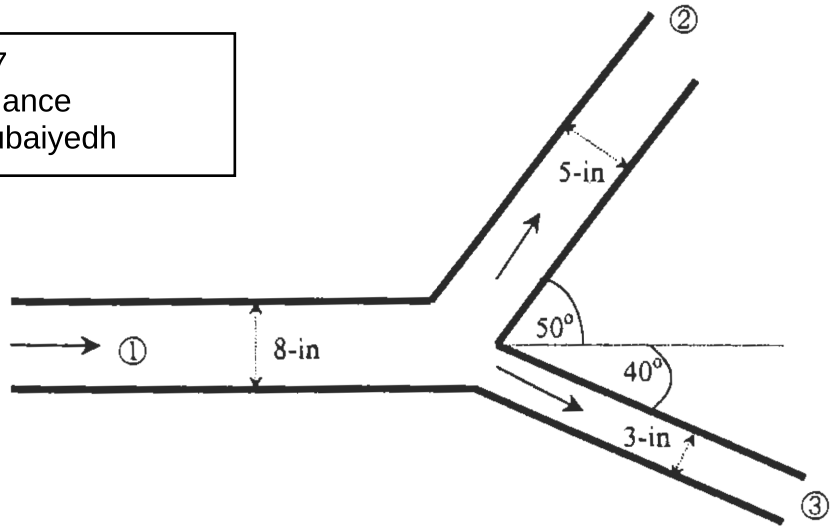
The figure below shows a horizontal flow separator (i.e., $z_1 = z_2 = z_3$). Water (ρ_w = 62.4 lb_m/ft^3) at points ② and ③ exits to the atmosphere. The volumetric flow rate and pressure at point ① are 18.87 ft^3/s and 64.7 psia, respectively. The diameters of the pipes are shown in the figure. Calculate the forces (magnitude and direction) required to keep the flow separator in place. Neglect frictional losses in your analysis.

Handout_7
Momentum Balance
Dr. Usamah Al-Mubaiyedh



$$u_1 = \frac{Q_1}{A_1} = \frac{18.87}{\frac{\pi}{4}\left(\frac{8}{12}\right)^2} = 54.05 \ \frac{ft}{s}$$

Bernoulli's Eq. ① → ②

$$\cancel{gz_1} + \frac{u_1^2}{2} + \frac{P_1}{\rho} = \cancel{gz_2} + \frac{u_2^2}{2} + \frac{P_2}{\rho}$$

$z_1 = z_2$

$$\Rightarrow u_2 = \sqrt{u_1^2 + 2\,\frac{P_1 - P_2}{\rho}} = 101.75 \ \frac{ft}{s}$$

