Interacting vs. Noninteracting Systems

- Consider a process with several invariables and several output variables. The process is said to be *interacting* if:

  o Each input affects more than one output.

  or

  o A change in one output affects the other outputs.

  Otherwise, the process is called *noninteracting*.

- As an example, we will consider the two liquid-level storage systems shown in Figs. 4.3 and 6.13.
- In general, transfer functions for interacting processes are more complicated than those for noninteracting processes.

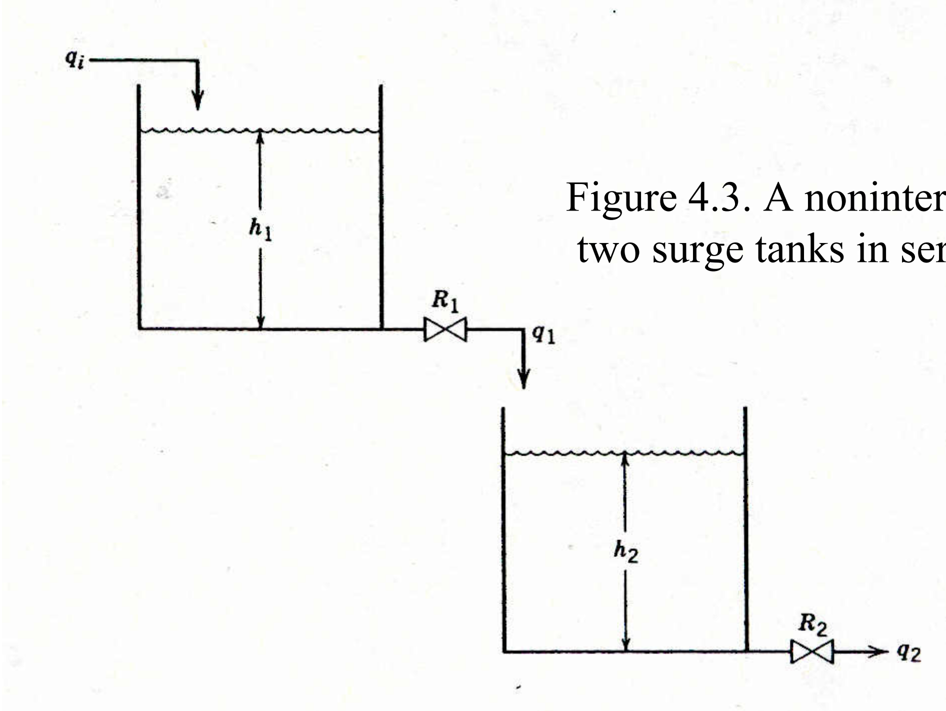

Figure 4.3. A noninteracting system: two surge tanks in series.

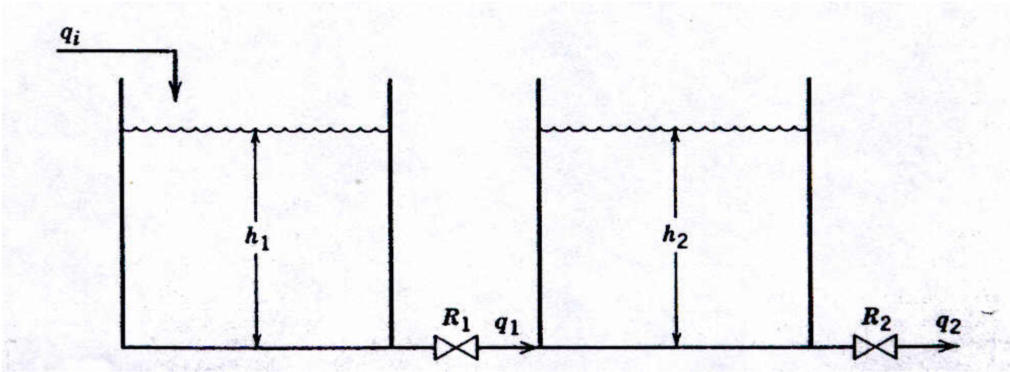

Figure 6.13. Two tanks in series whose liquid levels interact.

