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$$V\rho C\left[sT'(s)-T'(t=0)\right]=wC\left[T_i'(s)-T'(s)\right]-Q'(s) \quad (8)$$

$$T'(s)=\left(\frac{K}{\tau s+1}\right)Q'(s)+\left(\frac{1}{\tau s+1}\right)T_i'(s) \quad (10)$$

$$K \triangleq \frac{1}{wC} \text{ and } \tau \triangleq \frac{V\rho}{w} \quad (11)$$

$$T'(s)=G_1(s)Q'(s)+G_2(s)T_i'(s)$$

G_1 and G_2 are transfer functions and independent of the inputs, Q' and T_i'.

Note G_1 (process) has gain K and time constant τ.

G_2 (disturbance) has gain=1 and time constant τ.
gain = G(s=0). Both are first order processes.

If there is no change in inlet temperature ($T_i' = 0$), then $T_i'(s) = 0$.

System can be forced by a change in either T_i or Q (see Example 4.1).

Transfer Function Between Q' and T'

Suppose T_i is constant at the steady-state value. Then,

$T_i(t) = \bar{T}_i \Rightarrow T_i'(t) = 0 \Rightarrow T_i'(s) = 0.$ Then we can substitute into (10) and rearrange to get the desired TF:

$$\boxed{\frac{T'(s)}{Q'(s)} = \frac{K}{\tau s + 1}} \qquad (12)$$

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Transfer Function Between T' and T_i':

Suppose that Q is constant at its steady-state value:

$$Q(t) = \bar{Q} \Rightarrow Q'(t) = 0 \Rightarrow Q'(s) = 0$$

Thus, rearranging

$$\boxed{\frac{T'(s)}{T_i'(s)} = \frac{1}{\tau s + 1}} \qquad (13)$$

Comments:

1. The TFs in (12) and (13) show the *individual* effects of Q and T_i on T. What about *simultaneous* changes in both Q and T_i ?

   - Answer: See (10). The same TFs are valid for simultaneous changes.
   - Note that (10) shows that the effects of changes in both Q and T_i are *additive*. This always occurs for linear, dynamic models (like TFs) because the Principle of Superposition is valid.

2. The TF model enables us to determine the output response to any change in an input.

3. Use deviation variables to eliminate initial conditions for TF models.

Example: Stirred Tank Heater

$$K = 0.05 \qquad \tau = 2.0$$

$$T' = \frac{0.05}{2s+1} Q'$$

No change in T_i'

Q(0)= 1000cal/sec. Then, step change in Q(t):1500 cal/sec

$$Q' = \frac{500}{s}$$

$$T' = \frac{0.05}{2s+1}\frac{500}{s} = \frac{25}{s(2s+1)}$$

What is T′(t)?

$$T'(t) = 25[1 - e^{-t/\tau}] \longleftarrow T(s) = \frac{25}{s(\tau s+1)}$$

$$T'(t) = 25[1 - e^{-t/2}]$$

Solve Example 4.1

and

Solve Example 4.2

if you have any question ask me !

Properties of Transfer Function Models

1. Steady-State Gain

The steady-state of a TF can be used to calculate the steady-state change in an output due to a steady-state change in the input. For example, suppose we know two steady states for an input, *u*, and an output, *y*. Then we can calculate the steady-state gain, *K*, from:

$$K = \frac{\bar{y}_2 - \bar{y}_1}{\bar{u}_2 - \bar{u}_1} \qquad (4\text{-}38)$$

For a linear system, *K* is a constant. But for a nonlinear system, *K* will depend on the operating condition $(\bar{u}, \bar{y})$.

