Name of student: ______________________ ID#: ____________
Section: ____________

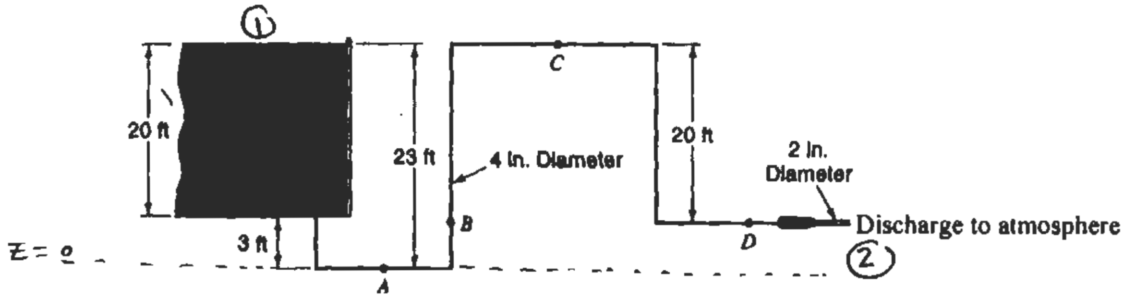
Q3 [25 points]

Assume that the level of water in the open tank remains the same and that there is no friction loss throughout the system. Determine:

a) The volumetric discharge flow rate from the nozzle.

b) The pressure and velocity at points A, B, C, and D.

25/25



• Energy bal between ① & ②:

$$\Delta\left(\frac{u^2}{2}\right) + g\Delta z + \cancel{\frac{\Delta P}{\rho}}^{0} + \cancel{w}^{0} + \cancel{f}^{0} = 0$$

$$\left(\frac{u_2^2}{2} - \cancel{\frac{u_1^2}{2}}^{0}\right) + g(z_2 - z_1) = 0$$ (+2)

• $\Rightarrow$ $\boxed{u_2 = \sqrt{2g(z_1 - z_2)}}$ where: $z_1 = 23$ ft, $z_2 = 3$ ft

$$u_2 = \sqrt{2\left(32.2\ \frac{ft}{s^2}\right)(23 - 3)\ ft}$$

• $\boxed{u_2 = 35.9\ ft/s}$ (+2)

• $\Rightarrow$ $Q = A_2 u_2$

$$Q = \frac{\pi}{4}\left[(2\ in)^2 * \left(\frac{1\ ft}{12\ in}\right)^2\right]\left(35.9\ \frac{ft}{s}\right)$$

$\boxed{Q = 0.783\ ft^3/s}$ (+2)

Name of student: ______________________ ID#: __________

Section: __________

- contin. eq. between D & 2 :

$$\cancel{\rho} A_D u_D = \cancel{\rho} A_2 u_2$$

$$\Rightarrow \boxed{u_D = \left(\frac{A_2}{A_D}\right) u_2}$$ +2

$$u_D = \frac{\cancel{\frac{\pi}{4}} d_2^2}{\cancel{\frac{\pi}{4}} d_D^2} u_2$$

+1.5

- $u_D = \left(\frac{2\,in}{4\,in}\right)^2 \left(35.9 \frac{ft}{s}\right) = \boxed{8.975 \; ft/s}$

- $\boxed{u_A = u_B = u_C = u_D} = 8.975 \; ft/s$

from contin. eq. +1.5

- apply Energy bal from 1 & A :

$$\left(\frac{u_A^2}{2} - \cancel{\frac{u_1^2}{2}}^{0}\right) + g(z_A - z_1) + \frac{P_A - P_1}{\rho} = 0$$

$$\Rightarrow \boxed{P_A = P_1 + \rho g (z_1 - z_A) - \frac{\rho}{2} u_A^2}$$ +2

$$P_A = P_{atm} + \rho \frac{g}{g_c}(z_1 - 0) - \frac{\rho}{2 g_c} u_A^2$$

$$P_A = 14.7 \frac{lb_f}{in^2} + \left(62.3 \frac{lb_m}{ft^3}\right)\left(32.2 \frac{ft}{s^2}\right)(23\,ft)\left(\frac{lb_f \cdot s^2}{32.2\, lb_m \cdot ft}\right)\left(\frac{1\,ft^2}{12^2\, in^2}\right)$$

$$-\left(\frac{1}{2}\right)(8.975)^2\left(\frac{ft^2}{s^2}\right)\left(62.3 \frac{lb_m}{ft^3}\right)\left(\frac{lb_f \cdot s^2}{32.2\, lb_m \cdot ft}\right)\left(\frac{1\,ft^2}{12^2\, in^2}\right)$$

$$\Rightarrow P_A = \left(14.7 \frac{lb_f}{in^2}\right) + \left(9.951 \frac{lb_f}{in^2}\right) - \left(0.541 \frac{lb_f}{in^2}\right) = \boxed{24.11 \frac{lb_f}{in^2}}$$