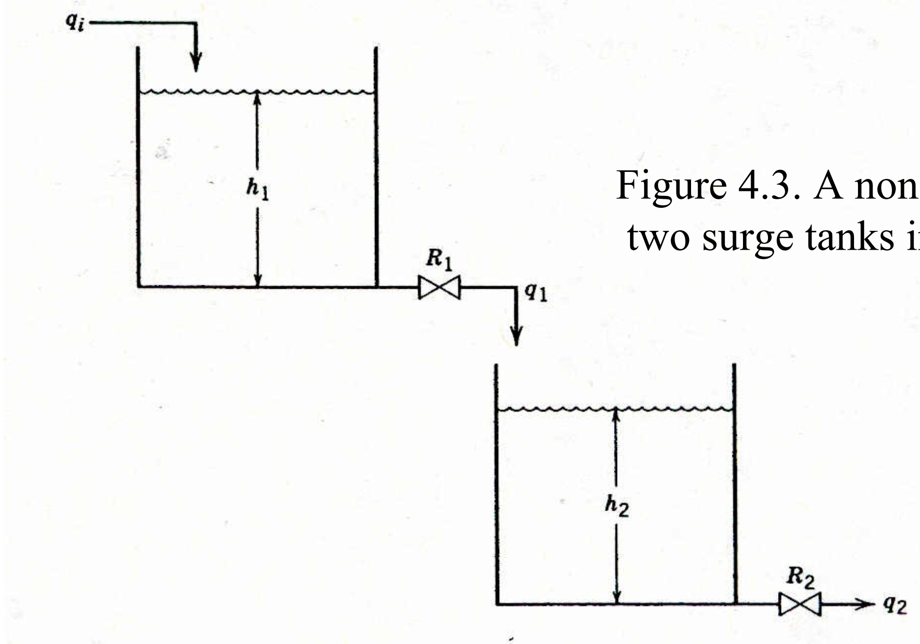

Figure 4.3. A noninteracting system: two surge tanks in series.

Mass Balance:

$$A_1 \frac{dh_1}{dt} = q_i - q_1 \tag{4-48}$$

Valve Relation:

$$q_1 = \frac{1}{R_1} h_1 \tag{4-49}$$

Substituting (4-49) into (4-48) eliminates q_1:

$$A_1 \frac{dh_1}{dt} = q_i - \frac{1}{R_1} h_1 \tag{4-50}$$

Putting (4-49) and (4-50) into deviation variable form gives

$$A_1 \frac{dh_1'}{dt} = q_i' - \frac{1}{R_1} h_1' \qquad (4\text{-}51)$$

$$q_1' = \frac{1}{R_1} h_1' \qquad (4\text{-}52)$$

The transfer function relating $H_1'(s)$ to $Q_{1i}'(s)$ is found by transforming (4-51) and rearranging to obtain

$$\frac{H_1'(s)}{Q_i'(s)} = \frac{R_1}{A_1 R_1 s + 1} = \frac{K_1}{\tau_1 s + 1} \qquad (4\text{-}53)$$

where K_1 □ R_1 and τ_1 □ $A_1 R_1$. Similarly, the transfer function relating $Q_1'(s)$ to $H_1'(s)$ is obtained by transforming (4-52).

$$\frac{Q_1'(s)}{H_1'(s)} = \frac{1}{R_1} = \frac{1}{K_1} \qquad (4\text{-}54)$$

The same procedure leads to the corresponding transfer functions for Tank 2,

$$\frac{H_2'(s)}{Q_2'(s)} = \frac{R_2}{A_2 R_2 s + 1} = \frac{K_2}{\tau_2 s + 1} \qquad (4\text{-}55)$$

$$\frac{Q_2'(s)}{H_2'(s)} = \frac{1}{R_2} = \frac{1}{K_2} \qquad (4\text{-}56)$$

where K_2 □ R_2 and τ_2 □ $A_2 R_2$. Note that the desired transfer function relating the outflow from Tank 2 to the inflow to Tank 1 can be derived by forming the product of (4-53) through (4-56).

