More General Transfer Function Models

- Poles and Zeros:
- The dynamic behavior of a transfer function model can be characterized by the numerical value of its poles and zeros.

- General Representation of ATF:

There are two equivalent representations:

$$G(s) = \frac{\sum_{i=0}^{m} b_i s^i}{\sum_{i=0}^{n} a_i s^i} \qquad (4\text{-}40)$$

$$G(s) = \frac{b_m(s - z_1)(s - z_2)\ldots(s - z_m)}{a_n(s - p_1)(s - p_2)\ldots(s - p_n)} \qquad (6\text{-}7)$$

where $\{z_i\}$ are the "zeros" and $\{p_i\}$ are the "poles".

- We will assume that there are no "pole-zero" calculations. That is, that no pole has the same numerical value as a zero.

- *Review:* $n \geq m$ in order to have a physically realizable system.

Example 6.2

For the case of a single zero in an overdamped second-order transfer function,

$$G(s) = \frac{K(\tau_a s + 1)}{(\tau_1 s + 1)(\tau_2 s + 1)} \qquad (6\text{-}14)$$

calculate the response to the step input of magnitude *M* and plot the results qualitatively.

Solution

The response of this system to a step change in input is

$$y(t) = KM\left(1 + \frac{\tau_a - \tau_1}{\tau_1 - \tau_2} e^{-t/\tau_1} + \frac{\tau_a - \tau_2}{\tau_2 - \tau_1} e^{-t/\tau_2}\right) \qquad (6\text{-}15)$$

Note that $y(t \to \infty) = KM$ as expected; hence, the effect of including the single zero does not change the final value nor does it change the number or location of the response modes. But the zero does affect how the response modes (exponential terms) are weighted in the solution, Eq. 6-15.

A certain amount of mathematical analysis (see Exercises 6.4, 6.5, and 6.6) will show that there are three types of responses involved here:

