CHE 203

HW # 7 SOLUTION

1) $\beta = \frac{-1}{\rho}\left(\frac{d\rho}{dT}\right)_P$; $\kappa = \frac{1}{\rho}\left(\frac{d\rho}{dP}\right)_T$

At constant temperature (T), the second equation can be written as:

$\frac{d\rho}{\rho} = \kappa dP$, Integration gives: $\ln\left(\frac{\rho_2}{\rho_1}\right) = \kappa \Delta P$

$\rho_2 = 1.01\,\rho_1$, $\kappa = 44.18 \times 10^{-6}$ bar^{-1}

$\Rightarrow \Delta P = \frac{\ln(1.01)}{\kappa} = \frac{0.00995}{44.18 \times 10^{-6}} = 225.2$ bar

but $\Delta P = P_2 - P_1 \Rightarrow P_2 = \Delta P + P_1 = 225.2 + 1$
$= 226.2$ bar

2) $\kappa = a + bP$; $a = 3.9 \times 10^{-6}$ atm^{-1}, $b = -0.1 \times 10^{-9}$ atm^{-2}

$P_1 = 1$ atm , $P_2 = 3000$ atm , $V = 1$ ft^3 (assume Const.)

Combine equations (1.3) and (3.3) for constant T:

$$W = V\int_{P_1}^{P_2}(a + bP)\cdot P\,dP = V\left[\frac{aP^2}{2} + \frac{bP^3}{3}\right]_1^{3000}$$

$$= 1\,\text{ft}^3\left[16.65 - 1.94997 \times 10^{-6}\right] = 16.65 \text{ ft}^3\,\text{a}$$

3) $P_1 = 8$ bar , $P_2 = 1$ bar , $T_1 = 600$ K

$R = 8.314$ J/mol·K , $C_p = \left(\frac{7}{2}\right)R$, $C_v = \left(\frac{5}{2}\right)R$

a) Constant V : $W = 0$ and $\Delta U = Q = C_v \Delta T$

$T_2 = T_1 \left(\frac{P_2}{P_1}\right) = 600\left(\frac{1}{8}\right) = 75$ K

$\Delta T = T_2 - T_1 = 75 - 600 = -525$ K

$\Delta U = Q = C_v \Delta T = \left(\frac{5}{2}\right) 8.314 \times -525 = -10.912$

$\therefore$ ΔU and $Q = \underline{-10.91 \frac{kJ}{mol}}$

$\Delta H = C_p \Delta T = \left(\frac{7}{2}\right) 8.314 \times -525 = \underline{15.28 \frac{kJ}{mol}}$

b) Constant T : $\Delta U = \Delta H = 0$ and $Q = W$

$W = RT_1 \ln\left(\frac{P_2}{P_1}\right) = 8.314 \times 600 \ln\left(\frac{1}{8}\right) \underset{= Q}{=} \underline{-10.37 \frac{kJ}{mol}}$

c) Adiabatic : $Q = 0$ and $\Delta U = W = C_v \Delta T$

$\gamma = \frac{C_p}{C_v}$, $T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{\gamma - 1}{\gamma}}$ $\Delta T = T_2 - T_1$

$\Delta U = C_v \Delta T$, $\Delta H = C_p \Delta T$

W and $\Delta U = \underline{-5.586 \frac{kJ}{mol}}$; $\Delta H = \underline{-7.821 \frac{kJ}{mol}}$

4) In all parts of this problem : $T_2 = T_1$ and $\Delta U = \Delta H = 0$, Also $Q = -W$ and all that remains is to calculate W. Symbol V is used for total volume in this problem.

$P_1 = 1$ bar, $P_2 = 12$ bar, $V_1 = 12\ m^3$, and $V_2 = 1\ m^3$

a) $W = nRT \ln\left(\frac{P_2}{P_1}\right) = P_1 V_1 \ln\left(\frac{P_2}{P_1}\right)$

$\underline{\underline{W = 2982\ kJ}}$

b) step 1: Adiabatic Compression to P_2

$$\gamma = \frac{C_P}{C_V} = \frac{(5/2)R}{(3/2)R} = \frac{5}{3}$$

$V_i = V_1 \left(\frac{P_1}{P_2}\right)$ (intermediate V)

$$W_1 = \frac{P_2 V_i - P_1 V_1}{\gamma - 1} \Rightarrow W_1 = 3063\ kJ$$

step 2: cool at constant P_2 to V_2

$$W_2 = -P_2 (V_2 - V_i) \Rightarrow W_2 = 2042\ kJ$$

$$\therefore W = W_1 + W_2 = 3063 + 2042$$
$$= \underline{\underline{5105\ kJ}}$$

c) step 1 : Adiabatic compression to V_2

$P_i = P_1 \left(\frac{V_1}{V_2} \right)^{\gamma}$ (intermediate P)

$W_1 = \frac{P_i V_2 - P_1 V_1}{\gamma - 1} \Rightarrow W_1 = 7635 \text{ kJ}$

step 2: No work ; Thus, $W = W_1 = 7635 \text{ kJ}$

d) step 1 : heat at constant V_1 to P_2, $W_1 = 0$

step 2 : cool at constant P_2 to V_2

$W_2 = -P_2 (V_2 - V_1)$, $W = W_2 \Rightarrow W = 13200 \text{ kJ}$

e) step 1 : cool at constant P_1 to V_2

$W_1 = -P_1 (V_2 - V_1) \Rightarrow W_1 = 1100 \text{ kJ}$

step 2 : heat at constant V_2 to P_2, $W_2 = 0$

$\Rightarrow W = W_1$

Thus, $W = 1100 \text{ kJ}$