Chapter 6

$$\frac{Q_2'(s)}{Q_i'(s)} = \frac{Q_2'(s)}{H_2'(s)} \frac{H_2'(s)}{Q_1'(s)} \frac{Q_1'(s)}{H_1'(s)} \frac{H_1'(s)}{Q_i'(s)} \qquad (4\text{-}57)$$

or

$$\frac{Q_2'(s)}{Q_i'(s)} = \frac{1}{K_2} \frac{K_2}{\tau_2 s+1} \frac{1}{K_1} \frac{K_1}{\tau_1 s+1} \qquad (4\text{-}58)$$

which can be simplified to yield

$$\frac{Q_2'(s)}{Q_i'(s)} = \frac{1}{(\tau_1 s+1)(\tau_2 s+1)} \qquad (4\text{-}59)$$

a second-order transfer function (does unity gain make sense on physical grounds?). Figure 4.4 is a block diagram showing *information flow* for this system.

Block Diagram for Noninteracting Surge Tank System

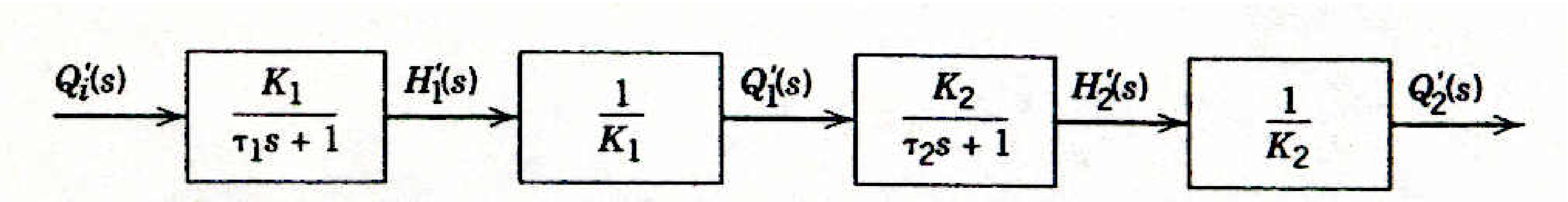

Figure 4.4. Input-output model for two liquid surge tanks in series.