Case a: $\tau_a > \tau_1$

Case b: $0 < \tau_a \leq \tau_1$

Case c: $\tau_a < 0$

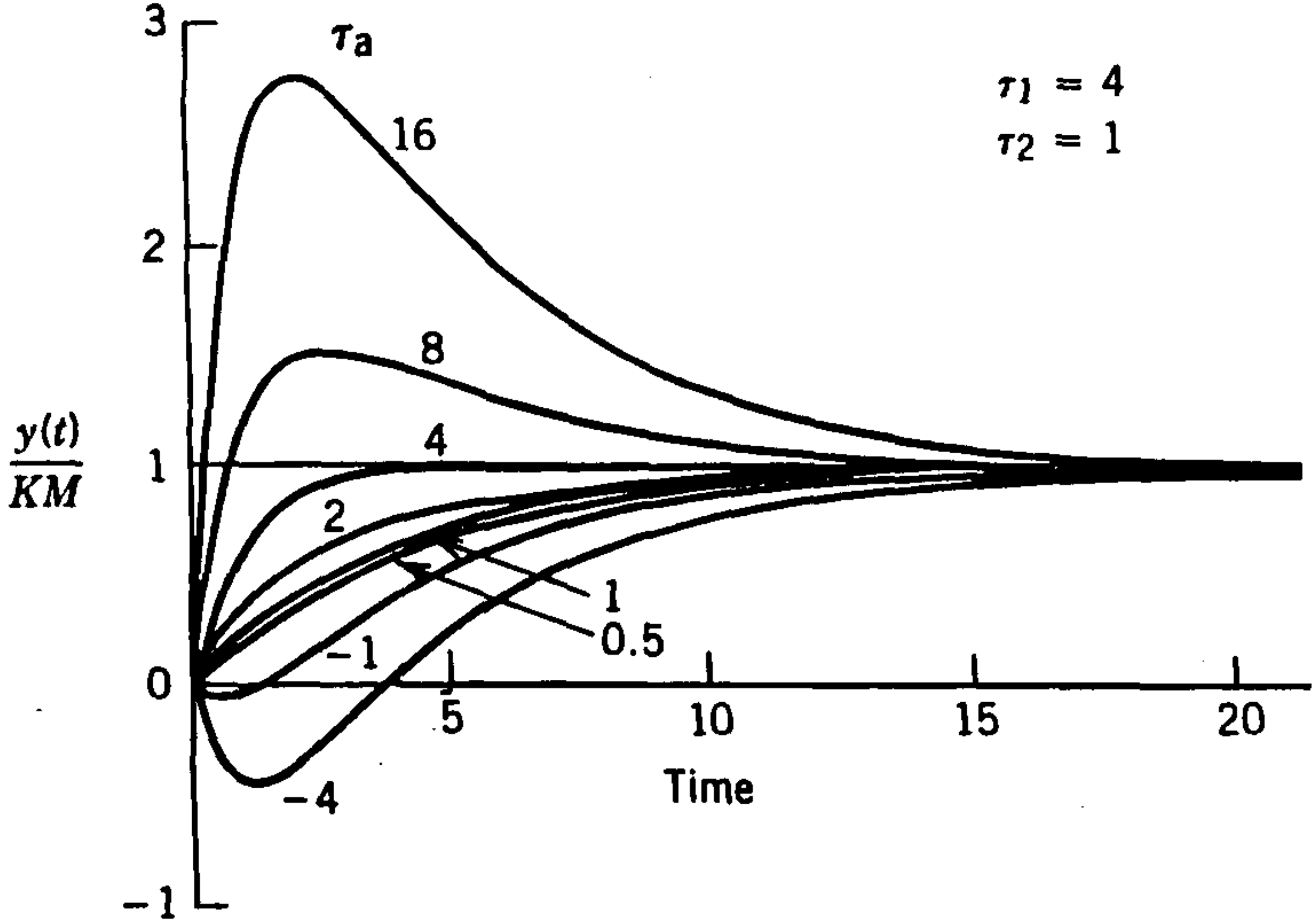


Figure 6.3. Step response of an overdamped second-order system (Eq. 6-14) with a single zero.

Reboiler duty change in distillation column.

① increase in tray froth → increase of liquid spill off
(fast dynamics)
Increase base level ←

② evaporation of liquid in the base → decrease base level
(slow dynamics)

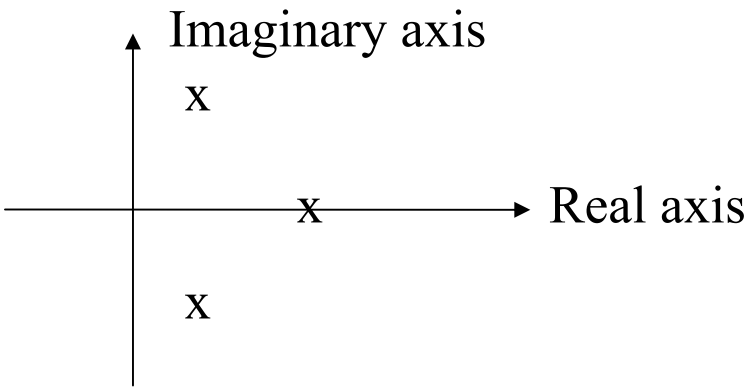
Summary: Effects of Pole and Zero Locations

1. Poles

- *Pole in "right half plane (RHP)"*: results in unstable system (i.e., unstable step responses)

