CHE 425

Engineering Economics and Design Principles

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CHAPTER 7

Engineering Economic Analysis

INTRODUCTION

The goal of any manufacturing company is to make money

Low-Value Raw Materials → Chemical Processing Company → High-Value Chemicals

INTRODUCTION (Cont.)

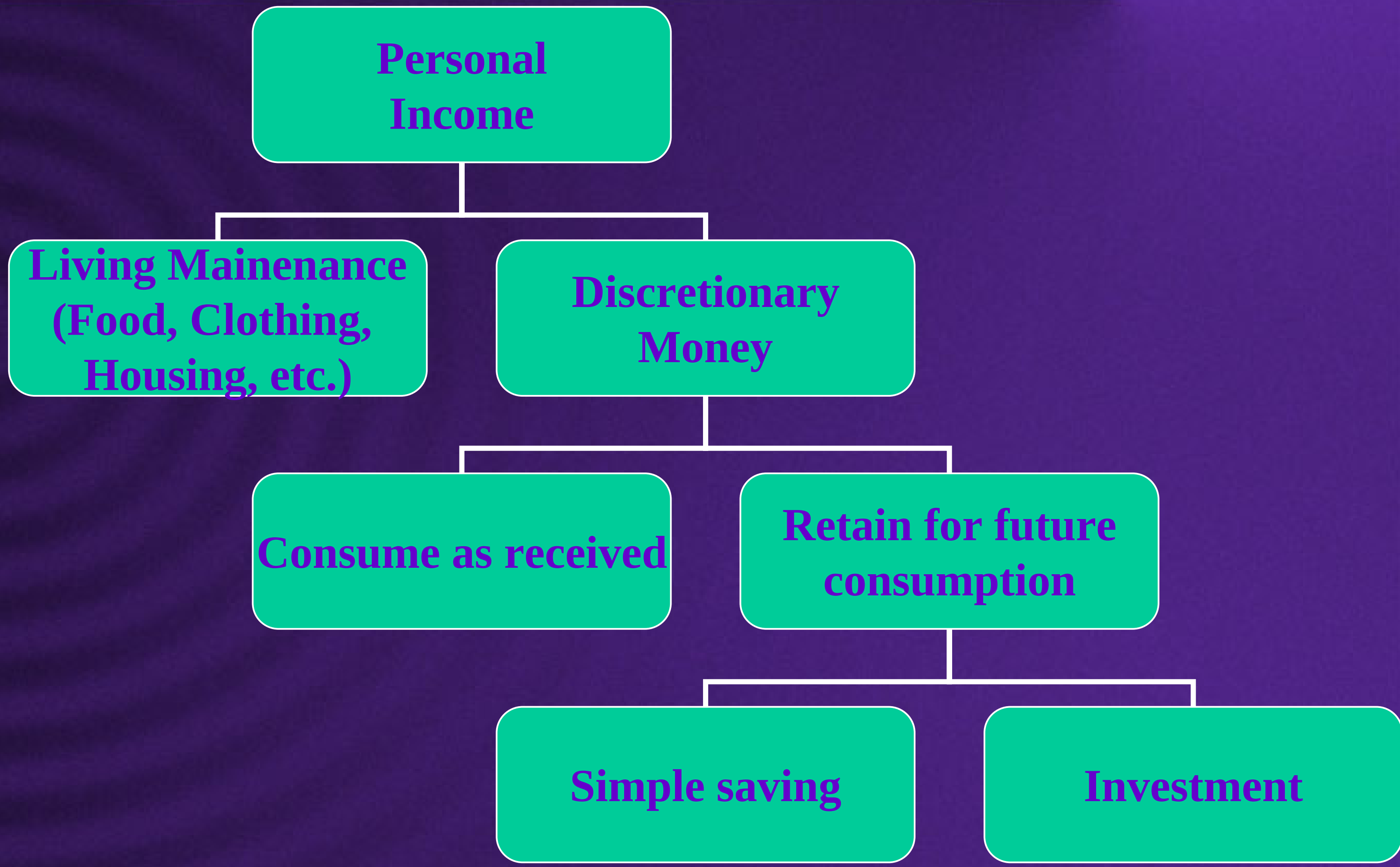
Purpose of Chapter

- To discuss the principles of economic analysis

Importance

- This chapter covers all of the major topics required for completion of the Fundamentals of Engineering (FE) examination.

TIME VALUE OF MONEY



Definitions

- *P* – Principal or Present Value (of an investment)
- F_n – Future Value (of an investment)
- *n* – Years (or other time unit) between *P* and *F*
- *i* – Interest Rate (based on time interval for *n*) per anum

Basis premise: Money when invested earns money.

$1 today is worth more than $1 in the Future.

Example 7.1

Upon graduation, you start your first job at $50,000/yr. You decide to set aside 10% or $5000/yr for retirement in forty years' time, and you assume that you will live twenty years after retiring. You have been offered an investment that will pay you $67,468/yr during your retirement years for the money you invest.

a. How much money would you have per year in retirement if you had saved the money, but not invested it, until retirement?

b. How does this compare with the investment plan offered?

c. How much money was produced from the investment?

a. Money saved: ($5000)(40) = $200,000

Income during retirement: $200,000/20 = $10,000/yr

b. Comparison: (Income from savings)/(Income from investments) = $10,000/$67,468 = 0.15

c. Money Produced = Money Received – Money Invested = ($67,468)(20) – $200,000 = $1,149,360

Money when invested earns money.

$1 today is worth more than $1 in the future.

DEFINION OF INVESTMENT

An *investment* is an agreement between two parties, whereby, one party, the *investor*, provides money, P, to a second party, the *producer*, with the expectation that the *producer* will return money, F, to the *investor* at some future specified date, where $F > P$. The terms used in describing the investment are:

P – Principal or Present Value

F – Future Value

n – Years between F and P

The amount of money earned from the investment

$$E = F - P \tag{7.1}$$

The yearly earnings rate is

$$i_s = \frac{E}{Pn} = \frac{(F-P)}{Pn} \tag{7.2}$$

where i_s is termed the simple interest rate.

From Equation 7.2, we have:

$$\frac{F}{P} = (1 + ni_s) \text{ or, in general, } \frac{F}{P} = f(n,i) \tag{7.3}$$

Example 7.2

You decide to put $1000 into a bank that offers a special rate if left in for two years. After two years you will be able to withdraw $1150.

a. Who is the producer?

b. Who is the investor?

c. What are the values of P, F, i_s , and n?

a. Producer: The bank has to produce \$150.00 after two years.

b. Investor: You invest \$1000 in an account at the beginning of the two-year period.

c. $P = \$1000$ (given)

$F = \$1150$ (given)

$n = 2$ years (given)