Dynamic Model of An Interacting Process

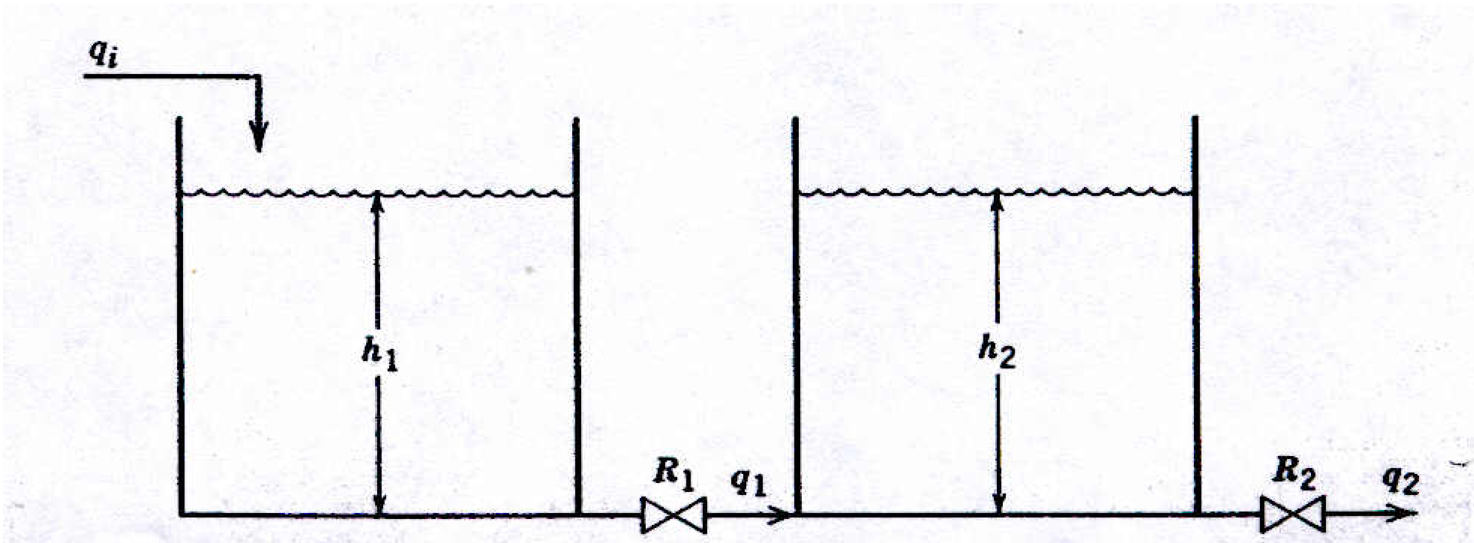

Figure 6.13. Two tanks in series whose liquid levels interact.

$$q_1 = \frac{1}{R_1}\left(h_1 - h_2\right) \qquad (6\text{-}70)$$

The transfer functions for the interacting system are:

$$\frac{H_2'(s)}{Q_i'(s)} = \frac{R_2}{\tau^2 s^2 + 2\zeta\tau s + 1} \qquad (6\text{-}74)$$

$$\frac{Q_2'(s)}{Q_i'(s)} = \frac{1}{\tau^2 s^2 + 2\zeta\tau s + 1}$$

$$\frac{H_1'(s)}{Q_i'(s)} = \frac{K_1'(\tau_a s + 1)}{\tau^2 s^2 + 2\zeta\tau s + 1} \qquad (6\text{-}72)$$

where

$$\tau = \sqrt{\tau_1 \tau_2}, \quad \zeta \,\square\, \frac{\tau_1 + \tau_2 + R_2 A_1}{2\sqrt{\tau_1 \tau_2}}, \text{ and } \tau_a \,\square\, R_1 R_2 A_2 / (R_1 + R_2)$$

In Exercise 6.15, the reader can show that $\zeta > 1$ by analyzing the denominator of (6-71); hence, the transfer function is overdamped, second order, and has a negative zero.

Model Comparison

- **Noninteracting system**

$$\frac{Q_2'(s)}{Q_i'(s)} = \frac{1}{(\tau_1 s + 1)(\tau_2 s + 1)} \qquad (4\text{-}59)$$

where $\tau_1 \triangleq A_1 R_1$ and $\tau_2 \triangleq A_2 R_2$.

- **Interacting system**

$$\frac{Q_2'(s)}{Q_i'(s)} = \frac{1}{\tau^2 s^2 + 2\zeta\tau s + 1}$$

where $\zeta > 1$ and $\tau \triangleq \sqrt{\tau_1 \tau_2}$

- **General Conclusions**

1. The interacting system has a slower response.
   (Example: consider the special case where $\tau = \tau_1 = \tau_2$.)
2. Which two-tank system provides the best damping of inlet flow disturbances?

Multiple-Input, Multiple Output (MIMO) Processes

- Most industrial process control applications involved a number of input (manipulated) and output (controlled) variables.
- These applications often are referred to as multiple-input/ multiple-output (MIMO) systems to distinguish them from the simpler single-input/single-output (SISO) systems that have been emphasized so far.
- Modeling MIMO processes is no different conceptually than modeling SISO processes.

- For example, consider the system illustrated in Fig. 6.14.
- Here the level *h* in the stirred tank and the temperature *T* are to be controlled by adjusting the flow rates of the hot and cold streams w_h and w_c, respectively.
- The temperatures of the inlet streams T_h and T_c represent potential disturbance variables.
- Note that the outlet flow rate *w* is maintained constant and the liquid properties are assumed to be constant in the following derivation.