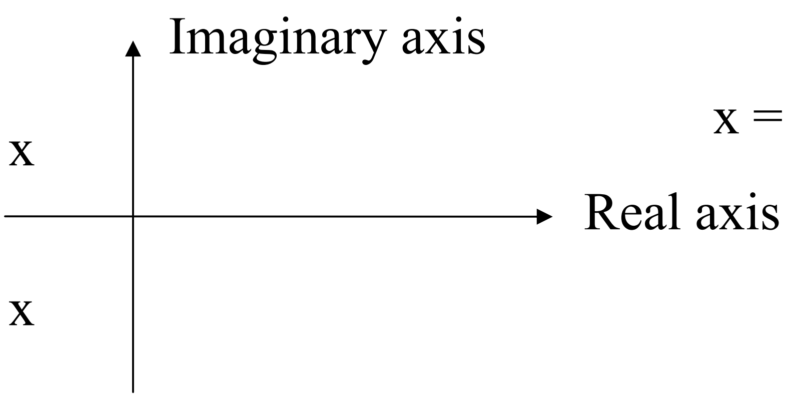
$$p = a + bj$$

$$\left(j = \sqrt{-1} \right)$$



x = unstable pole

- *Complex pole*: results in oscillatory responses



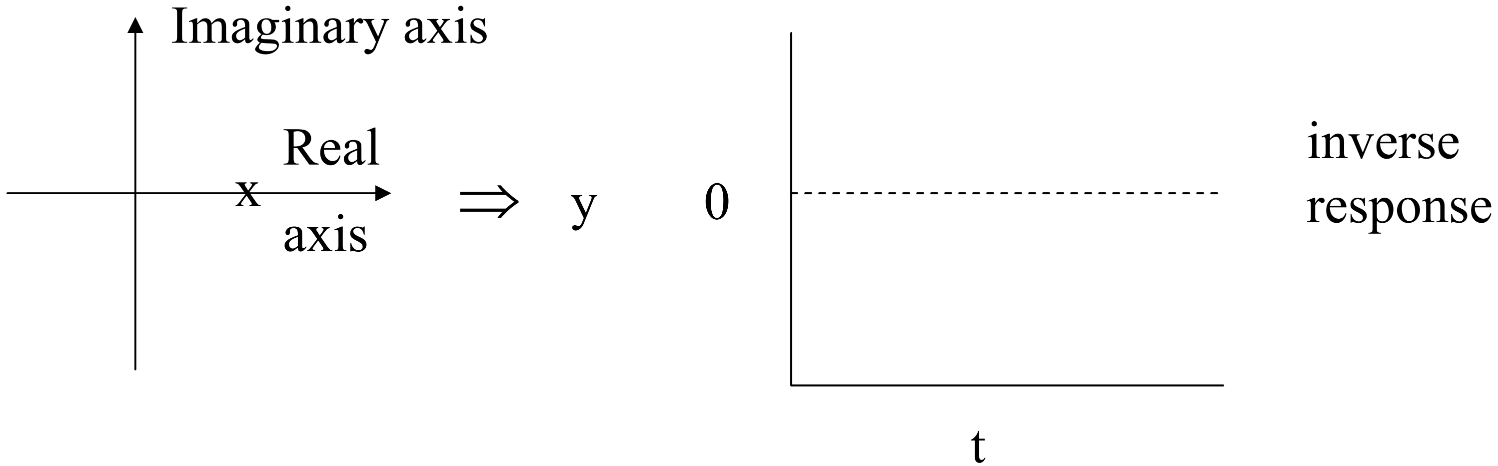
x = complex poles

- Pole at the origin (1/s term in TF model): results in an "integrating process"

2. Zeros

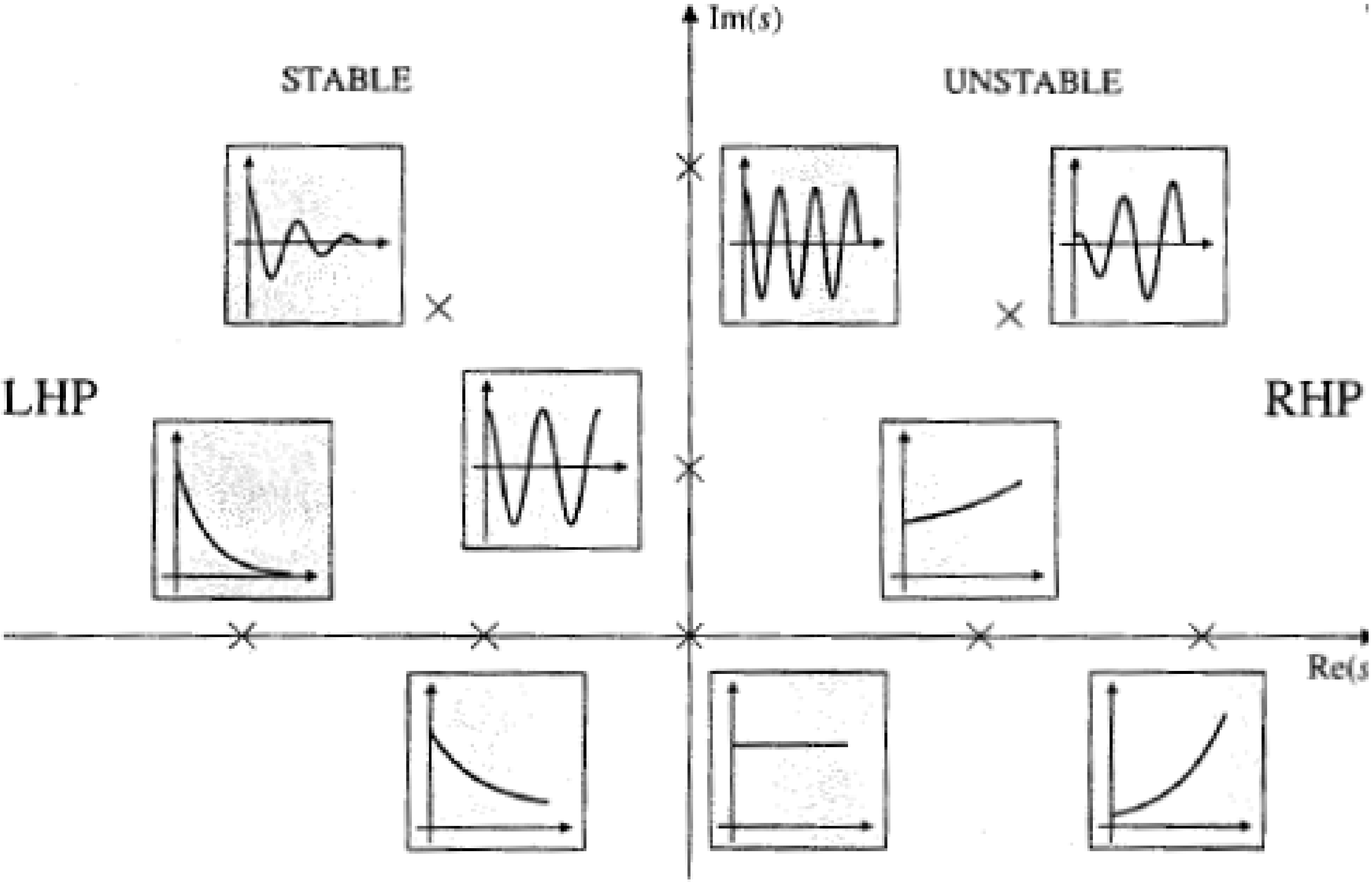
Note: Zeros have no effect on system stability.

