^2} - 1 \right)$

Flow rate $Q = A u_1 = C_D A \sqrt{\frac{2(p_1 - p_2)}{\rho \left(\frac{A^2}{a^2} - 1 \right)}} = C_D A \sqrt{\frac{2(p_m - p_I) g \Delta h}{\rho_I \left(\frac{A^2}{a^2} - 1 \right)}}$

$m = \text{mercury}$ $I = \text{isopentane}$

Substituting numbers, $A = \frac{\pi}{4} \left(\frac{6}{12} \right)^2 = 0.196 \text{ ft}^2$, $C_D = 0.98$

$$Q = 0.98 \times 0.196 \sqrt{\frac{2(13.57 \times 62.4 - 38.75) \times 32.2 \times \left(\frac{20}{12} \right)}{38.75 \left(\frac{6^4}{34} - 1 \right)}}$$

$= 0.98 \times 0.196 \times 12.22 = 2.35 \text{ ft}^3/\text{sec}$

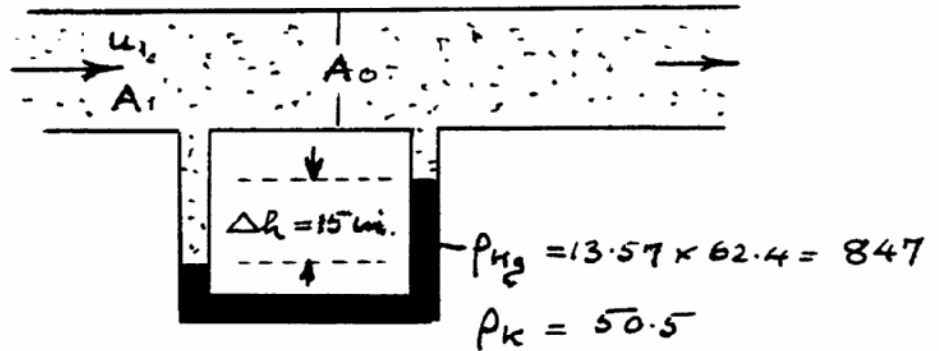
$p_1 - p_2 = \frac{\alpha}{32.2 \times 144} = 9.35 \text{ psi}$

$= 1053 \text{ gpm}$

2.13

Orifice Plate

Use the orifice-plate equation $u_1 = C_D \sqrt{\frac{2(p_1 - p_2)}{\rho \left(\frac{A_1^2}{A_0^2} - 1\right)}}$



$$A_1 = \frac{\pi}{4} \left(\frac{2}{12}\right)^2 = 0.0218 \text{ ft}^2, \quad u_1 = \frac{560}{60 \times 50.5 \times 0.0218} = 8.48 \frac{\text{ft}}{\text{sec}}$$

Got most orifice plate, $C_D = 0.62$, which can be used as first trial.

$$8.48 = 0.62 \sqrt{\frac{2 \times \frac{15}{12} (847 - 50.5) \times 32.2}{50.5 \left(\frac{A_1^2}{A_0^2} - 1\right)}}$$

whence

$$\frac{A_1}{A_0} = \frac{D_1^2}{D_0^2} = 2.79, \quad \frac{D_1}{D_0} = 1.67, \quad \underline{D_0 = 1.2 \text{ in dia}}$$

Check on value of C_D

$$Re_0 = \frac{D_1}{D_0} Re_1 = \frac{1.67 \times 8.48 \times 50.5 \times \frac{2}{12}}{3.18 \times \frac{1}{3600}} = 1.35 \times 10^5$$

From chart, for $\frac{D_0}{D_1} = 0.60$, $Re = 1.35 \times 10^5$

$C_D = 0.62$ Assumption was OK