$$\begin{bmatrix} T'(s) \\ H'(s) \end{bmatrix} = \begin{bmatrix} \dfrac{(\bar{T}_h - \bar{T})/\bar{w}}{\tau s + 1} & \dfrac{(\bar{T}_c - \bar{T})/\bar{w}}{\tau s + 1} & \dfrac{\bar{w}_h/\bar{w}}{\tau s + 1} & \dfrac{\bar{w}_c/\bar{w}}{\tau s + 1} \\ \dfrac{1/A\rho}{s} & \dfrac{1/A\rho}{s} & 0 & 0 \end{bmatrix} \begin{bmatrix} W'_h(s) \\ W'_c(s) \\ T'_h(s) \\ T'_c(s) \end{bmatrix}$$

(6-88)

Chapter 6

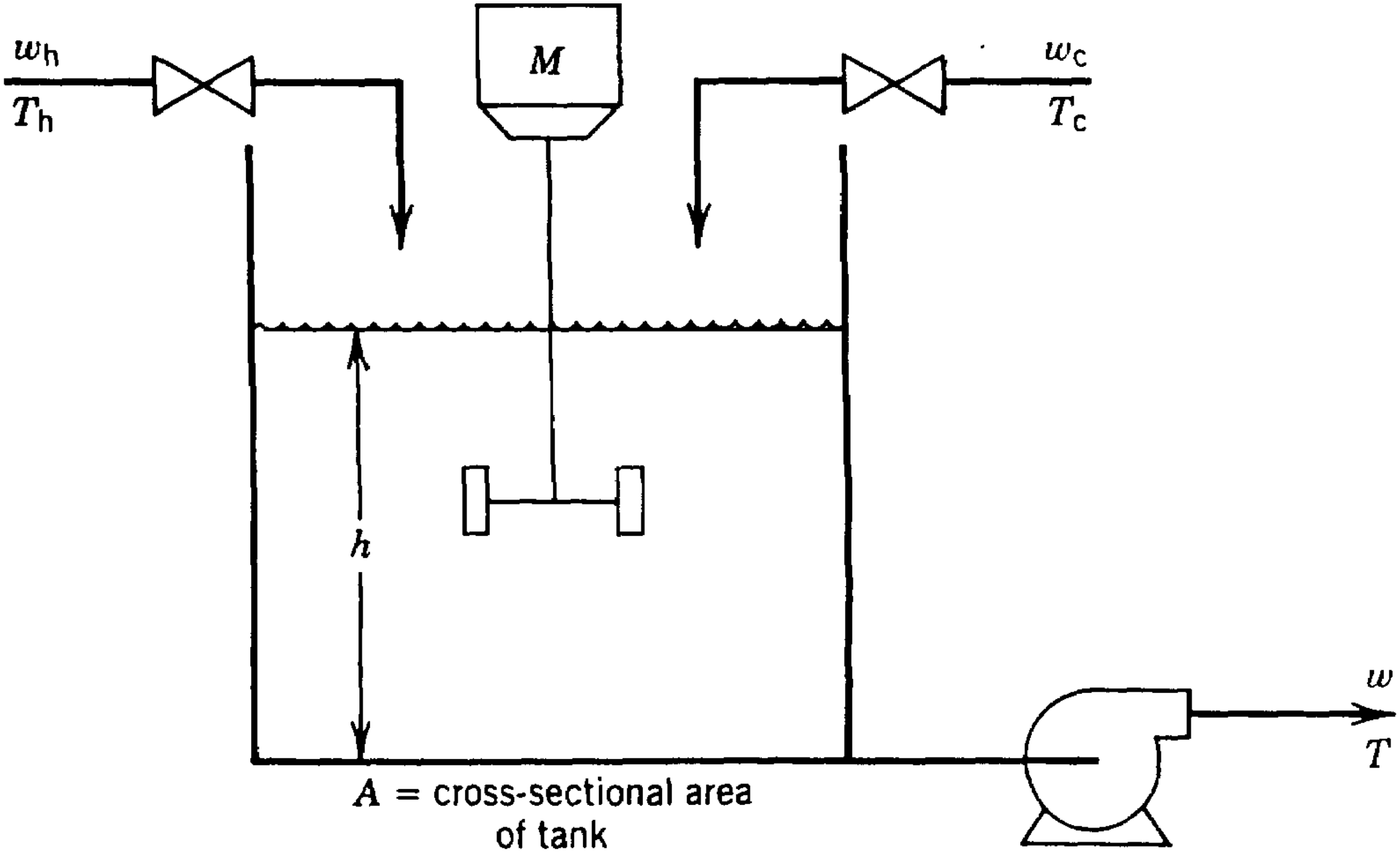

Figure 6.14. A multi-input, multi-output thermal mixing process.

