Dynamic Behavior

In analyzing process dynamic and process control systems, it is important to know how the process responds to changes in the process inputs.

A number of standard types of input changes are widely used for two reasons:

1. They are representative of the types of changes that occur in plants.

2. They are easy to analyze mathematically.

1. Step Input

A sudden change in a process variable can be approximated by a step change of magnitude, *M*:

$$U_s \triangleq \begin{cases} 0 & t < 0 \\ M & t \geq 0 \end{cases} \qquad (5\text{-}4)$$

The step change occurs at an arbitrary time denoted as $t = 0$.

- *Special Case:* If $M = 1$, we have a "unit step change". We give it the symbol, $S(t)$.

- *Example of a step change:* A reactor feedstock is suddenly switched from one supply to another, causing sudden changes in feed concentration, flow, etc.

Example:

The heat input to the stirred-tank heating system in Chapter 2 is suddenly changed from 8000 to 10,000 kcal/hr by changing the electrical signal to the heater. Thus,

$$Q(t) = 8000 + 2000S(t), \qquad S(t) \,\square\, \text{unit step}$$

and

$$Q'(t) = Q - \bar{Q} = 2000S(t), \qquad \bar{Q} = 8000 \text{ kcal/hr}$$

2. Ramp Input

- Industrial processes often experience “drifting disturbances”, that is, relatively slow changes up or down for some period of time.
- The rate of change is approximately constant.

We can approximate a drifting disturbance by a *ramp input:*

$$U_R(t) \triangleq \begin{cases} 0 & t < 0 \\ at & t \geq 0 \end{cases} \qquad (5\text{-}7)$$

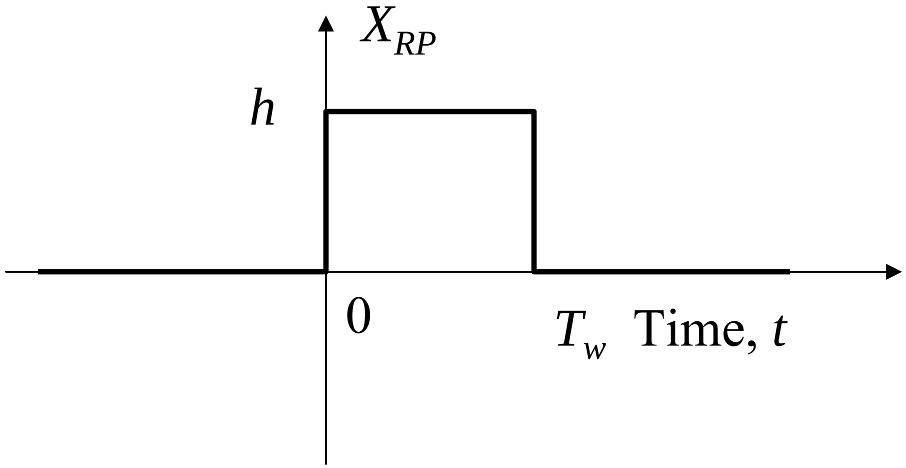
Examples of ramp changes:

1. Ramp a setpoint to a new value. (Why not make a step change?)
2. Feed composition, heat exchanger fouling, catalyst activity, ambient temperature.

3. Rectangular Pulse

It represents a brief, sudden change in a process variable:

$$U_{RP}(t) \triangleq \begin{cases} 0 & \text{for } t < 0 \\ h & \text{for } 0 \le t < t_w \\ 0 & \text{for } t \ge t_w \end{cases} \qquad (5\text{-}9)$$



Examples:

