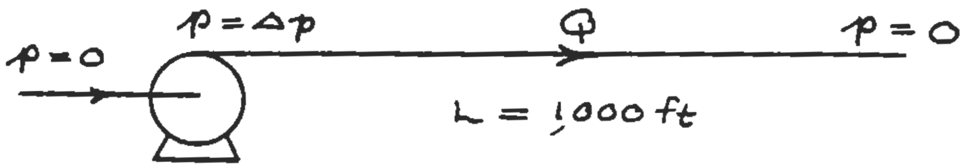
4.2
Pump and Pipeline



Pipe friction $\quad \Delta p = 2 f_F \rho u_m^2 \frac{L}{D}$

(gained across pump and lost in the pipe)

Substitute $Q = \frac{\pi D^2}{4} u_m$ gives $\Delta p = \frac{32 f_F \rho L Q^2}{\pi^2 D^5}$

Head Δh gained across the pump and lost in the pipe

$$\Delta h = \frac{\Delta p}{\rho g} = \frac{32 f_F L Q^2}{\pi^2 g D^5} = \frac{32 \times 0.00475 \times 1000\, Q^2}{\pi^2 \times 32.2 \times \left(\frac{2.067}{12}\right)^2} = 3{,}154\, Q^2$$

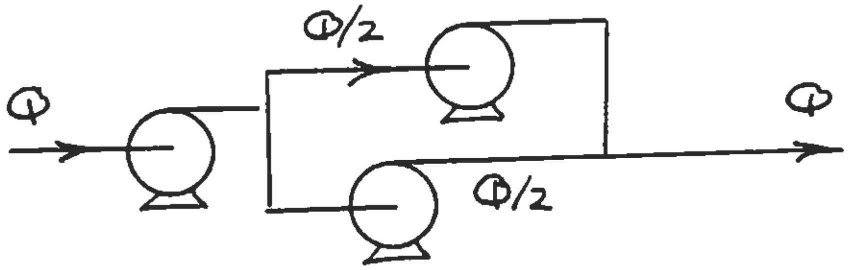
Solution either by drawing $\Delta h = 3154 Q^2$ on the pump curve and finding the point of intersection, or by trial and error:

Q (ft^3/s)	Δh_{pipe} (ft)	Δh_{pump} (ft)
0.12	45.4	66
0.16	80.7	53
0.14	61.8	61

Hence $\quad Q \doteq 0.14\ \frac{ft^3}{s}$

4.5-1

Pumps in Series and Parallel

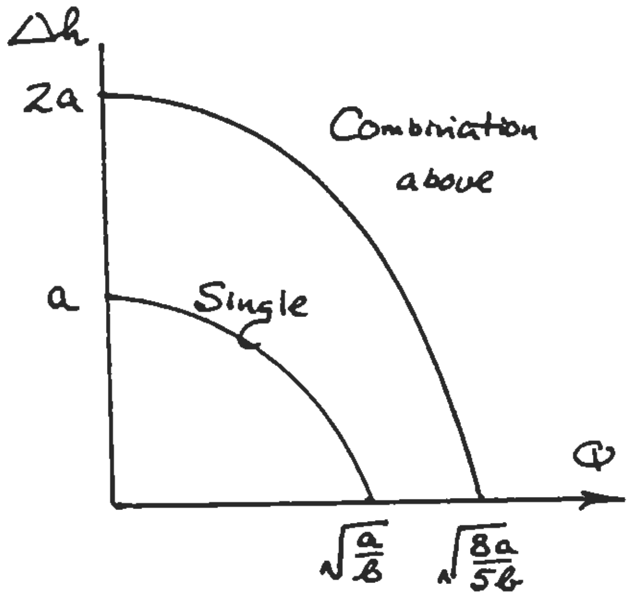


Single pump

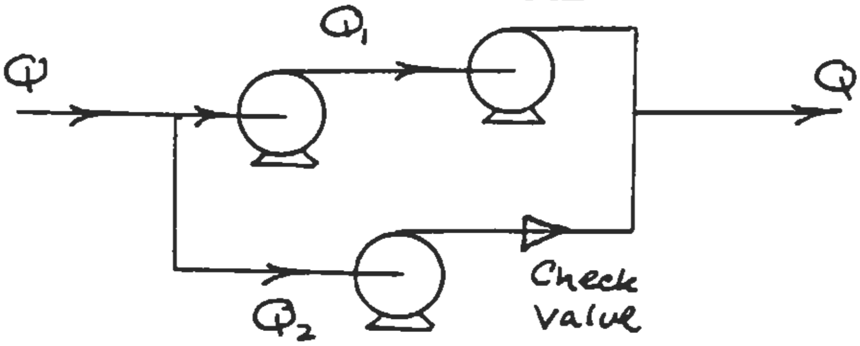
$$\Delta h = a - bQ^2$$

Above combination

$$\Delta h = a - bQ^2 + a - b\left(\frac{Q}{2}\right)^2 = 2a - \frac{5}{4}bQ^2$$



4.5-2



Δh

$$\Delta h = 2(a - b Q_1^2) \qquad (1)$$

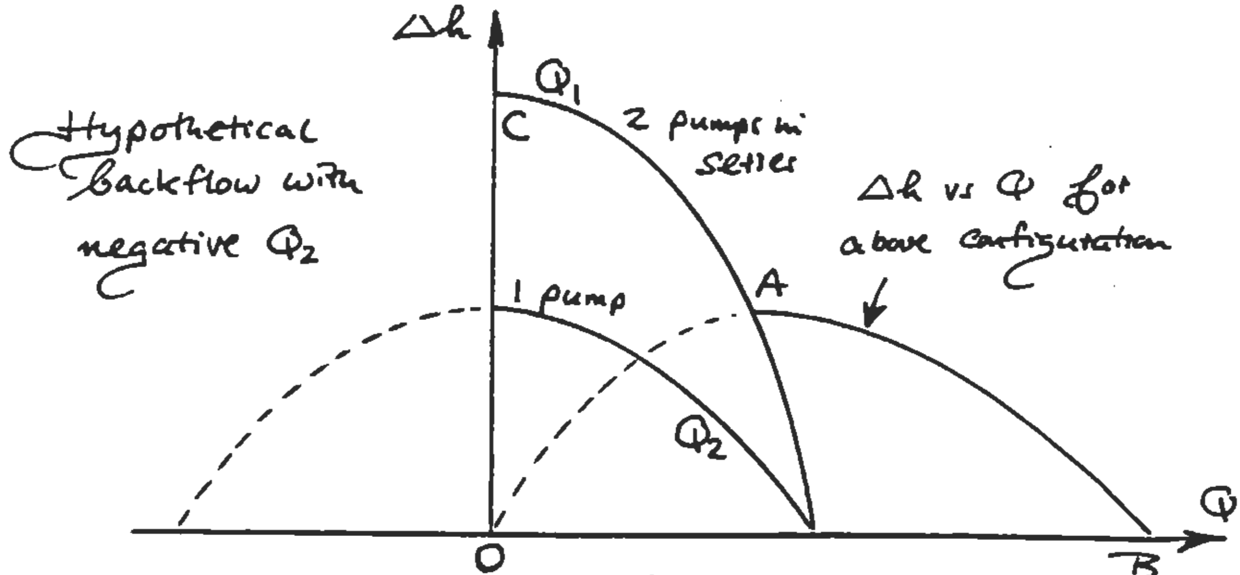
$$= a - b Q_2^2 \qquad (2)$$

$$Q = Q_1 + Q_2 \qquad (3)$$

Algebra gives $Q_1 = -Q + \sqrt{2Q^2 + a/b}$

$$\Delta h = 2bQ\left[2\sqrt{2Q^2 + a/b} - 3Q\right], \qquad (4)$$

Which (surprisingly) gives $\Delta h = 0$ for $Q = 0$! The paradox arises because we are applying (4) in a region that — because of the check valve — is physically impossible.



4.5-3

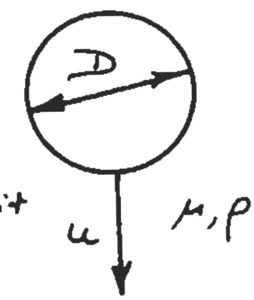
The hypothetical prediction of equation (4) along OA arises mathematically with a positive Q_1 and a negative Q_2. That is the head generated by the two pumps in series is supposedly strong enough to cause a backflow in the other pump — impossible in practice because of the check valve.