From Equation 7.2,

$i_s = (\$1150 - \$1000)/(\$1000)/(2) = 0.075$ or 7.5% per year

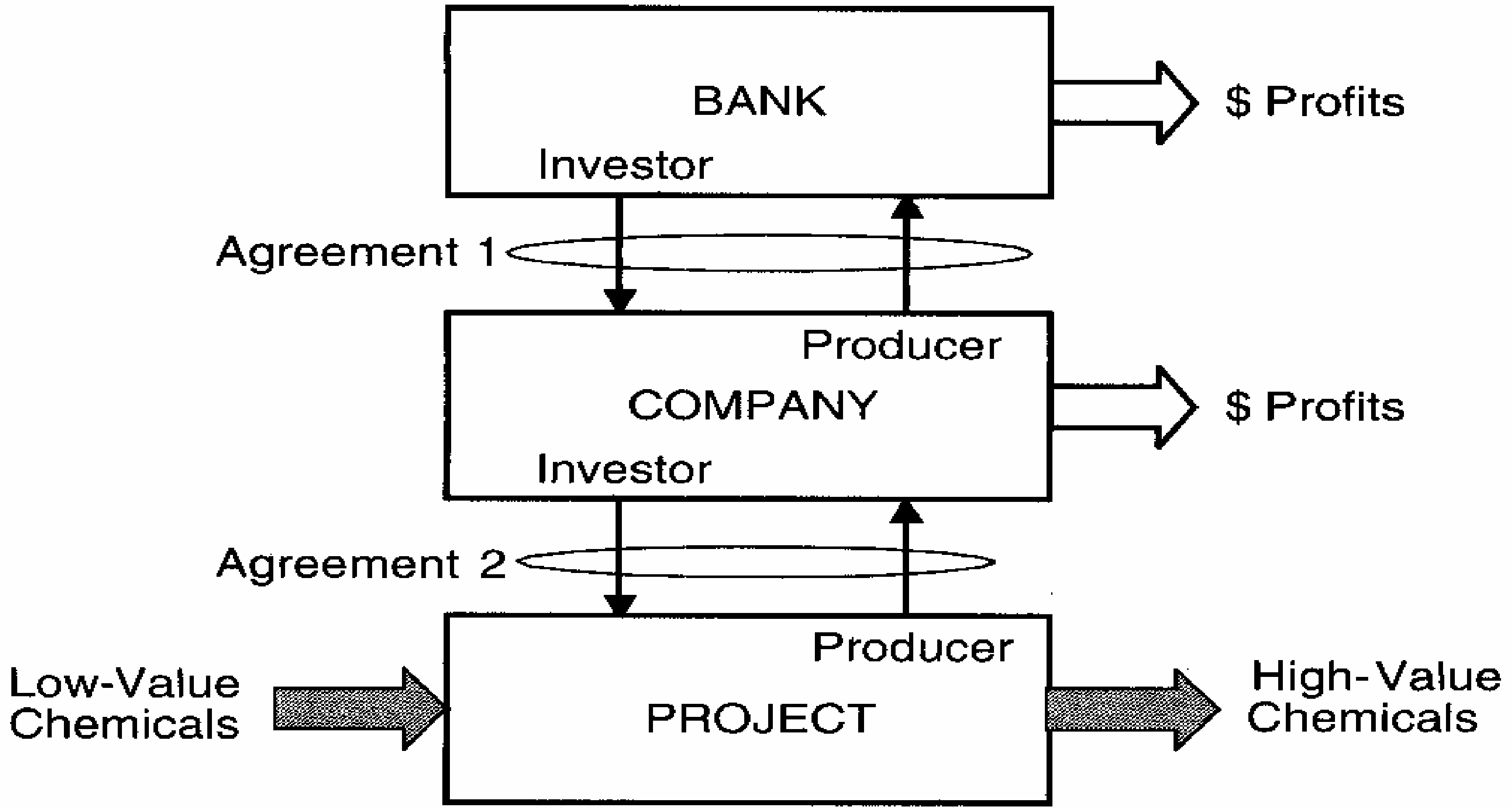


Figure 7.1 A Typical Financing Scheme for a Chemical Plant

Example 7.3

You estimate that in two years time you will need \$1150 in order to replace the linoleum in your kitchen. Consider two choices:

1. Wait two years to take action, or
2. Invest \$1000 now (assume that interest is offered by the bank at the same rate as given in Example 7.1)

What would you do (explain your answer)?

Solution: Consider investing the \$1000 today as it will provide the \$1150 in two years. The key is that the dollar I have today is worth 15% more than a dollar I will have in two years' time.

TIME VALUE OF MONEY

SR100 today is worth more than SR100 in the future.

Interest

- Simple Interest – Annual Basis
  - Interest paid in any year = Pi_s
    - Pi_s – Fraction of investment paid as interest per year
  - After n years total interest paid = $Pi_s n$
  - Total investment is worth = $P + Pi_s n$
  - Could earn interest on earned interest

Interest

- Compound Interest

At time 0 we have P

At the end of Year 1, we have $F_1 = P\,(1 + i)$

At the end of Year 2, we have $F_2 = P\,(1 + i)^2$

At the end of Year n, we have $F_n = P\,(1 + i)^n$

or $P = F_n\,/\,(1 + i)^n$

Interest

SIMPLE INTEREST

$$F_n = P(1 + i_s n) \quad (7.4)$$

COMPOUND INTEREST

$$F_n = P(1 + i)^n \quad (7.5)$$

Example

- How much would i need to invest at 8 % p.a. to yield $5000 in 10 years

$$i = 0.08$$

$$n = 10$$

$$F_{10} = 5000$$

$$P = \frac{5000}{(1 + 0.08)^{10}} = \$2315.97$$

Example

Example 7.6

I need to borrow a sum of money (*P*) and have two loan alternatives:

- **a.** I borrow from my local bank who will lend me money at an interest rate of 7% p.a. and pay compound interest.
- **b.** I borrow from "Honest Sam" who offers to loan me money at 7.3% p.a. using simple interest.

In both cases, I need the money for three years. How much money would I need in three years to pay off this loan? Consider each option separately.

Example (cont.)

Bank: From Equation 7.5 for $n = 3$ and $i = 0.07$ we get

$$F_3 = (P)(1 + 0.07)^3 = 1.225\ P$$

Sam: From Equation 7.4 for $n = 3$ and $i = 7.3$ we get

$$F_3 = (P)(1 + (3)(0.073)) = 1.219\ P$$

Even though Sam stated a higher interest rate to be paid, I would borrow the money from Sam because $1.219P < 1.225P$. This was because Sam used simple interest, and the bank used compound interest.

What if Interest Rate Changes with Time?

If we have an investment over a period of years, and the interest rate changes each year, then the appropriate calculation for compound interest is given by

$$F_n = P\prod_{j=1}^{n}(1 + i_j) = P(1 + i_1)(1 + i_2)\cdots(1 + i_n) \quad (7.7)$$

Different Time Basis for Interest Calculations

- Relates to statement "Your loan is 6 % p.a. compounded monthly"
- Define actual interest rate per compounding period as r
  - i_{nom} = Nominal annual interest rate
  - m = Number of compounding periods per year (12)

Different Time Basis for Interest Calculations (cont.)

- i_{eff} = Effective annual interest rate

$$r = \frac{i_{nom}}{m}$$

- Look at condition after 1 year

$$F_1 = P\left(1 + i_{eff}\right)$$

$$i_{eff} = \left(1 + \frac{i_{nom}}{m}\right)^m - 1$$

Example

- Invest $1000 at 10 % p.a. compounded monthly. How much do I have in 1 year, 10 years?

$$F_1 = P\left(1+\frac{i_{nom}}{m}\right)^m = 1000\left(1+\frac{0.10}{12}\right)^{12} = \$1104.71$$

$$i_{eff} = \left(1+\frac{0.10}{12}\right)^{12} - 1 = 0.1047$$

$$F_{10} = P\left(1+i_{eff}\right)^{10} = \$2707.04$$

Example (cont.)

- As m decreases i_{eff} increases
- Is there a limit as *m* goes to infinity
  - Yes – continuously compounded interest
  - Derivation – pp. 229-230
  - i_{eff} (continuous) = $e^{i_{nom}} - 1$

Example 7.9

What is the effective annual interest rate for an investment made at a nominal rate of 8% p.a. compounded continuously?

