1.2
Units Convetsion

Viscosity

$$\mu = 10 \text{ Centipoise} = \underbrace{10 \times 0.01 \times 10^{-1} \frac{kg}{m\,s}}_{1 \text{ poise}} = 0.01 \frac{kg}{m\,s}$$

$$= 0.01 \times \frac{0.3048}{0.4536} = 6.72 \times 10^{-3} \frac{lbm}{ft\,s}$$

$$\frac{kg}{m\ s} \quad \frac{lbm}{kg} \quad \frac{m}{ft}$$

(Useful conversion factors: 1 cp = 0.000672 lbm/ft s
= 2.42 lbm/ft hr)

Density

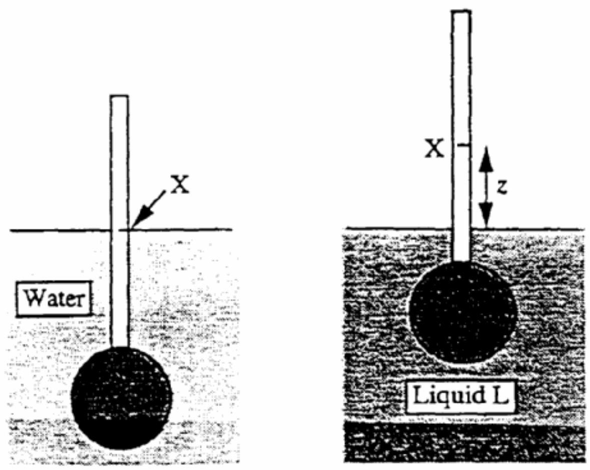
$$\rho = 0.8 \times 1 \times \frac{(100)^3}{1000} = 800 \frac{kg}{m^3}$$

$$\frac{g}{cm^3} \quad \frac{kg}{g} \quad \left(\frac{cm}{m}\right)^3$$

$$= 0.8 \times 62.4 = 49.9 \frac{lbm}{ft^3}$$

1.14

Hydrometer



Since the same weight (that of the hydrometer) is supported by the displaced liquid in both cases:

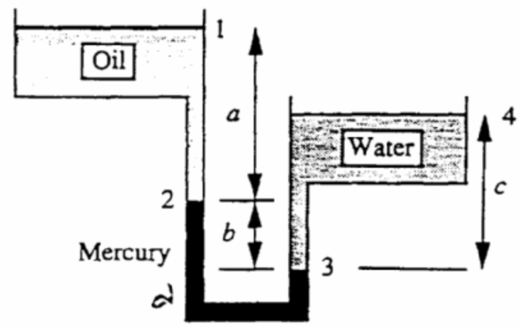
$$\underset{\text{mass of hydrometer}}{Mg} = V \underset{\uparrow \text{ Density of water}}{\rho_w} g = (V - Az)\rho_w s g$$

Cancellation of $\rho_w g$ and solution for s gives

$$s = \frac{1}{1 - \dfrac{Az}{V}}$$

1.15

Three-Liquid Manometer



From hydrostatics:

$$p_4 = p_1 + \rho_O a g + \rho_M b g - \rho_W c g = p_1$$

Cancel p_1 and divide by $\rho_W g$, with $s = \frac{\rho}{\rho_W}$

$$s_O a + s_M b = c$$

$$b = \frac{c - s_O a}{s_M} = \frac{48 - 0.8 \times 72}{13.6} = -0.706$$

Thus the diagram is incorrect as drawn, and the mercury rises on the right by 0.706 in.

1.30

Pressures in Oil and Gas Well

Variation of pressure with elevation for an ideal gas:

$$\frac{dp}{dz} = -\rho g = -\frac{Mp}{RT} g$$

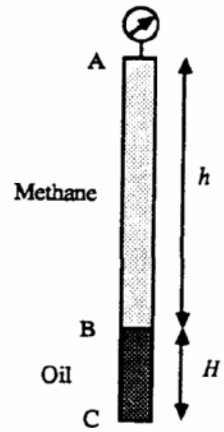


Fig. 1 Well containing oil and methane.

Separate variables and integrate:

$$\int_{p_A}^{p_B} \frac{dp}{p} = -\frac{Mg}{RT} \int_h^0 dz$$

$$\ln \frac{p_B}{p_A} = +\frac{Mgh}{RT} = \frac{16 \times 32.2 \times 10{,}000}{10.73 \times 560 \times 32.2 \times 144}$$

$$= 0.185$$

$$[=] \quad \frac{lb_m}{mole} \frac{ft}{s^2} ft \frac{in^2 \ mole \ {}^\circ R}{lb_f \ ft^3} \frac{lb_f \ s^2}{{}^\circ R \ lb_m \ ft} \frac{ft^2}{in^2} \quad \left(\begin{array}{c}\text{all units}\\ \text{cancel}\end{array}\right)$$

$$p_B = p_A e^{0.185} = 1014.7 \times 1.203 = \underline{\underline{1221 \text{ psia}}}$$

Pressure increase in the oil section

$$p_C - p_B = \rho_o g H = \frac{0.75 \times 62.4 \times 32.2 \times 2000}{32.2 \times 144}$$

$$= \underline{\underline{650 \text{ psi}}}$$

1.34

Blake – Kozeny Equation

$$\frac{p_1 - p_2}{L} = 150 \frac{\mu v_0}{D_p^2} \frac{(1-\varepsilon)^2}{\varepsilon^3}$$

Check dimensions

$$\underbrace{\frac{M}{LT^2}\ \frac{1}{L} \qquad \frac{M}{LT}\ \frac{L}{T}\ \frac{1}{L^2} = \frac{M}{L^2T^2}}$$

Same, hence ε is dimensionless. Also, examination of $(1-\varepsilon)$ gives same conclusion assuming "1" is dimensionless.

Numerical values

$$\frac{\varepsilon^3}{(1-\varepsilon)^2} = \frac{150\,\mu v_0 L}{(p_1-p_2) D_p^2} = \frac{150 \times \overset{\frac{g}{cm\,s}}{0.22} \times \overset{\frac{ft}{s}}{0.1} \times 6 \times \overset{\frac{cm}{ft}}{30.48}}{\underset{\frac{g}{lb_m}}{453.6} \times \underset{\frac{lb_f}{in^2}}{75} \times \underset{in^2}{0.1^2} \times \underset{\frac{lb_m\,ft}{lb_f\,s^2}}{32.2}}$$

$$= 0.05509$$

Rearrange to $\varepsilon = f(\varepsilon) = \left[0.05509 \times (1-\varepsilon)^2\right]^{1/3}$

Successive substitution

i	ε_i	$f(\varepsilon_i)$
1	0.5000	0.2397
2	0.2397	0.3170
3	0.3170	0.3014
4	0.3014	0.2996
5	0.2996	0.3001
6	0.3001	0.3000

Porosity (void fraction)

$\varepsilon = 0.300$