4.16-1

Dimensional Analysis for Ship Model

$$F = \psi(\rho, \mu, g, L, u)$$

$$\Psi(F, \rho, \mu, g, L, u) = 0$$

Choose ρ, u, L as primary variables. Dimensionless groups for the remaining variables F, μ, g are then:

$$\Pi_1 = \frac{F}{\rho^a u^b L^c} \; [=] \; \frac{ML/T^2}{\left(\frac{M}{L^3}\right)^a \left(\frac{L}{t}\right)^b L^c}$$

Equate powers of M, L, T

$$\left.\begin{aligned} M&: \quad 1 = a \\ L&: \quad 1 = -3a + b + c \\ T&: \quad -2 = -b \end{aligned}\right\}$$

Hence $a = 1$, $b = 2$, $c = 2$

$$\Pi_1 = \frac{F}{\rho u^2 L^2}$$

Likewise, $\Pi_2 = \dfrac{\mu}{\rho^a u^b L^c} = \dfrac{\mu}{\rho u L}$

$$\Pi_3 = \frac{g}{\rho^a u^b L^c} = \frac{gL}{u^2}$$

Buckingham "Pi" Theorem

$$\psi(\Pi_1, \Pi_2, \Pi_3) = 0$$

$$\frac{F}{\rho u^2 L^2} = \psi\left(\underbrace{\frac{\rho u L}{\mu}}_{\substack{Re \\ \text{Reynolds No.}}}, \underbrace{\frac{u^2}{gL}}_{\substack{Fr \\ \text{Froude No.}}}\right)$$

To preserve dynamical similarity, we must tow the model with the same Re and Fr as for the full-size ship. Within practical constraints, this is impossible, e.g.

	Full-Size	Model
u	20 ft/sec	? u_{model}
L	500 ft	10 ft (for example)

Since water is the only reasonable medium in both cases, ρ and μ are irrelevant.

To maintain same Re

$$20 \times 500 = u_{model} \times 10 \quad \text{or} \quad u_{model} = 1000 \ \frac{\text{ft}}{\text{sec}} \ (!)$$

4.18

Dimensional Analysis for Disk Torque

$$T = \Psi(\rho, \mu, \omega, D)$$

$$\Psi(T, \rho, \mu, \omega, D)$$

Choose D, ρ, ω as primary variables. Dimensionless groups for T and μ are:

$$\Pi_1 = \frac{T}{D^a \rho^b \omega^c} = \frac{ML^2/T^2}{L^a \left(\frac{M}{L^3}\right)^b \left(\frac{1}{T}\right)^c}$$

Equate powers of L, M, T

$$\left.\begin{aligned} L:&\quad 2 = a - 3b \\ M:&\quad 1 = b \\ T:&\quad -2 = -c \end{aligned}\right\}$$

Hence $a = 5,\ b = 1,\ c = 2$

$$\Pi_1 = \frac{T}{\rho \omega^2 D^5}$$

Likewise,

$$\Pi_2 = \frac{\mu}{D^a \rho^b \omega^c} = \frac{\mu}{D^2 \rho \omega}$$

more conventional this way round (Re)

Hence, plot

$$\frac{T}{\rho \omega^2 D^5} \quad \text{versus} \quad \frac{\rho D^2 \omega}{\mu}$$

Dimensional Analysis of Centrifugal Pumps

$$\Psi(Q, P, \rho, N, D) = 0$$

$$Q \to \frac{L^3}{T}, \quad P \to \frac{ML^2}{T^3}, \quad \rho \to \frac{M}{L^3}, \quad N \to \frac{1}{T}, \quad D \to L$$

Power $\quad \Pi_1 = \dfrac{P}{\rho^a N^b D^c} \; [=] \; \dfrac{M L^2 T^{-3}}{\left(\frac{M}{L^3}\right)^a \left(\frac{1}{T}\right)^b (L)^c}$

$$\left.\begin{aligned} M:&\quad 1 = a \\ L:&\quad 2 = -3a + c \\ T:&\quad -3 = -b \end{aligned}\right\} \begin{aligned} a &= 1 \\ b &= 3 \\ c &= 5 \end{aligned} \qquad \Pi_1 = \frac{P}{\rho N^3 D^5}$$

Flow rate $\quad \Pi_2 = \dfrac{Q}{\rho^a N^b D^c} \; [=] \; \dfrac{L^3 T^{-1}}{\left(\frac{M}{L^3}\right)^a \left(\frac{1}{T}\right)^b L^c}$

$$\left.\begin{aligned} M:&\quad 0 = a \\ L:&\quad 3 = -3a + c \\ T:&\quad -1 = -b \end{aligned}\right\} \begin{aligned} a &= 0 \\ b &= 1 \\ c &= 3 \end{aligned} \qquad \Pi_2 = \frac{Q}{N D^3}$$

Buckingham Pi Theorem

$$\frac{P}{\rho N^3 D^5} = \psi\left(\frac{Q}{N D^3}\right)$$

<u>4.21-2</u>

<u>Speed for model</u>

$$\frac{Q}{ND^3} = \frac{100}{N_1\,(0.5)^3} = \frac{1000}{750\,(1.5)^3}$$

Hence $\underline{\underline{N_1 = 2{,}025 \text{ rpm}}}$

<u>Power for full-size pump</u>

$$\frac{P}{\rho N^3 D^5} = \frac{1.2}{62.4 \times 2025^3 \times 0.5^5} = \frac{P_2}{50 \times 750^3 \times 1.5^5}$$

Hence $\underline{\underline{P_2 = 11.87 \text{ HP}}}$