Home Work # 2

Chapter 2

2.1

a) Overall mass balance:

$$\frac{d(\rho V)}{dt} = w_1 + w_2 - w_3 \quad (1)$$

Energy balance:

$$C\frac{d\left[\rho V(T_3 - T_{ref})\right]}{dt} = w_1 C(T_1 - T_{ref}) + w_2 C(T_2 - T_{ref}) - w_3 C\left(T_3 - T_{ref}\right) \quad (2)$$

Because ρ = constant and $V = \overline{V}$ = constant, Eq. 1 becomes:

$$w_3 = w_1 + w_2 \quad (3)$$

b) From Eq. 2, substituting Eq. 3

$$\rho C\overline{V}\frac{d(T_3 - T_{ref})}{dt} = \rho C\overline{V}\frac{dT_3}{dt} = w_1 C(T_1 - T_{ref}) + w_2 C(T_2 - T_{ref}) - (w_1 + w_2) C\left(T_3 - T_{ref}\right) \quad (4)$$

Constants C and T_{ref} can be cancelled:

$$\rho\overline{V}\frac{dT_3}{dt} = w_1 T_1 + w_2 T_2 - (w_1 + w_2)T_3 \quad (5)$$

The simplified model now consists only of Eq. 5.

Degrees of freedom for the simplified model:

Parameters : ρ, $\overline{V}$

Variables : w_1, w_2, T_1, T_2, T_3
$N_E = 1$
$N_V = 5$

Thus, $N_F = 5 - 1 = 4$

Because w_1, w_2, T_1 and T_2 are determined by upstream units, we assume they are known functions of time:

$$w_1 = w_1(t)$$
$$w_2 = w_2(t)$$
$$T_1 = T_1(t)$$
$$T_2 = T_2(t)$$

Thus, N_F is reduced to 0.

2.2

Energy balance:

$$C_p \frac{d\left[\rho V(T - T_{ref})\right]}{dt} = wC_p(T_i - T_{ref}) - wC_p(T - T_{ref}) - UA_s(T - T_a) + Q$$

Simplifying

$$\rho V C_p \frac{dT}{dt} = wC_p T_i - wC_p T - UA_s(T - T_a) + Q$$
$$\rho V C_p \frac{dT}{dt} = wC_p(T_i - T) - UA_s(T - T_a) + Q$$

b) T increases if T_i increases and vice versa.

T decreases if w increases and vice versa if $(T_i - T) < 0$. In other words, if $Q > UA_s(T-T_a)$, the contents are heated, and $T > T_i$.

2.3

a) Mass Balances:

$$\rho A_1 \frac{dh_1}{dt} = w_1 - w_2 - w_3 \qquad (1)$$

$$\rho A_2 \frac{dh_2}{dt} = w_2 \qquad (2)$$

Flow relations:

Let P_1 be the pressure at the bottom of tank 1.

Let P_2 be the pressure at the bottom of tank 2.

Let P_a be the ambient pressure.

Then

$$w_2 = \frac{P_1 - P_2}{R_2} = \frac{\rho g}{g_c R_2}(h_1 - h_2) \qquad (3)$$

$$w_3 = \frac{P_1 - P_a}{R_3} = \frac{\rho g}{g_c R_3} h_1 \qquad (4)$$

b) Seven parameters: ρ, A_1, A_2, g, g_c, R_2, R_3

Five variables : h_1, h_2, w_1, w_2, w_3

Four equations

Thus $N_F = 5 - 4 = 1$

1 input = w_1 (specified function of time)
4 outputs = h_1, h_2, w_2, w_3

2.4

Assume constant liquid density, ρ . The mass balance for the tank is

$$\frac{d(\rho A h + m_g)}{dt} = \rho(q_i - q)$$

Because ρ, A, and m_g are constant, this equation becomes

$$A\frac{dh}{dt} = q_i - q \tag{1}$$

The square-root relationship for flow through the control valve is

$$q = C_v\left(P_g + \frac{\rho g h}{g_c} - P_a\right)^{1/2} \tag{2}$$

From the ideal gas law,

$$P_g = \frac{(m_g / M)RT}{A(H-h)} \tag{3}$$

where T is the absolute temperature of the gas.

Equation 1 gives the unsteady-state model upon substitution of q from Eq. 2 and of P_g from Eq. 3:

$$A\frac{dh}{dt} = q_i - C_v\left[\frac{(m_g / M)RT}{A(H-h)} + \frac{\rho g h}{g_c} - P_a\right]^{1/2} \tag{4}$$

Because the model contains P_a, operation of the system is not independent of P_a. For an open system $P_g = P_a$ and Eq. 2 shows that the system is independent of P_a.

2.10

Assume that the feed contains only A and B, and no C. Component balances for A, B, C over the reactor give.

$$V\frac{dc_A}{dt} = q_i c_{Ai} - qc_A - Vk_1 e^{-E_1/RT} c_A \tag{1}$$

$$V\frac{dc_B}{dt} = q_i c_{Bi} - qc_B + V(k_1 e^{-E_1/RT} c_A - k_2 e^{-E_2/RT} c_B) \tag{2}$$

$$V\frac{dc_C}{dt} = -qc_C + Vk_2 e^{-E_2/RT} c_B \tag{3}$$

An overall mass balance over the jacket indicates that $q_c = q_{ci}$ because the volume of coolant in jacket and the density of coolant are constant.

Energy balance for the reactor:

$$\frac{d\left[(Vc_A M_A S_A + Vc_B M_B S_B + Vc_C M_C S_C)T\right]}{dt} = (q_i c_{Ai} M_A S_A + q_i c_{Bi} M_B S_B)(T_i - T)$$

$$-UA(T - T_c) + (-\Delta H_1)Vk_1 e^{-E_1/RT} c_A + (-\Delta H_2)Vk_2 e^{-E_2/RT} c_B \tag{4}$$

where M_A, M_B, M_C are molecular weights of A, B, and C, respectively
S_A, S_B, S_C are specific heats of A, B, and C.
U is the overall heat transfer coefficient
A is the surface area of heat transfer

Energy balance for the jacket:

$$\rho_j S_j V_j \frac{dT_c}{dt} = \rho_j S_j q_{ci}(T_{ci} - T_c) + UA(T - T_c) \tag{5}$$

where:

ρ_j, S_j are density and specific heat of the coolant.
V_j is the volume of coolant in the jacket.

Eqs. 1 - 5 represent the dynamic model for the system.