Name of student: ______________________ ID#: ________
Section: ________

* apply Energy bal from A to B:

$$\cancel{\Delta\left(\frac{u^2}{2}\right)}^{0} + g\Delta Z + \frac{\Delta P}{\rho} + \cancel{W}^{0} + \cancel{F}^{0} = 0$$

$$g(Z_B - Z_A) + \frac{P_B - P_A}{\rho} = 0$$

$$\Rightarrow \boxed{P_B = P_A + \rho g (Z_A - Z_B)}$$

+2

$$P_B = 24.11\,\frac{lb_f}{in^2} + \left(62.3\,\frac{lb_m}{ft^3}\right)\left(32.2\,\frac{ft}{s^2}\right)(0-3\,ft)\left(\frac{lb_f\cdot s^2}{32.2\,lb_m\cdot ft}\right)\left(\frac{1\,ft^2}{12^2\,in^2}\right)$$

$$\Rightarrow P_B = 24.11\,\frac{lb_f}{in^2} - 1.298\,\frac{lb_f}{in^2} = \boxed{22.81\,\frac{lb_f}{in^2}}$$

+2

Apply Energy bal from A to C:

$$\cancel{\Delta\left(\frac{u^2}{2}\right)}^{0} + g\Delta Z + \frac{\Delta P}{\rho} = 0$$

$$g(Z_C - Z_A) + \frac{P_C - P_A}{\rho} = 0$$

+2

$$\Rightarrow \boxed{P_C = P_A + \rho g (Z_A - Z_C)}$$

$$P_C = 24.11 + (62.3)(32.2)(-23)\left(\frac{1}{32.2}\right)\left(\frac{1}{12^2}\right) = \boxed{14.15\,\frac{lb_f}{in^2}}$$

+2

Apply Energy bal from B to D:

$$\cancel{\Delta\left(\frac{u^2}{2}\right)}^{0} + g\,\cancel{\Delta Z}^{0} + \frac{\Delta P}{\rho} = 0$$

$$\Rightarrow \boxed{P_D = P_B = 22.81\,\frac{lb_f}{in^2}}$$

+2

Name of student: ______________________ ID#: __________

Section: __________

25/25

Q4 [25 Points]

The pressure rise (ΔP) across a pump is a function of the fluid density ρ, the angular velocity of the impeller ω, the impeller diameter D, the volumetric flow rate Q, and the fluid viscosity μ.

Apply dimensional analysis and select ρ, D, and ω as the primary variables, in order to define three dimensionless groups for the above problem.

$$(\Delta P) = f(\rho, \omega, D, Q, \mu)$$

Define fundamental dimensions

Length : L

mass : M

time : t

+1

write the dimensions of all variables in terms of fundamental dimensions:

$$\Delta P = \frac{M}{Lt^2}$$

$$\rho = \frac{M}{L^3}$$

$$\omega = \frac{1}{t}$$

$$D = L$$

$$Q = \frac{L^3}{t}$$

$$\mu = \frac{M}{Lt}$$

+3

select primary variables : (given in the question)

$\boxed{\begin{matrix}\rho \\ D \\ \omega\end{matrix}}$ because

$D \to L$

$1/\omega \to t$

$\rho D^3 \to M$

1.

Name of student: ______________________ ID#: __________
Section: __________

4. define dimensionless ratio for the remaining variables (ΔP, Q, μ)

For ΔP:

$$\pi_1 = \frac{\Delta P}{\rho^a D^b \omega^c}$$

dimension of ΔP = dimension of $(\rho^a D^b \omega^c)$

$$\Rightarrow M L^{-1} t^{-2} = \left(\frac{M}{L^3}\right)^a (L)^b \left(\frac{1}{t}\right)^c$$

for M : $1 = a \longrightarrow \boxed{a=1}$

for t : $-2 = -c \longrightarrow \boxed{c=2}$

for L : $-1 = -3a + b$

$-1 = -3(1) + b \longrightarrow \boxed{b=2}$

$$\Rightarrow \boxed{\pi_1 = \frac{\Delta P}{\rho D^2 \omega^2}}$$

+7

---

for Q :

$$\pi_2 = \frac{Q}{\rho^a D^b \omega^c}$$

$$\Rightarrow \frac{L^3}{t} = \left(\frac{M}{L^3}\right)^a (L)^b \left(\frac{1}{t}\right)^c$$

for M : $0 = a \longrightarrow \boxed{a=0}$

for t : $-1 = -c \longrightarrow \boxed{c=1}$

for L : $3 = -3a + b \longrightarrow \boxed{b=3}$

$$\Rightarrow \boxed{\pi_2 = \frac{Q}{D^3 \omega}}$$

+7

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for μ :

$$\pi_3 = \frac{\mu}{\rho^a D^b \omega^c}$$

$$\rightarrow \frac{M}{Lt} = \left(\frac{M}{L^3}\right)^a (L)^b \left(\frac{1}{t}\right)^c \Rightarrow \boxed{\pi_3 = \frac{\mu}{\rho D^2 \omega}}$$

for M : $1 = a \longrightarrow \boxed{a=1}$

for t : $-1 = \cancel{-3a+b}\ -c \longrightarrow \boxed{c=1}$

for L : $-1 = -3a + b \longrightarrow b = 2$

+7