From Equation 7.10 for $i_{nom} = 0.08$ we obtain

$$i_{eff} = e^{0.08} - 1 = 0.0833 \text{ or } 8.33\% \text{ p.a.}$$

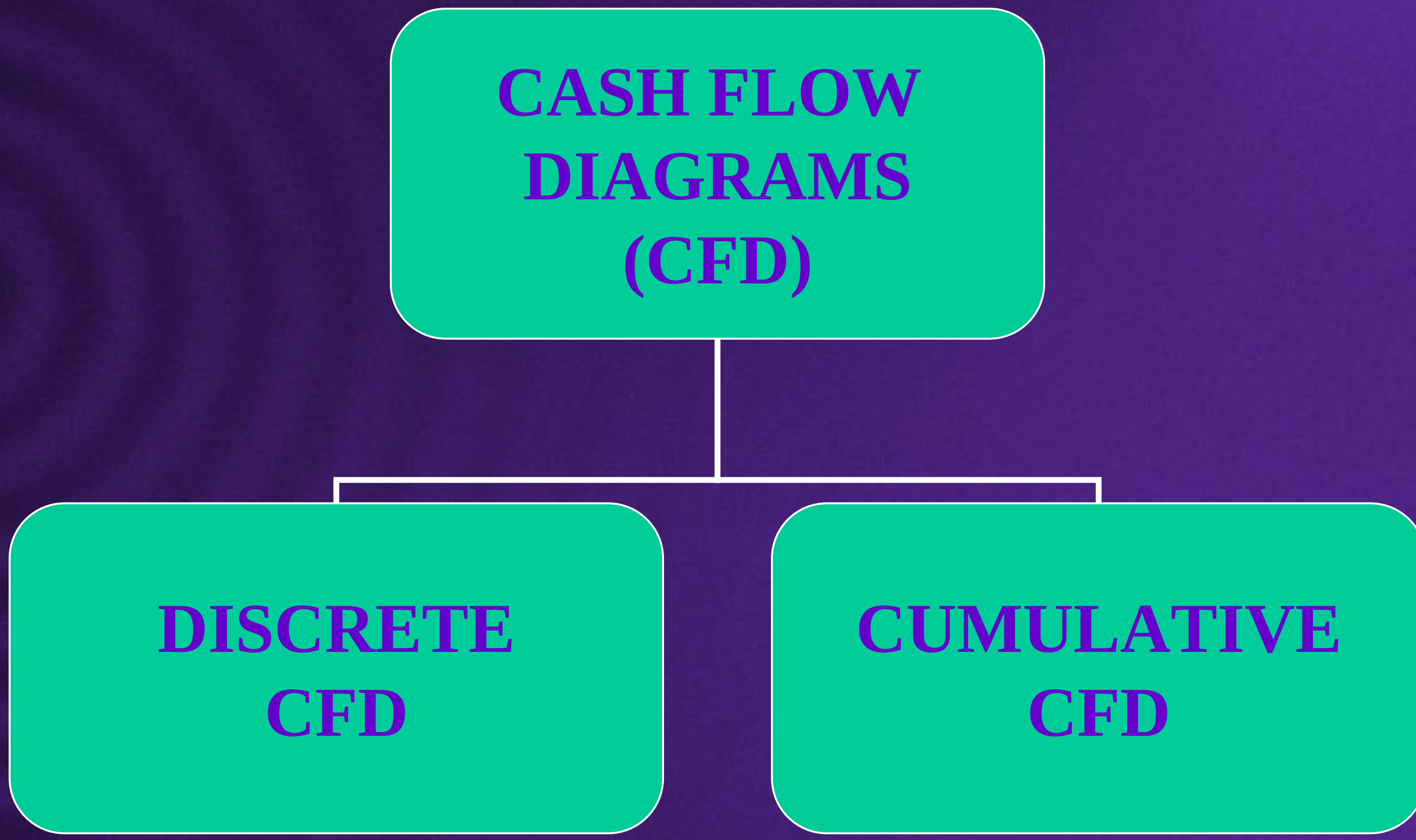
Note: We can see, by comparison with Example 7.8, that by compounding continuously little was gained over monthly compounding.

IN COMPARING ALTERNATIVES, THE EFFECTIVE ANNUAL RATE AND NOT THE NOMINAL ANNUAL RATE OF INTEREST MUST BE USED

Cash Flow Diagrams (CFD)

- Represent timings and approximate magnitude of investment on a CFD
  - *x*-axis is time and *y*-axis is magnitude
  - both positive and negative investments are possible.
- In order to determine direction (sign) of cash flows, we must define what system is being considered.

TYPES OF CFDs



Discrete CFD

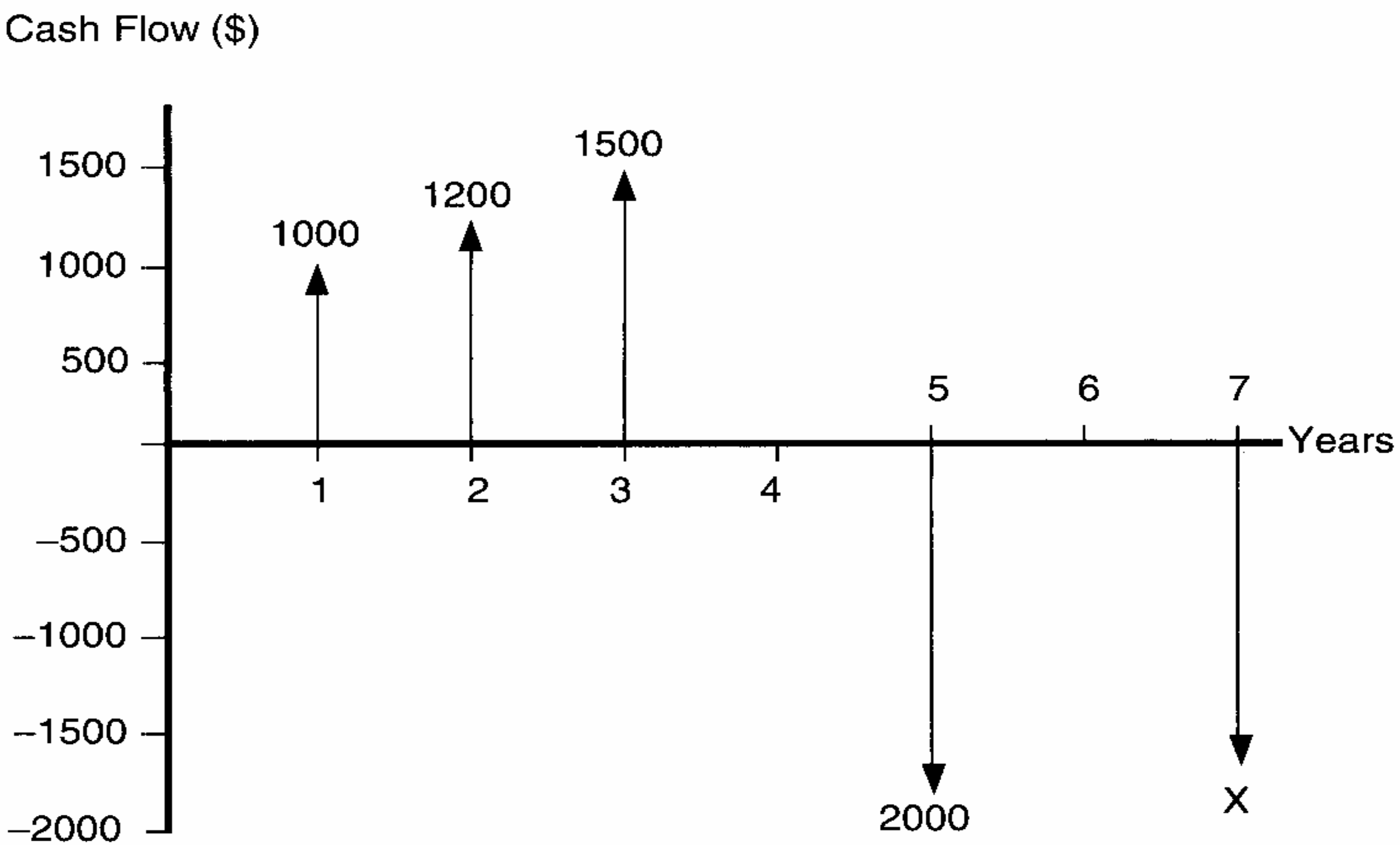
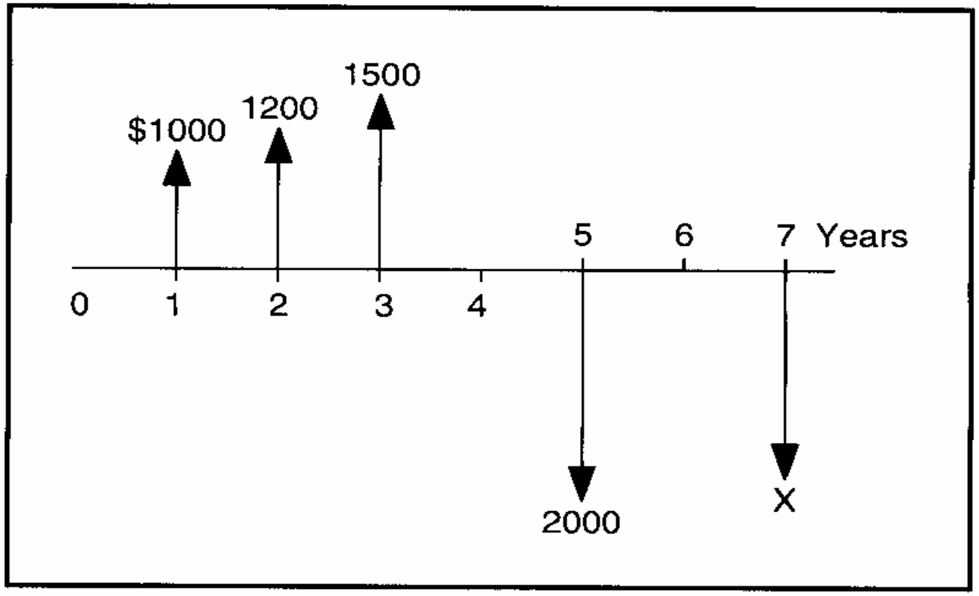


