Example

Handout_10
Packed Beds
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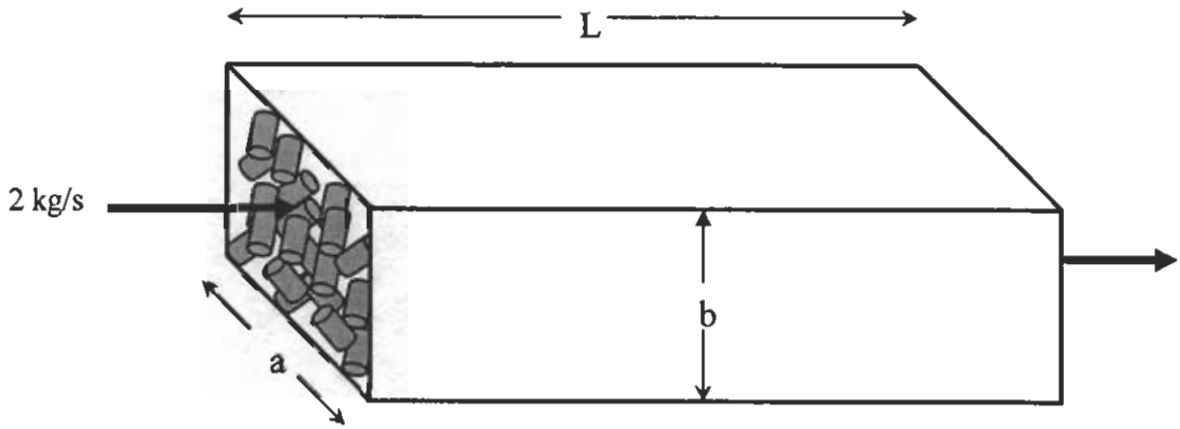
Water (ρ = 1000 kg/m^3, μ = 1*10^{-3} kg/ m s) flows through a horizontal duct that has a rectangular cross section with a mass flow rate 2 kg/s, see figure below. The duct is packed with 9000000 small cylinders with diameters $D_{cylinder}$ = 1 mm and lengths $L_{cylinder}$ = 2 mm. Find the pressure drop across the duct.



L = 1 m
a = 0.2 m
b = 0.1 m

$$-\frac{\Delta P}{\rho u_0^2}\frac{D_p}{L}\frac{\epsilon^3}{1-\epsilon} = \left[\frac{150}{Re} + 1.75\right] + \left[\frac{g\Delta z}{u_0^2}\frac{D_p}{L}\frac{\epsilon^3}{1-\epsilon}\right]$$

0 horizontal

$$Re = \frac{\rho u_0 D_p}{(1-\epsilon)\mu}$$

$$\Rightarrow \frac{-\Delta P}{\rho u_0^2}\frac{D_p}{L}\frac{\epsilon^3}{1-\epsilon} = \frac{150(1-\epsilon)\mu}{\rho u_0 D_p} + 1.75$$

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$$-\Delta P = \frac{150\, u_0 L (1-\epsilon)^2 \mu}{D_p^2\ \epsilon^3} + 1.75\ \frac{\rho u_0^2 L (1-\epsilon)}{D_p\ \epsilon^3}$$

$$u_0 = \frac{Q}{A} = \frac{m/\rho}{A} = \frac{2\ kg/s\ /\ 1000\ kg/m^3}{(0.1)(0.2)} = 0.1\ \frac{m}{s}$$

(0.1 ↑ b, 0.2 ↑ a)

$D_p \longrightarrow$ effective diameter of particles

$D_p = 6/a_v$ (see table 4.2 in book)

$$a_v = \frac{\text{surface area of particle}}{\text{Volume of particle}}$$

$$= \frac{\left(\frac{\pi}{4} D_{cylindr}^2\right) * 2 + \pi D_{cylinder} L_{cylinder}}{\frac{\pi}{4} D_{cylinder}^2 * L_{cylinder}}$$

$$= \frac{\frac{\pi}{4}(0.001)^2 * 2 + \pi (0.001)(0.002)}{\frac{\pi}{4}(0.001)^2 * (0.002)} = \frac{7.854 * 10^{-6} m}{1.571 * 10^{-9} m}$$

$$= 5000\ m^{-1}$$

$$\Rightarrow D_p = 1.2 * 10^{-3}\ m$$

$$\epsilon = \frac{\text{Volume of duct} - \text{total volume of particles}}{\text{Volume of duct}} \quad (3)$$

$$= \text{fraction of void (not occupied by particle.}$$

$$= \frac{(0.1)(0.2)(1) - 9000000 * 1.571 * 10^{-9}}{(0.1)(0.2)(1)}$$

$$= 0.293$$

$$\Rightarrow -\Delta P = \frac{(150)(0.1)(1)(1-0.293)^2 (1*10^{-3})}{(1.2*10^{-3})^2 (0.293)^3} +$$

$$\frac{(1.75)(1000)(0.1)^2(1)(1-0.293)}{(1.2*10^{-3})(0.293)^3}$$

$$= (206997.3 + 409895.7) \frac{kg\ m}{m^2\ s^2}$$

$$= 616893 \left(\frac{N}{m^2}\right)$$

↖ Pascals

$$= 6 \text{ atm}$$