1. Reactor feed is shut off for one hour.
2. The fuel gas supply to a furnace is briefly interrupted.

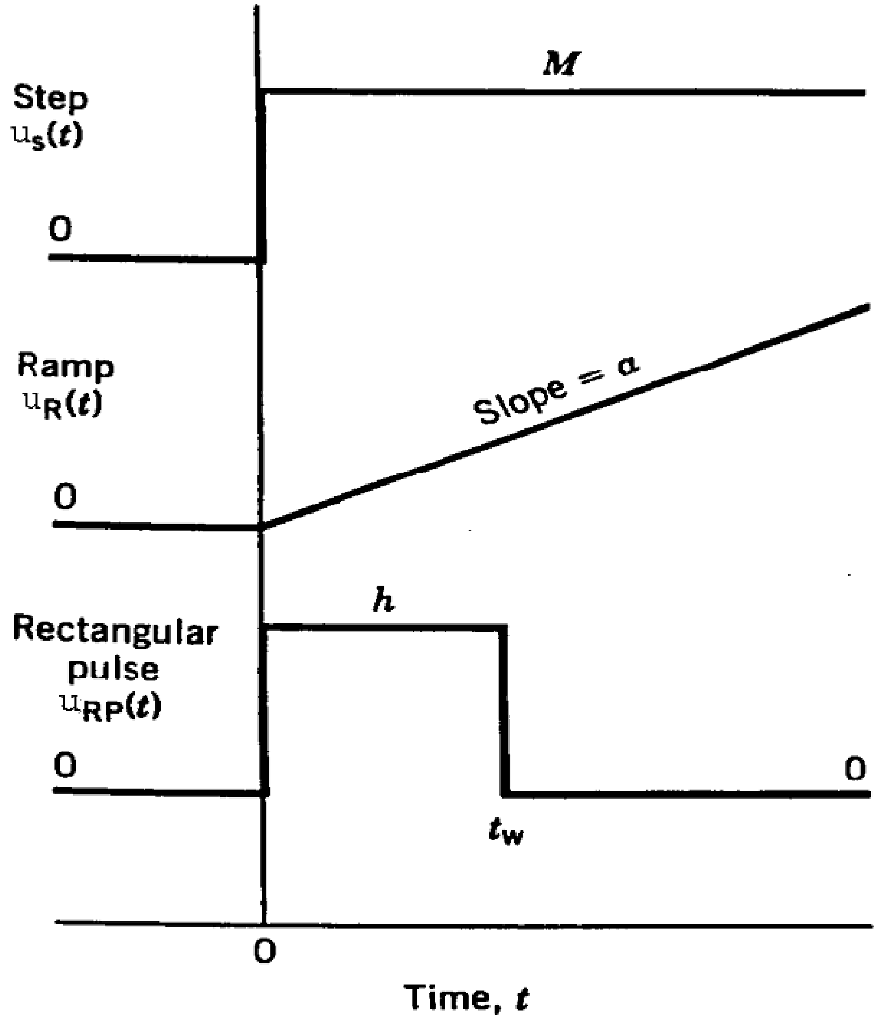


Figure 5.2. Three important examples of deterministic inputs.

4. Sinusoidal Input

Processes are also subject to periodic, or cyclic, disturbances. They can be approximated by a sinusoidal disturbance:

$$U_{\sin}(t) \triangleq \begin{cases} 0 & \text{for} \quad t < 0 \\ A\sin(\omega t) & \text{for} \quad t \geq 0 \end{cases} \qquad (5\text{-}14)$$

where: A = amplitude, ω = angular frequency

Examples:

1. 24 hour variations in cooling water temperature.
2. 60-Hz electrical noise (in the USA)

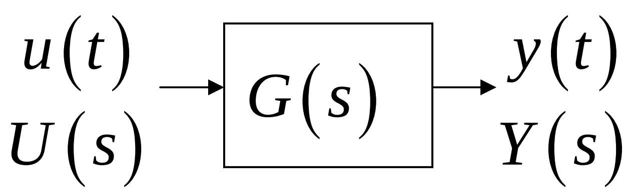
5. Impulse Input

- Here, $U_I(t) = \delta(t)$.
- It represents a short, transient disturbance.

Examples:

1. Electrical noise spike in a thermo-couple reading.
2. Injection of a tracer dye.

- Useful for analysis since the response to an impulse input is the inverse of the TF. Thus,



Here,

$$Y(s) = G(s)U(s) \qquad (1)$$

The corresponding time domain express is:

$$y(t) = \int_0^t g(t-\tau)u(\tau)d\tau \qquad (2)$$

where:

$$g(t) \triangleq \mathcal{L}^{-1}\left[G(s)\right] \qquad (3)$$

Suppose $u(t) = \delta(t)$. Then it can be shown that:

$$y(t) = g(t) \qquad (4)$$

Consequently, $g(t)$ is called the "impulse response function".