Figure 7.2 An Example of a Representative Discrete Cash Flow Diagram (CFD)

Example 7.10

I borrow \$1000, \$1200, and \$1500 from a bank (at 8% p.a. effective interest rate) at the end of years 1, 2, and 3, respectively. At the end of year 5, I make a payment of \$2000, and at the end of year 7, I pay off the loan in full. The CFD for this exchange from my point of view (producer) is given to the right.

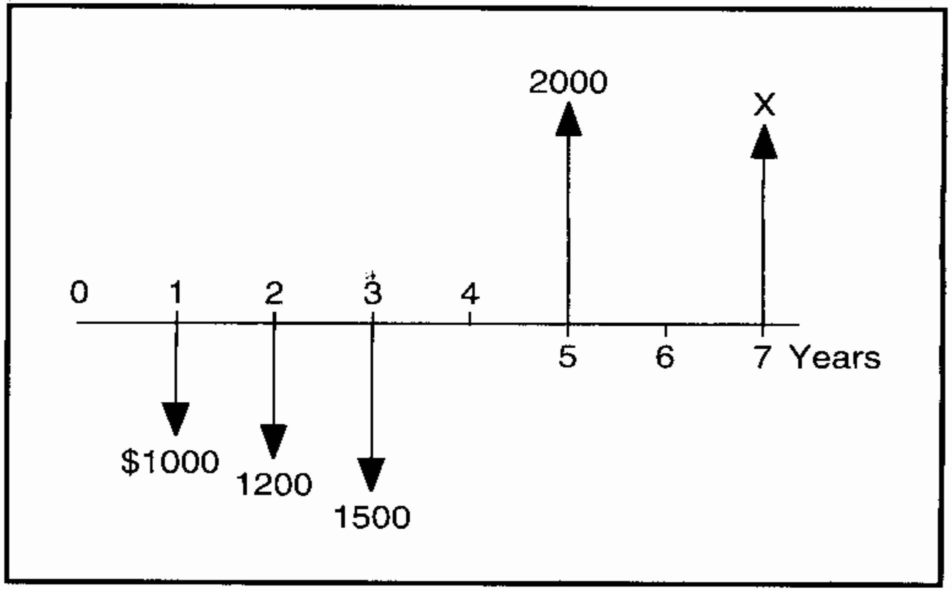


Note: This figure is the short-hand version of the one presented in Figure 7.2 used to introduce the CFD.

Draw a discrete cash flow diagram for the investor.

The bank represents the investor. From the investor's point of view, the initial three transactions are negative and the last two are positive.

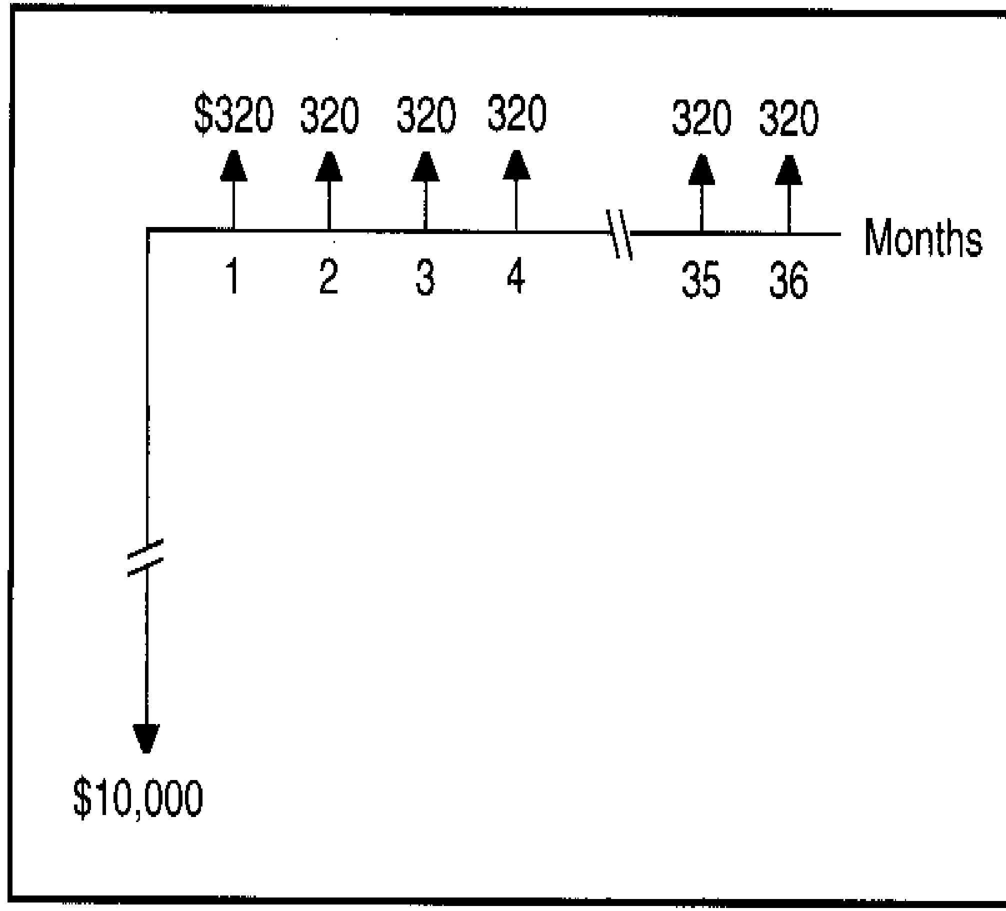
The figure to the right represents the CFD for the bank. It is the mirror image of the one given above in the problem statement.



Example 7.11

You borrow $10,000 from a bank to buy a new car and agree to make 36 equal monthly payments of $320 each to repay the loan. Draw the discrete CFD for the investor in this agreement.

