A stainless steel sphear has a diameter of 1 mm and a density of 7870 kg/m$^3$. Calculate the terminal velocity of the sphear for the following two cases:

a) The sphear is droped in a polymer of density 1052 kg/m$^3$ and viscosity 0.1 kg /(m s).
b) The sphear is droped in kerosine of density 765.52 kg/m$^3$ and viscosity 0.00818 kg /(m s).



$$C_D = \frac{4}{3}\frac{gD}{U_t^2}\frac{\rho_s-\rho_f}{\rho_f}$$

$$= \frac{0.0131}{U_t^2}\frac{7870-\rho_f}{\rho_f}$$

$\boxed{a}$ $\rho_f = 1052$ kg/m$^3$

$\mu_f = 0.1 \ \frac{kg}{m\,s}$

$$\Rightarrow C_D = \frac{0.0847}{U_t^2}$$

to find $C_D$:

assume laminar flow ($Re<1$)

$$\Rightarrow C_D = \frac{24}{Re}$$

$$\Rightarrow \frac{24\mu}{\rho_f U_t D} = \frac{0.0847}{U_t^2}$$

solve for $U_t = 0.0371 \ \frac{m}{s}$

check $Re = \frac{\rho_f U_t D}{\mu} = 0.39$

$Re<1 \Rightarrow$ assumption was correct $\boxed{U_t = 0.071 \frac{m}{s}}$

$\boxed{b}$ $\rho_f = 765.52$ kg/m$^3$

$\mu_f = 0.00818$ kg/(m.s)

$$C_D = \frac{0.1216}{U_t^2}$$

assume laminar ($Re<1$) $C_D = \frac{24}{Re}$

$$\Rightarrow \frac{24\mu}{\rho_f U_t D} = \frac{0.1216}{U_t^2}$$

$\Rightarrow U_t = 0.474 \ \frac{m}{s}$

check $Re = \frac{\rho_f U_t D}{\mu} = 44.4$

$Re>1 \Rightarrow$ laminar assumption is wrong.

assume transition $1<Re<1000$

$$C_D = \frac{18}{Re^{0.6}}$$

$$\Rightarrow \frac{18\,\mu^{0.6}}{\rho_f^{0.6}\,U_t^{0.6}\,D^{0.6}} = \frac{0.1216}{U_t^2}$$

$\Rightarrow U_t = 0.1970 \ \frac{m}{s}$

check $Re = \frac{\rho_f U_t D}{\mu} = 18.44$

$1 < Re \leq 1000$

$\Rightarrow$ transition assumption was correct

$$\Rightarrow \boxed{U_t = 0.197 \ \frac{m}{s}}$$

2.05

A sphere of 2.2 mm diameter is dropped in water ($\rho_w$ = 1000 kg/m³, $\mu_w$ = $1{\cdot}10^{-3}$ kg/m/s). The sphere falls a distance of 10.5 cm in 21 seconds. Assuming steady state conditions:

(a) Calculate the density of the sphere.
(b) If the above sphere is dropped in a fluid of a specific gravity of 0.81 and its terminal velocity was measured as 20.5 cm/s, calculate the viscosity of the fluid.

$\boxed{a}$ $D = 2.2 \text{ mm} = 2.2 * 10^{-3} \text{ m}$

$\rho_f = 1000 \frac{kg}{m^3} \qquad \mu = 1 * 10^{-3} \frac{kg}{m\,s}$

$$u_t = \frac{10.5}{21} \frac{cm}{s} * \frac{m}{100\,cm} = 5 * 10^{-3} \frac{m}{s}$$

$$Re = \frac{(1000)(5*10^{-3})(2.2*10^{-3})}{1*10^{-3}} = 11$$

$\Rightarrow$ transition Range $\Rightarrow C_D = 18\, Re^{-0.6}$
$= 18 * 0.237$
$= 4.27$

$$C_D = \frac{4}{3} \frac{g D}{u_t^2} \left( \frac{\rho_s - \rho_f}{\rho_f} \right) = 4.27$$

$$\Rightarrow \rho_s = \frac{(4.27)(3)\, u_t^2\, \rho_f}{4\, g\, D} + \rho_f$$

$$\boxed{\rho_s = 1003.7 \frac{kg}{m^3}}$$

$\boxed{b}$ $\rho_f = (0.81)(1000) = 810 \frac{kg}{m^3}$

$$C_D = \frac{4}{3} \frac{gD}{U_t^2} \frac{\rho_s - \rho_f}{\rho_f} \qquad (4.14)$$

$$= \frac{4}{3} \frac{(9.8)(2.2 * 10^{-3})}{(2.05 * 10^{-2})^2} \frac{1003.7 - 810}{1003.7}$$

$$\boxed{C_D = 13.2}$$

We know $C_D$ but don't know Re

<u>assume laminar</u> $(Re < 1) \Rightarrow C_D = \frac{24}{Re}$

check $\frac{24}{Re} = 13.2 \Rightarrow Re = 1.82$

$Re > 1 \Rightarrow$ laminar assumption was wrong

<u>assume transition</u> $(1 < Re \leq 1000)$ $C_D = 18\, Re^{-0.6}$

check $\frac{18}{Re^{0.6}} = 13.2 \Rightarrow Re = \left(\frac{18}{13.2}\right)^{\frac{1}{0.6}} = 1.68$

$1 < 1.68 \leq 1000 \Rightarrow$ transition assumption was correct

$$Re = \frac{\rho_f U_t D}{\mu} = 1.68$$

$$\Rightarrow \mu = \frac{(810)(2.05 * 10^{-2})(2.2 * 10^{-3})}{1.68} = 0.022 \left(\frac{kg}{m \cdot s}\right)$$