First-Order System

The standard form for a first-order TF is:

$$\frac{Y(s)}{U(s)} = \frac{K}{\tau s + 1} \qquad (5\text{-}16)$$

where:

K □ steady-state gain

τ □ time constant

Consider the response of this system to a step of magnitude, *M*:

$$U(t) = M \text{ for } t \geq 0 \qquad \Rightarrow U(s) = \frac{M}{s}$$

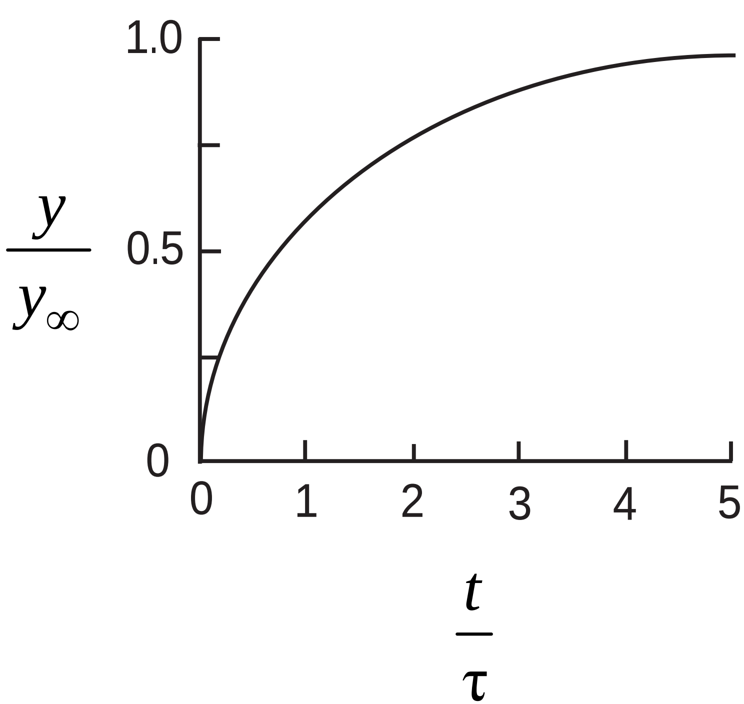
Substitute into (5-16) and rearrange,

$$Y(s) = \frac{KM}{s(\tau s + 1)} \qquad (5\text{-}17)$$

Take L^{-1} (cf. Table 3.1),

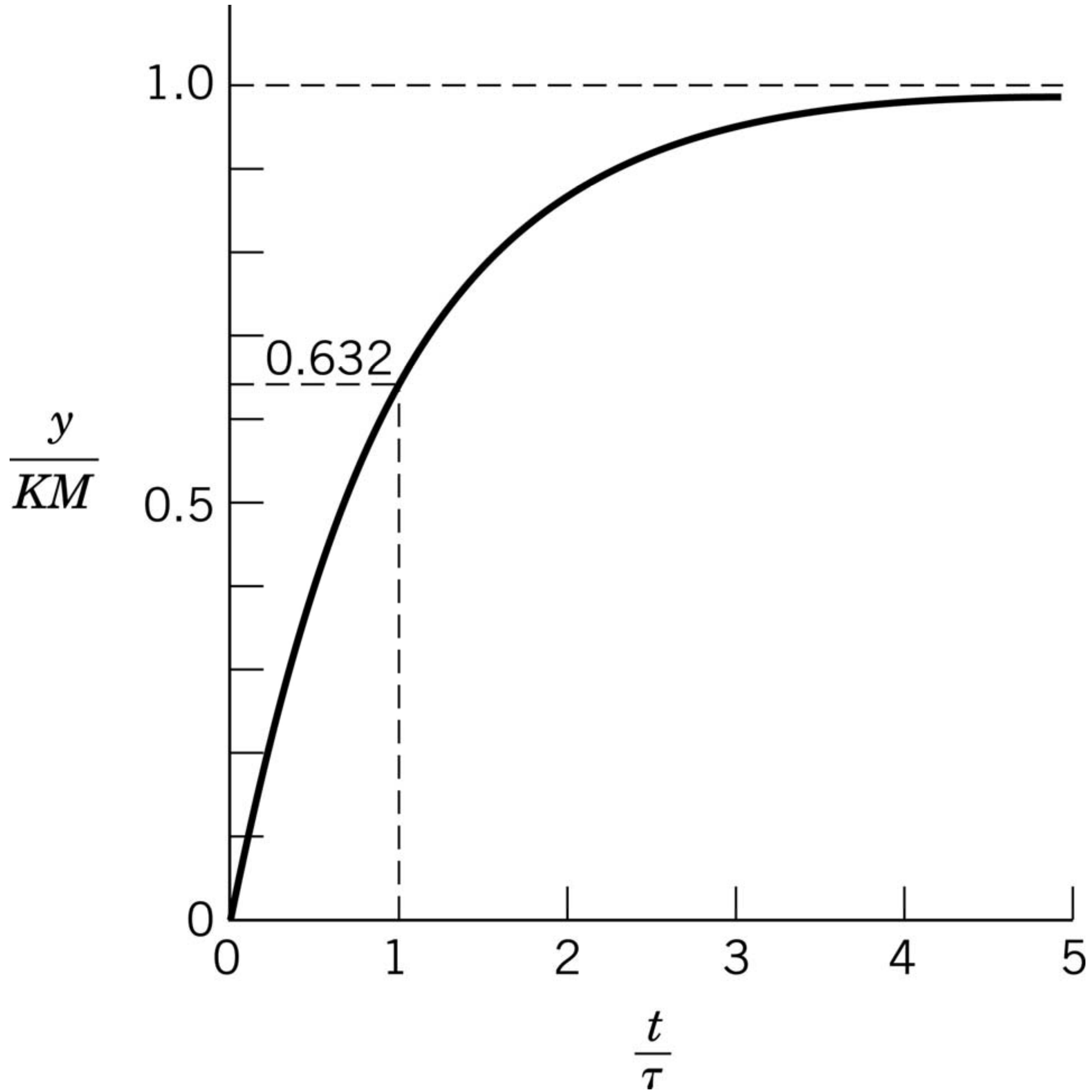
$$\boxed{y(t) = KM\left(1 - e^{-t/\tau}\right)} \tag{5-18}$$