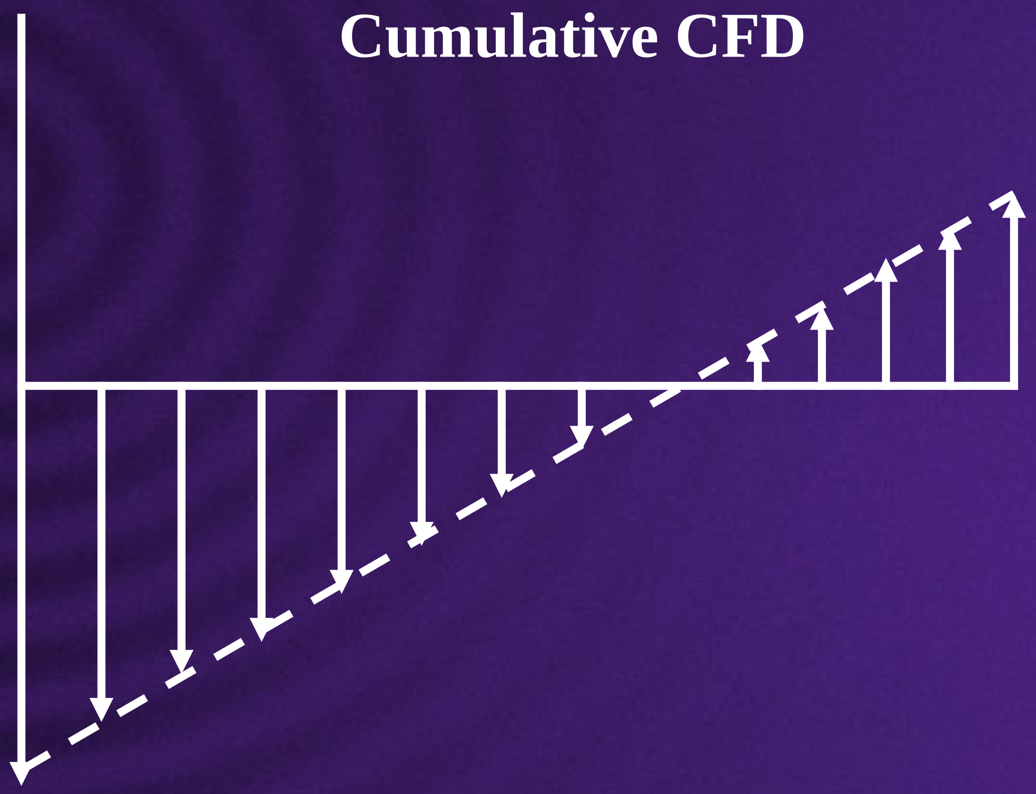
The bank is the investor. The discrete CFD for this investment is shown on the right.



Notes:

1. There is a break in both the time scale and in the investment at time = 0 (the initial investment).
2. From your point of view, the cash flow diagram would be the mirror image of the one shown.

Cumulative CFD

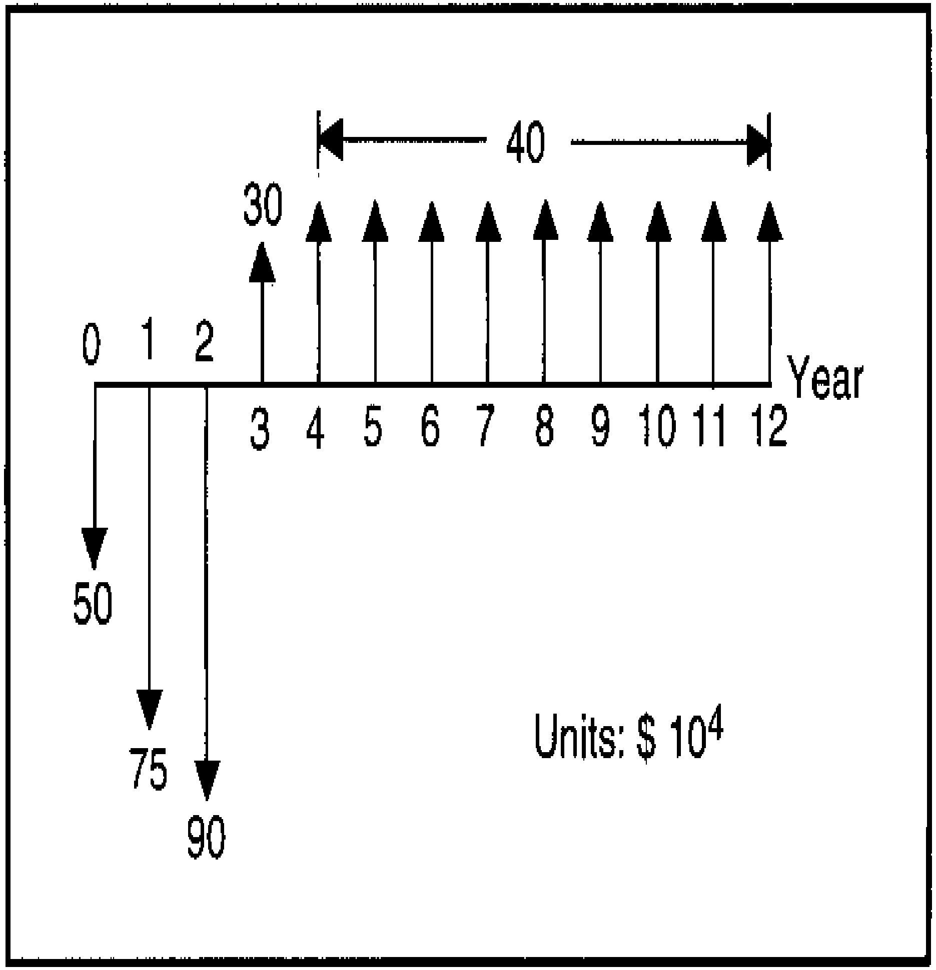


Cumulative CFD (cont.)

Example 7.12

The yearly cash flows estimated for a project involving the construction and operation of a chemical plant producing a new product are provided in the discrete CFD given on the right. Using this information, construct a cumulative CFD.

The numbers shown in the worksheet below were obtained from this diagram.