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Calculation of *K* from the TF Model:

If a TF model has a steady-state gain, then:

$$\boxed{K = \lim_{s \to 0} G(s)} \qquad (14)$$

- This important result is a consequence of the Final Value Theorem

- *Note*: Some TF models do *not* have a steady-state gain (e.g., integrating process in Ch. 5)

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2. Order of a TF Model

Consider a general n-th order, linear ODE:

$$a_n \frac{d^n y}{dt^n} + a_{n-1} \frac{dy^{n-1}}{dt^{n-1}} + \ldots a_1 \frac{dy}{dt} + a_0 y = b_m \frac{d^m u}{dt^m} +$$

$$b_{m-1} \frac{d^{m-1} u}{dt^{m-1}} + \ldots + b_1 \frac{du}{dt} + b_0 u \qquad (4\text{-}39)$$

Take L, assuming the initial conditions are all zero. Rearranging gives the TF:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{\sum_{i=0}^{m} b_i s^i}{\sum_{i=0}^{n} a_i s^i} \qquad (4\text{-}40)$$

Definition:

The order of the TF is defined to be the order of the denominator polynomial.

Note: The order of the TF is equal to the order of the ODE.

Physical Realizability:

For any physical system, $n \geq m$ in (4-38). Otherwise, the system response to a step input will be an impulse. This can't happen.

Example:

$$a_0 y = b_1 \frac{du}{dt} + b_0 u \quad \text{and step change in } u \qquad (4\text{-}41)$$

n=1 > m=0

Impulse response to step change. Physically impossible.

2nd order process

General 2nd order ODE:

$$a\frac{d^2y}{dt^2}+b\frac{dy}{dt}+y=Ku$$

Laplace Transform: $\left[as^2+bs+1\right]\cdot Y(s)=KU(s)$

$$G(s)=\frac{Y(s)}{U(s)}=\frac{K}{as^2+bs+1}$$

2 roots $s_{1,2}=\dfrac{-b\pm\sqrt{b^2-4a}}{2a}$

$\dfrac{b^2}{4a}>1$: real roots

$\dfrac{b^2}{4a}<1$: imaginary roots

Examples

1. $$\frac{2}{3s^2+4s+1} \qquad \frac{b^2}{4a}=\frac{16}{12}=1.333>1$$

$$3s^2+4s+1=(3s+1)(s+1)=3(s+{}^1\!/_3)(s+1)$$

$$transforms\ to\ e^{-t/3}, e^{-t}\ \ (real\ \ roots)$$

(no oscillation)

2. $$\frac{2}{s^2+s+1} \qquad \frac{b^2}{4a}=\frac{1}{4}<1$$

$$e^{-bt}\sin\omega t \overset{L}{\longleftrightarrow} \frac{\omega}{(s+b)^2+\omega^2}$$

$$\frac{2}{s^2+s+1}=\frac{2}{(s+0.5)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$s^2+s+1=(s+0.5+\frac{\sqrt{3}}{2}j)(s+0.5-\frac{\sqrt{3}}{2}j)$$

$$transforms\ to\ e^{-0.5t}\cos\frac{\sqrt{3}}{2}t,\ e^{-0.5t}\sin\frac{\sqrt{3}}{2}t$$

(oscillation)

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3. Additive Property

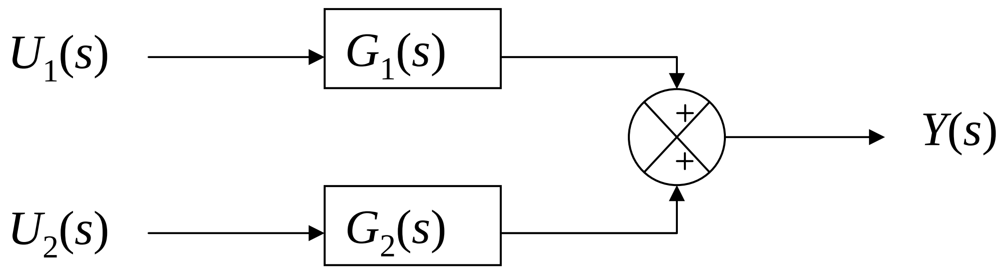
Suppose that an output is influenced by two inputs and that the transfer functions are known:

$$\frac{Y(s)}{U_1(s)} = G_1(s) \quad \text{and} \quad \frac{Y(s)}{U_2(s)} = G_2(s)$$