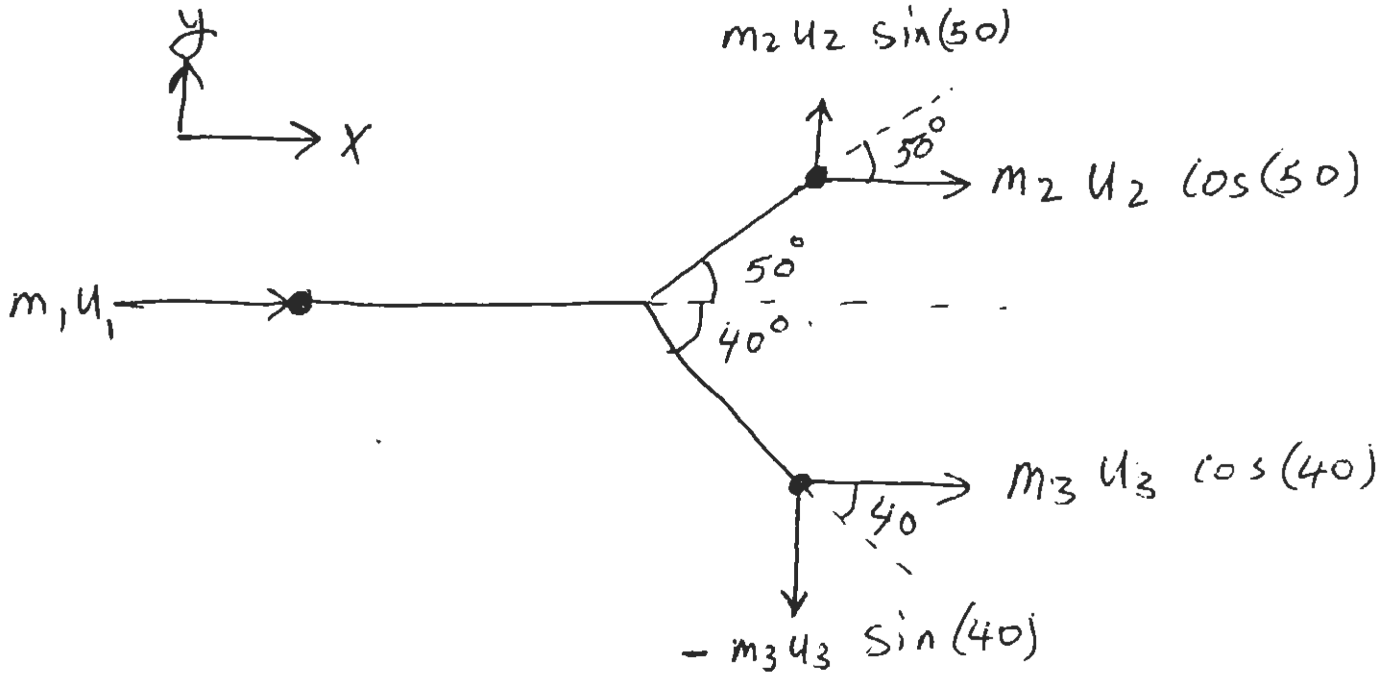
Bernoullis Eq. ① → ③

$$\cancel{gz_1} + \frac{u_1^2}{2} + \frac{P_1}{\rho} = \cancel{gz_3} + \frac{u_3^2}{2} + \frac{P_3}{\rho}$$