The complete picture is therefore the combination CA – AB.

4.7

Terminal Velocity of Hailstones

The following form is useful when the velocity is unknown



$$C_D Re^2 = \frac{4}{3}\frac{g \rho_f D^3}{\mu^2}(\rho_s - \rho_f)$$

negligible

$$= \frac{4}{3}\ \frac{32.2 \times 0.0765 \times \left(\frac{1}{12}\right)^3 \times 57.2}{\left(\frac{0.0435}{3600}\right)^2}$$

$$= 7.45 \times 10^8$$

first estimate from chart

C_D	0.4	0.45	0.45
$Re = (7.45 \times 10^8 / C_D)^{1/2}$	43,200	40,700	

$$Re = \frac{\rho_f u_t D}{\mu}$$

Terminal Velocity

$$u_t = \frac{\mu Re}{\rho_f D}$$

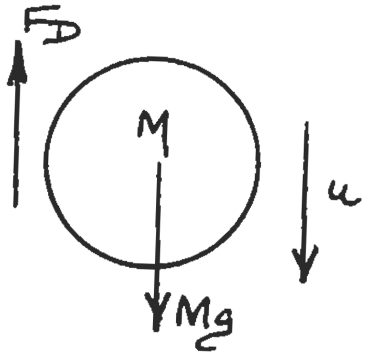
$$= \frac{0.0435 \times 40{,}700}{0.0765 \times \left(\frac{1}{12}\right) \times 3{,}600}$$

$$u_t = 77.1\ \frac{ft}{s}$$

$$\frac{lbm}{ft\ hr}\ \frac{ft^3}{lbm}\ \frac{1}{ft}\ \frac{hr}{s}$$

4.8

Hot-Air Balloon Emergency



At terminal velocity,
weight is exactly
balanced by drag force:

$$F_D = Mg$$

$$C_D = \frac{F_D}{\frac{1}{2}\rho u^2 A_p} = \frac{Mg}{\frac{1}{2}\rho u^2 \frac{\pi D^2}{4}}$$

Solve for velocity

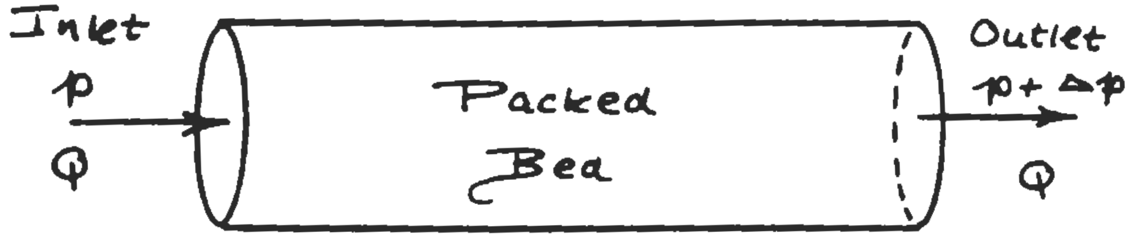
$$u^2 = \frac{8Mg}{C_D \rho \pi D^2} = \frac{8 \times 500 \times 32.2}{0.44 \times 0.0774 \times \pi \times 80^2}$$

$$u = 13.7 \text{ ft/s}$$

Hence the balloonist should use the parachute.

4.14

Pressure Drop in Ion-Exchange Bed



The Ergun equation consists of a laminar term and a turbulent term:

$$-\Delta p = aQ + bQ^2$$

$$4 = 10a + 100b$$

$$10 = 20a + 400b$$

Hence $a = 0.3$ and $b = 0.01$

Maximum flow rate is the solution of

$$54 = 0.3\, Q_{max} + 0.01\, Q_{max}^2$$

$$Q_{max}^2 + 30\, Q_{max} - 5{,}400 = 0$$

$$Q_{max} = \frac{1}{2}\left[-30 \underset{\uparrow \text{reject}}{\pm} \sqrt{30^2 + 4 \times 5{,}400}\right]$$

$$\underline{\underline{Q_{max} = 60 \text{ gpm}}}$$