Then the response to changes in both U_1 and U_2 can be written as:

$$Y(s) = G_1(s)U_1(s) + G_2(s)U_2(s)$$

The graphical representation (or *block diagram*) is:



4. Multiplicative Property

Suppose that,

$$\frac{Y(s)}{U_2(s)} = G_2(s) \text{ and } \frac{U_2(s)}{U_3(s)} = G_3(s)$$

Then,

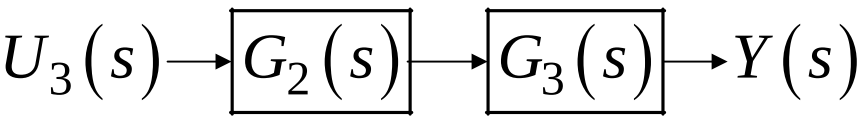
$$Y(s) = G_2(s)U_2(s) \; and \; U_2(s) = G_3(s)U_3(s)$$

Substitute,

$$Y(s) = G_2(s)G_3(s)U_3(s)$$

Or,

$$\frac{Y(s)}{U_3(s)} = G_2(s)G_3(s)$$



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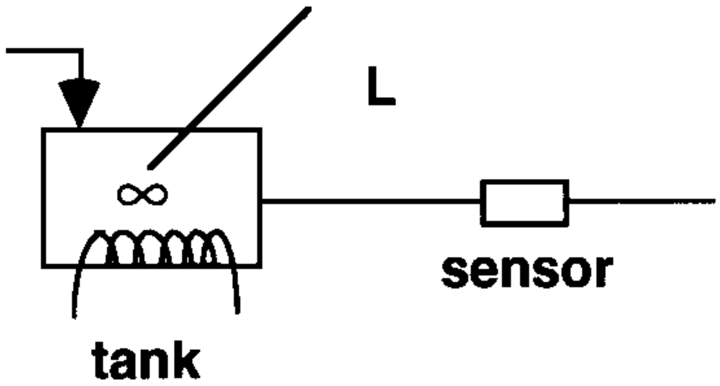
Example 1:

Place sensor for temperature downstream from heated tank (transport lag)



Distance L for plug flow,

Dead time $\theta = \dfrac{L}{V}$

V = fluid velocity

Tank: $G_1 = \dfrac{T(s)}{U(s)} = \dfrac{K_1}{1+\tau_1 s}$

Sensor: $G_2 = \dfrac{T_s(s)}{T(s)} = \dfrac{K_2 e^{-\theta s}}{1+\tau_2 s}$ $\quad K_2 \leq 1,$ $\quad \tau_2$ is very small (neglect)

Overall transfer function:

$$\frac{T_s}{U} = \frac{T_s}{T} \cdot \frac{T}{U} = G_2 \cdot G_1 = \frac{K_1 K_2 e^{-\theta s}}{1+\tau_1 s}$$

Solve Example 4.3

and

Solve Example 4.4

if you have any question
ask me !