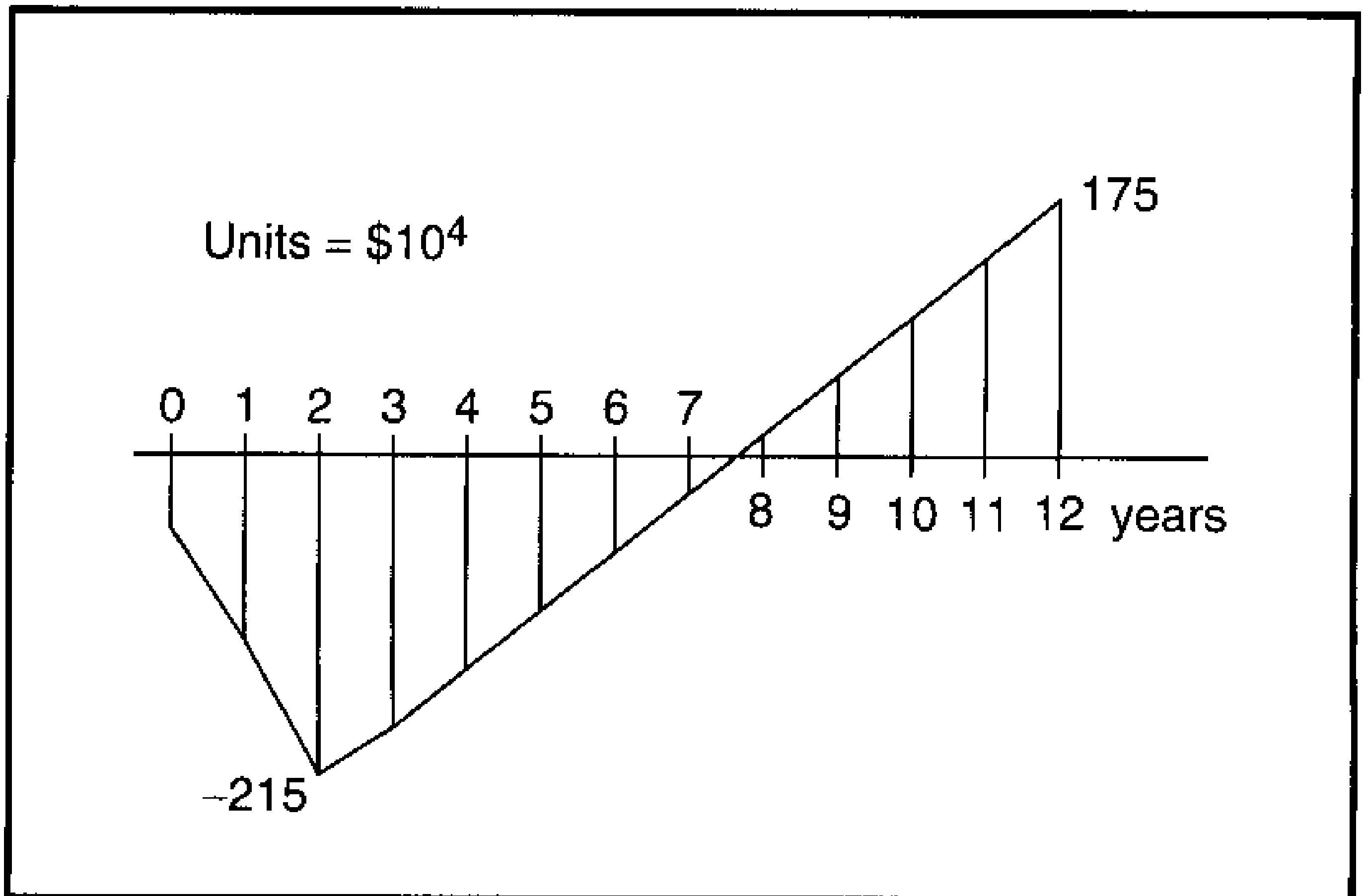


Cumulative CFD (cont.)

Year	Cash Flow ($) (from discrete CFD)	Cumulative Cash Flow (calculated)
0	–500,000	–500,000
1	–750,000	–1,250,000
2	–900,000	–2,150,000
3	300,000	–1,850,000
4	400,000	–1,450,000
5	400,000	–1,050,000
6	400,000	–650,000
7	400,000	–250,000
8	400,000	150,000
9	400,000	550,000
10	400,000	950,000
11	400,000	1,350,000
12	400,000	1,750,000

Cumulative CFD (cont.)

The cumulative cash flow diagram is plotted below.



Calculations from CFD

When cash flows occur at different times, each cash flow must be brought forward (or backward) to the same point in time and then compared.

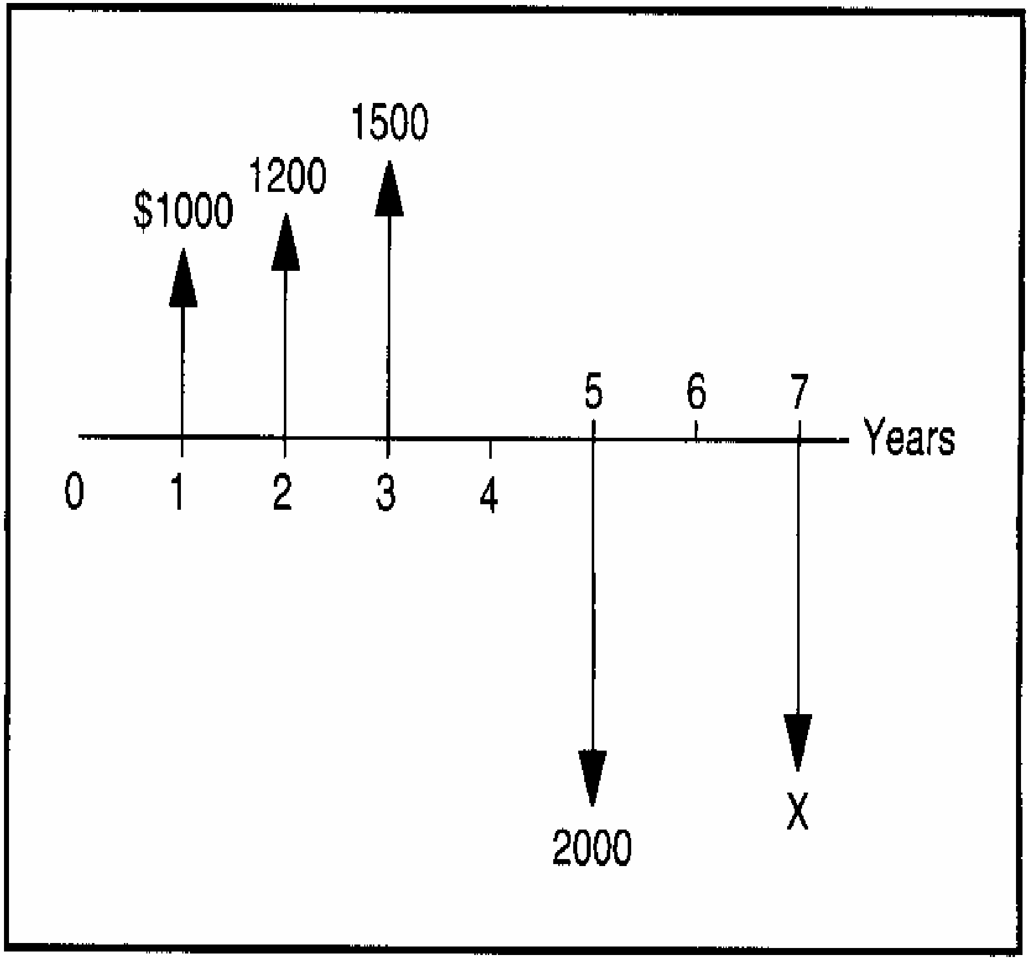
Example 7.13

The CFD obtained from Example 7.10 (for the borrower) is copied below. The annual interest rate paid on the loan is 8% p.a.

In year 7, the remaining money owed on the loan is paid off.

a. Determine the amount, X, of the final payment.

b. Compare the value of X with the value that would be owed if there were no interest paid on the loan.



With the final payment at the end of year 7, no money is owed on the loan. If we sum all the positive and negative cash flows adjusted for the time of the transactions, this adjusted sum must equal zero.

We select as the base time the date of the final payment.

a. From Equation (7.5) for $i = 0.08$ we obtain:

For withdrawals:

\$1000 end of year 1: $F_6 = (\$1000)(1 + 0.08)^6 = \$1586.87$
\$1200 end of year 2: $F_5 = (\$1200)(1 + 0.08)^5 = \$1763.19$
\$1500 end of year 3: $F_4 = (\$1500)(1 + 0.08)^4 = \$2040.73$

Total withdrawals = \$5390.79

For repayments:

$\$2000$ end of year 5: $F_2 = -(\$2000)(1+0.08)^2 = -\$2332.80$

$\$X$ end of year 7: $F_0 = -(\$X)(1+0.08)^0 = -\$X$

Total repayments $= -\$(2332.80 + X)$

Summing the cash flows and solving for X yields

$0 = \$5390.79 - \$(2332.80 + X)$

$X = \$3057.99 \approx \$3058$

b. For $i = 0.00$

Withdrawals = \$1000 + \$1200 + \$1500 = \$3700

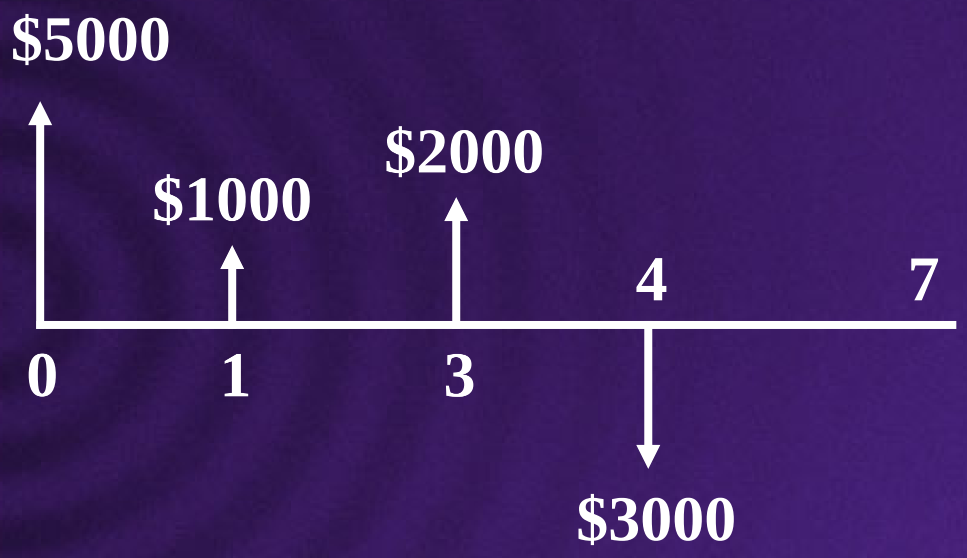
Repayments = – \$(2000 + X)

0 = \$3700 – \$(2000 + X)

X = \$1700

Note: Because of the interest paid to the bank, the borrower repaid a total of \$1358 (\$3058 – \$1700) more than was borrowed from the bank seven years earlier.

Calculations with Cash Flow Diagrams



- Invest 5K, 1K, 2K at End of Years 0, 1, 3, and take 3K at End of Year 4

Example 1

- How much in account at end of Year 7 if i = 8% p.a.

$$F_7 = 5{,}000(1+0.08)^7 + 1000(1+0.08)^6 + 2000(1+0.08)^4 - 3000(1+0.08)^3$$

$$F_7 = \$9097.84$$

- What would investment at Year 0 be to get this amount at Year 7

$$P = \frac{9097.84}{(1.08)^7} = 5308.50$$

Example 2

- What should my annual monthly car payment be if interest rate is 8% p.a. compounded monthly?

Example 2 cont.

- Compare at $n = 60$

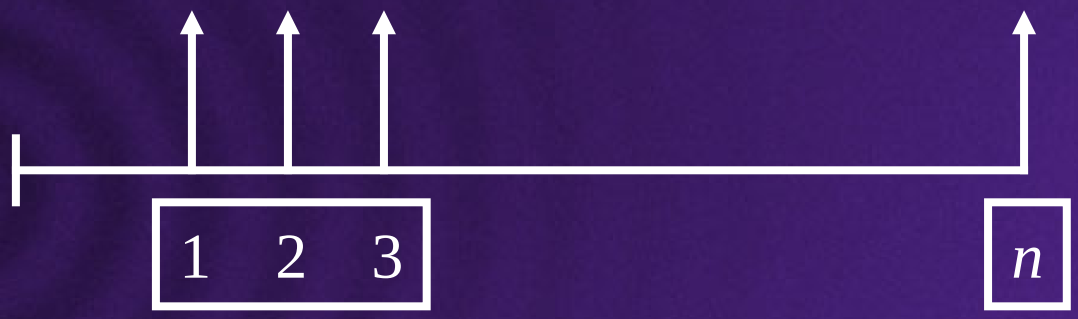
$$F_{60} = A\left[\frac{\left(1+\frac{0.08}{12}\right)^{60}-1}{\frac{0.08}{12}}\right] = 73.47A$$

$$F_{60} = -20{,}000\left[\left(1+\frac{0.08}{12}\right)^{60}\right] = -29796.90$$

$$73.47A - 29796.90 = 0$$

$$A = \$405.53$$

Annuities



Uniform series of equally spaced – equal value cash flows

Annuities

- What is future value $F_n = ?$

$$F_n = A(1+i)^{n-1} + A(1+i)^{n-2} + ...A$$

- Geometric progression

$$F_n = S_n = A\left[\frac{(1+i)^n - 1}{i}\right]$$

Discount Factors

- Just a shorthand symbol for a formula in *i* and *n*

$$P = \frac{F}{(1+i)^n} \Rightarrow \left(\frac{P}{F}, i, n\right) = \frac{1}{(1+i)^n}$$

$$\Rightarrow P = F\left(\frac{P}{F}, i, n\right) = F\left(\frac{1}{(1+i)^n}\right)$$

$$\Rightarrow A \to P \Rightarrow \left(\frac{P}{F}, i, n\right) = \frac{(1+i)^n - 1}{i(1+i)^n}$$

Table 7.1

Example 7.14

Example 7.14

You have just won \$2,000,000 in the Texas Lottery as one of seven winners splitting up a jackpot of \$14,000,000. It has been announced that each winner will receive \$100,000/year for the next 20 years. What is the equivalent present value of your winnings if you have a secure investment opportunity providing 7.5% p.a.?

From Table 7.1, Equation 7.14, for $n = 20$ and $i = 0.075$

$$P = (\$100{,}000)[(1+0.075)^{20} - 1]/[(0.075)(1+0.075)^{20}]$$

$$P = \$1{,}019{,}000$$

A present value of \$1,019,000 is equivalent to a 20-year annuity of \$100,000/yr when the effective interest rate is 7.5%.

Example 7.15

Consider Example 7.11, involving a car loan. The discrete CFD from the bank's point of view was shown previously.

What interest rate is the bank charging for this loan?

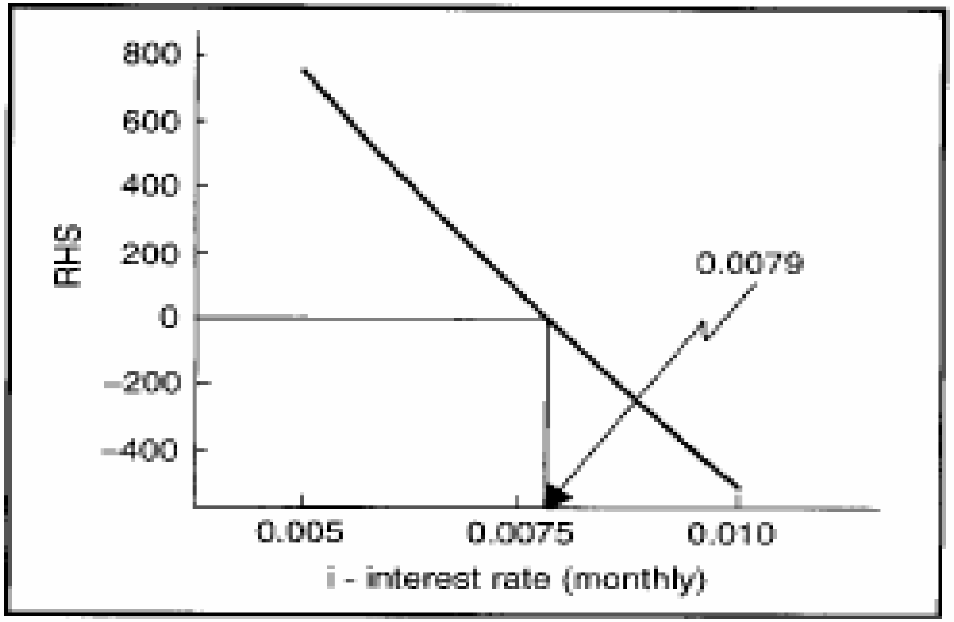
You have agreed to make 36 monthly payments of $320. The time selected for evaluation is the time at which the final payment is made. At this time, the loan will be fully paid off. This means that the future value of the $10,000 borrowed is equivalent to a $320 annuity over 36 payments.

$$(\$10{,}000)(F/P, i, n) = (\$320)(F/A, i, n)$$

Substituting the equations for the discount factors given in Table 7.1, with $n = 36$ months, we get:

$$0 = -(10{,}000)(1+i)^{36} + (320)[(1+i)^{36} - 1]/i$$

This equation cannot be solved explicitly for i. We solve this equation by plotting the value of the right-hand side of the equation shown above for various interest rates. This equation could also be solved using a numerical technique. From the graph, the interest rate that gives a value of zero represents the answer. From the graph on the previous page the rate of interest is $i = 0.0079$.