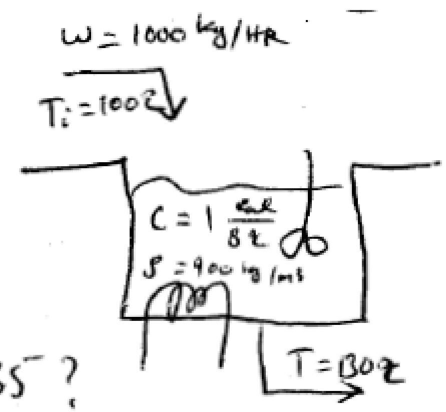
Let $y_\infty \triangleq$ steady-state value of $y(t)$. From (5-18), $y_\infty = KM$.



t	$\frac{y}{y_\infty}$
0	0
τ	0.632
2τ	0.865
3τ	0.950
4τ	0.982
5τ	0.993

Note: Large τ means a slow response.


1.0
0.632
$\frac{y}{KM}$
0.5
0
0
1
2
3
4
5
$\frac{t}{\tau}$



Ex

① Steady state Q ?

② +30% of Q, how long to get to SS ?

③ +20°C of T_i ; how long for T to reach 135 ?

① $\bar{Q} = \bar{w}c(\bar{T} - \bar{T}_i) = 3 \times 10^7 \frac{cal}{HR}$

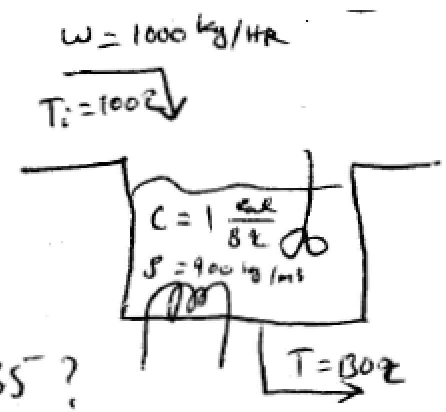
② $K = \frac{1}{wc} = 10^{-6} \frac{°C}{cal/HR}$ & $\tau = \frac{V\rho}{w} = 1.8$ hr

to reach SS it takes $5\tau = 9$ hr

also, F.V.T. $\lim_{s \to 0} s \frac{K}{\tau s + 1} \frac{0.3\bar{Q}}{s} = 9°C$

⇒ final T = 130 + 9 = 139°C after +30% in Q

Or by looking into gain: +30% in Q → +0.3Q K in out



Ex

① Steady state Q ?

② +30% of Q, how long to get to SS ?

③ +20°C of T_i ; how long for T to reach 135 ?

③ by looking to gain : final T is 150

$$\Rightarrow \frac{135-130}{150-130} = \frac{5}{20} = 25\%$$

from fig 5.4 $\rightarrow \frac{t}{\tau} = 0.3 \Rightarrow t = 0.54$ hr

OR $\frac{y(t)}{KM} = 1 - e^{-t/\tau} \rightarrow \frac{5}{(1)(20)} = 1 - e^{-t/1.8}$

$\Rightarrow t = 0.52$ hr