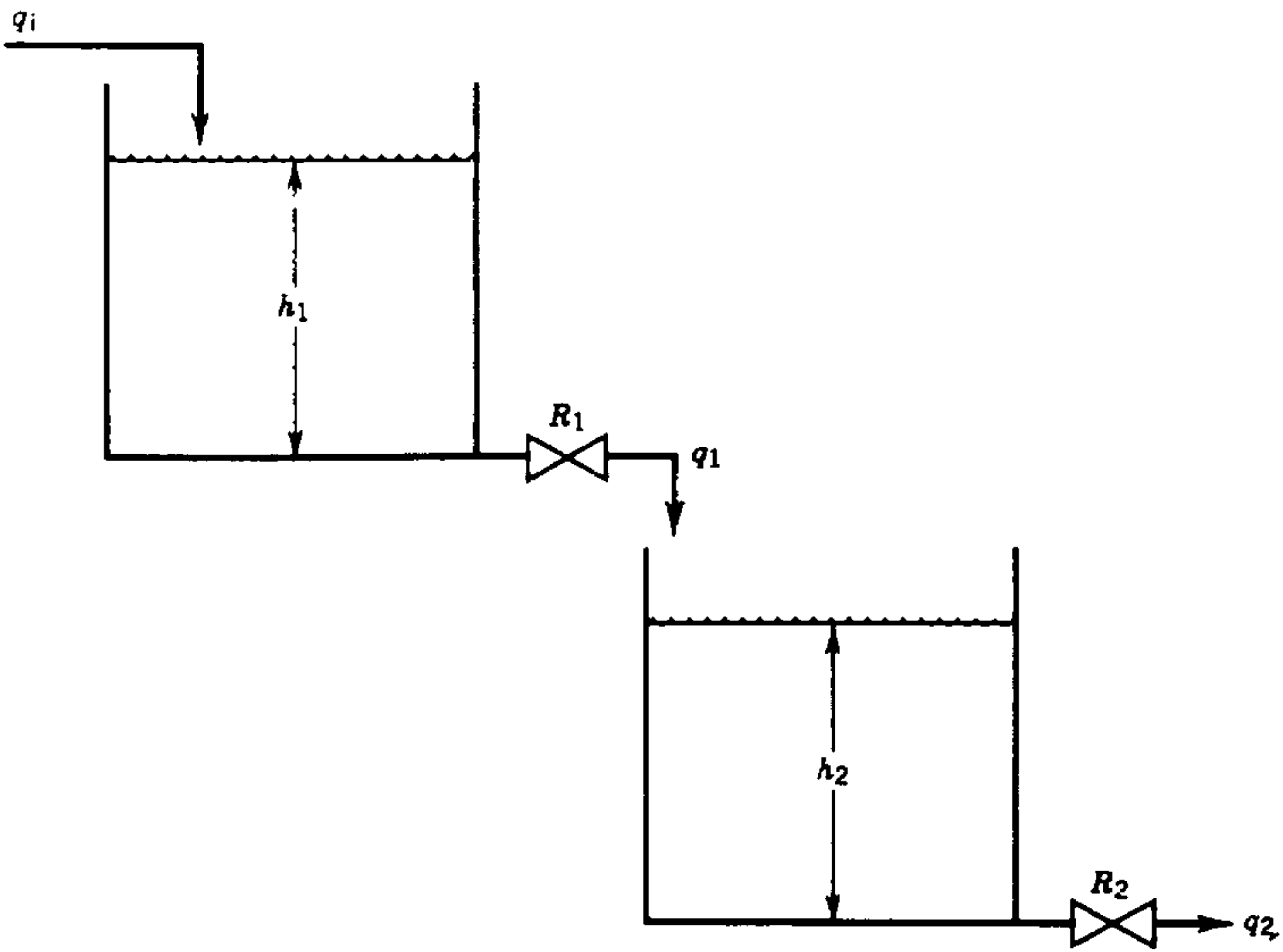


Figure 4.3. Schematic diagram of two liquid surge tanks in series.

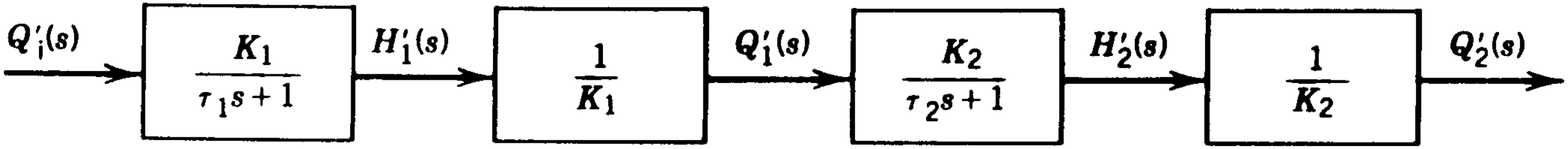


Figure 4.4. Input–output model for two liquid surge tanks in series.