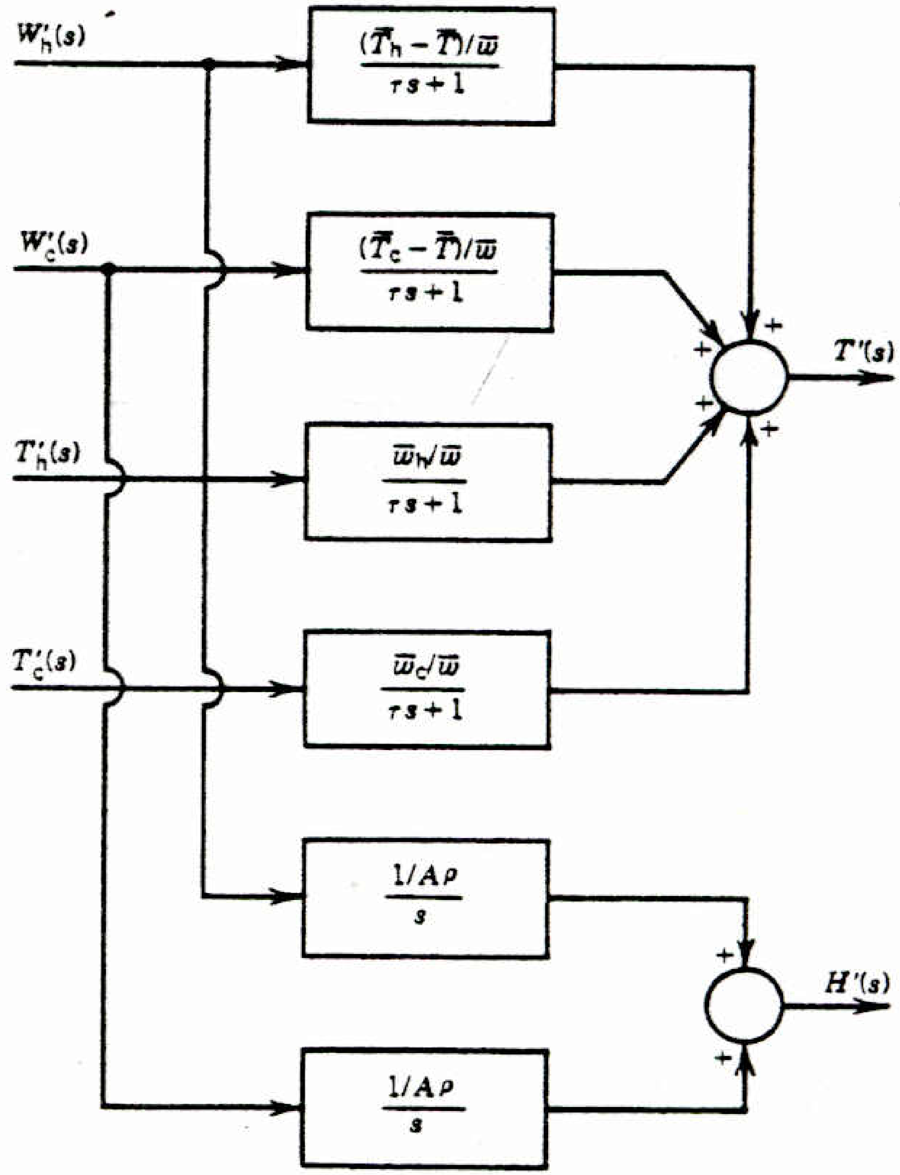

Figure 6.18. Block diagram of the MIMO mixing system with variable level.