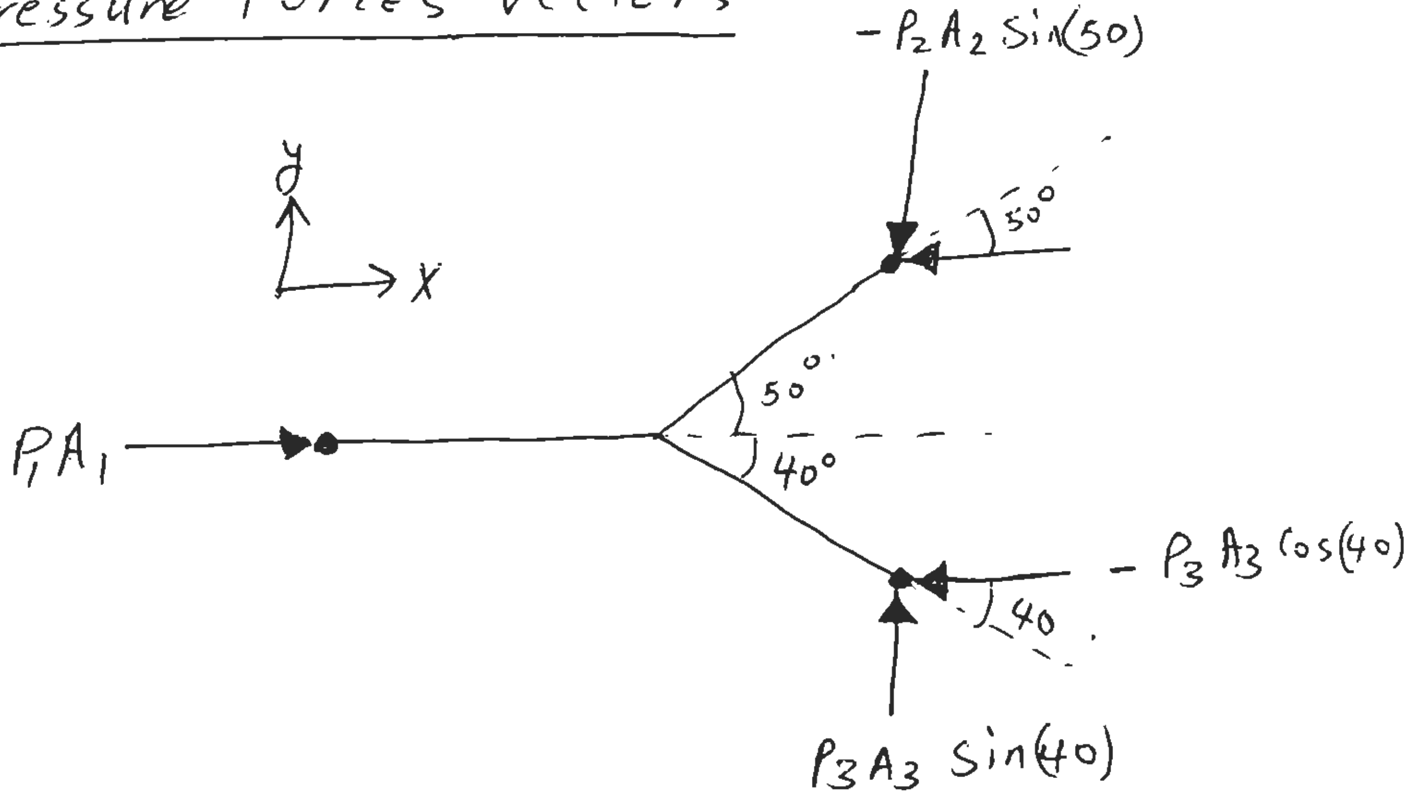
$z_1 = z_3$

$$\Rightarrow u_3 = \sqrt{u_1^2 + 2\,\frac{P_1 - P_3}{\rho}} = 101.75 \ \frac{ft}{s}$$

Momentum Vectors



Pressure Forces vectors



Momentum Balance X-direction

$$\dot{M}_{in}\big|_x - \dot{M}_{out}\big|_x + \sum F_x = 0$$

$$\left[m_1 u_1\right] - \left[m_2 u_2 \cos(50) + m_3 u_3 \cos(40)\right]$$

$P_2 = 0$ gaug $\qquad P_3 = 0$ gaug

$$+ \left[-\cancel{P_2 A_2}^{\,0} \cos(50)\right] + \left[-\cancel{P_3 A_3}^{\,0} \cos(40)\right] + \left[P_1 A_1\right]$$

$$+ B_x = 0$$

B_x ↑ force we need to apply in X-direction

$m_1 = \rho u_1 A_1 \qquad m_2 = \rho u_2 A_2 \qquad m_3 = \rho u_3 A_3$

$$\Rightarrow \boxed{B_x = -1976.6 \text{ lbf}}$$

Momentum Balance Y-direction

$$\dot{M}_{in}\big|_y - \dot{M}_{out}\big|_y + \sum F_y = 0$$

$$\left[0\right] - \left[m_2 u_2 \sin(50) - m_3 u_3 \sin(40)\right]$$

$P_2 = 0$ gaug $\qquad P_3 = 0$ gaug

$$+ \left[-\cancel{P_2 A_2}^{\,0} \sin(50)\right] + \left[\cancel{P_3 A_3}^{\,0} \sin(40)\right] + B_y = 0$$

B_y ↑ force we need to Apply in y-direction

$$B_y = +1462.6 \text{ lbf}$$

$\Rightarrow$ We need to apply

-1976.6 lbf in x-directi[on]

$+1462.6$ lbf in y-direction

$\Rightarrow$ Resultant force $= \sqrt{B_x^2 + B_y^2} = B_R$

$$B_R = 2459 \text{ lbf}$$

Angle

$$\tan(\theta) = \frac{B_y}{B_x} \Rightarrow \theta = \tan^{-1}\left(\frac{B_y}{B_x}\right)$$

$$= -36.5^\circ$$