- *Zero in RHP:* results in an inverse response to a step change in the input



- *Zero in left half plane:* may result in "overshoot" during a step response (see Fig. 6.3).

Chapter 6


Im(s)
STABLE
UNSTABLE
LHP
RHP
Re(s

Inverse Response Due to Two Competing Effects

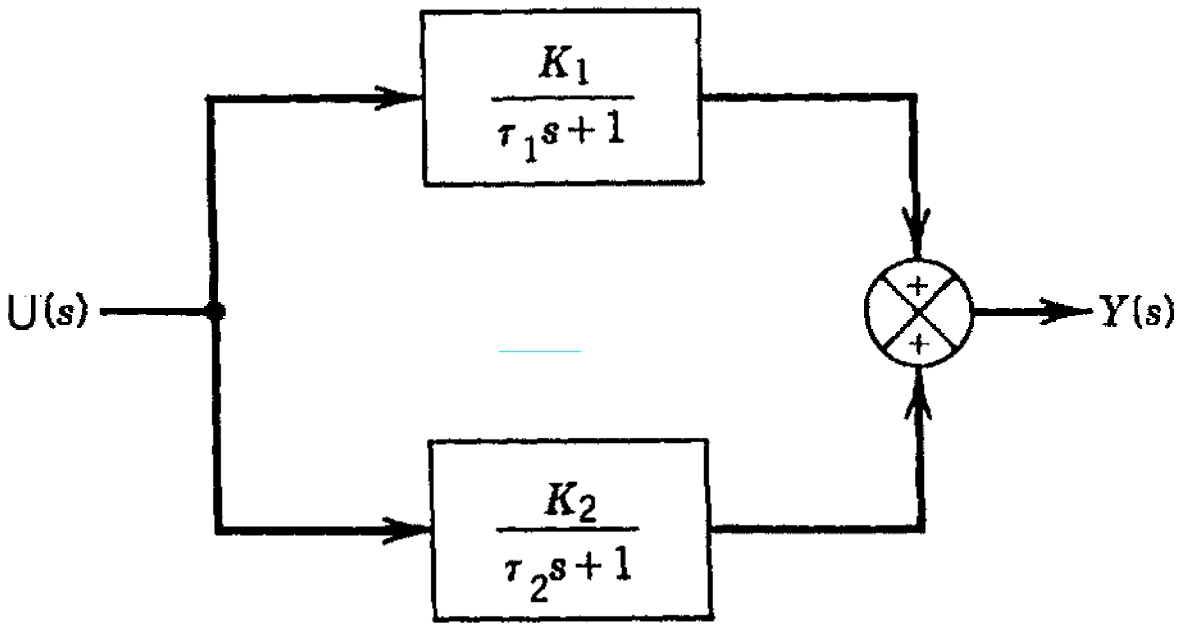


Figure 6.4. Two first-order process elements acting in parallel.

An inverse response occurs if:

$$-\frac{K_2}{K_1} > \frac{\tau_2}{\tau_1} \qquad (6\text{-}22)$$

Ex 6.3 $G(s) = \dfrac{-0.5s+1}{(4s+1)(s+1)}$

this can be split into 2 first order systems by Eq (6-18 & 6-20) ($K=1$; $\tau_1=4$; $\tau_2=1$; $\tau_a=-0.5$)

$$G(s) = \underbrace{\frac{1.5}{4s+1}}_{\text{slow}} + \underbrace{\frac{-0.5}{s+1}}_{\text{fast}}$$

Slow & +ve gain } work in parallel
fast & -ve gain }