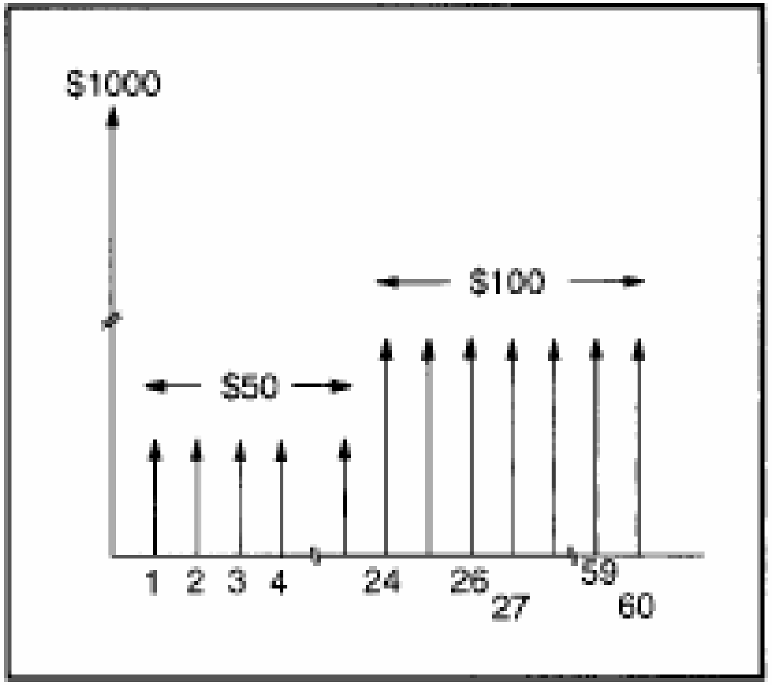
The nominal annual interest rate is $(12)(0.00786) = 0.095$ (9.5%).

Example 7.16

I invest money in a savings account that pays a nominal interest rate of 6% p.a. compounded monthly. I open the account with a deposit of $1000 and then deposit $50 at the end of each month for a period of two years followed by a monthly deposit of $100 for the following three years. What will the value of my savings account be at the end of the five-year period?

First, draw a discrete CFD (shown to the right).

Although this CFD looks rather complicated, we can break it down into 3 easy sub-problems:



1. The initial investment
2. The 24 monthly investments of $50
3. The 36 monthly investments of $100

Each of these investments is brought forward to the end of month 60.

$F = (\$1000)(F/P, 0.005, 60) + (\$50)(F/A, 0.005, 24)(F/P, 0.005, 36) + (\$100)(F/A, 0.005, 36)$

Note: the effective monthly interest rate is $0.06/12 = 0.005$

$$F = (\$1000)(1.005)^{60} + (\$50)\frac{(1.005^{24} - 1)}{0.005}(1.005)^{36} + (\$100)\frac{(1.005^{36} - 1)}{0.005} = \$6804.16$$

Example 7.17

In Example 7.1, we introduced an investment plan for retirement. It involved investing \$5000/year for 40 years leading to retirement. The plan then provided \$67,468/year for twenty years of retirement income.

a. What yearly interest rate was used in this evaluation?
b. How much money was invested in the retirement plan before withdrawals began?

a. The evaluation is performed in two steps:

Step 1: Find the value of the \$5000 annuity investment at the end of the 40 years.

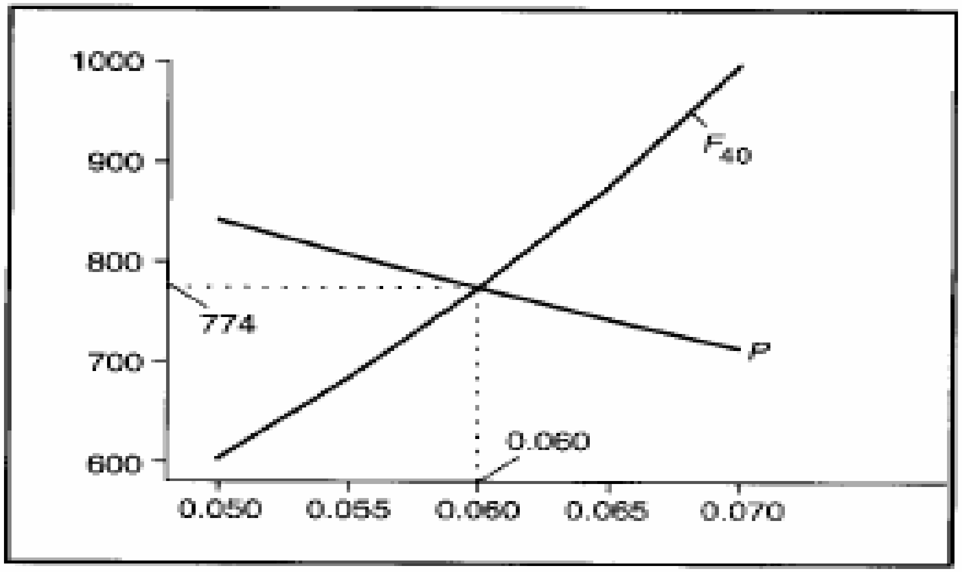
Step 2: Evaluate the interest rate of an annuity that will pay out this amount in 20 years at \$67,468/year.

Step 1: From Equation 7.11, Table 7.1, for $A = \$5000$ and $n = 40$,

$F_{40} = (A)(F/A, n, i) = (\$5000)[(1+i)^{40}-1]/i$

Step 2: From Equation 7.14, Table 7.1, for $A = \$67,468$ and $n = 20$,

$P = (A)(P/A, n, i) = (\$67,468)[(1+i)^{20} - 1]/[(i)(1+i)^{20}]$



Set $F_{40} = P$ and solve for i. From the graph above, we get $i = 0.060$

b. With $i = 0.060$, we have from the graph $F_{40} = \$774,000$

Note: The interest rate of 6.0% p.a. represents a relatively low interest rate, involving small risk.

Depreciation

- Total Capital Investment = Fixed Capital + Working Capital
  - Fixed Capital – All costs associated with new construction, but Land cannot be depreciated
  - Working Capital – Float of money to start operations

$$TCI = FCI_L + Land + WC$$

Definitions

- Salvage Value
  - Value of FCI_L at end of project
  - Often = 0
- Life of Equipment
  - n – Set by IRS
    - Not related to actual equipment life
- Total Capital for Depreciation
  - $FCI_L - S$

3 Basic Methods for Depreciation

- Straight Line
- Sum of Years Digits (SOYD)
- Double Deruning Balance (DDB)

Straight Line

$$d_k^{SL} = \left(\frac{FCI_L - S}{n} \right)$$

n = # of years

Sum of Years Digits (SOYD)

$$d_k^{SOYD} = \frac{\left[(n+1-k)(FCI_L - S)\right]}{\frac{1}{2}n(n+1)}$$

SOYD

Double Declining Balance (DDB)

$$d_k^{DDB} = \frac{2}{n}\left[FCI_L - \sum_{j=0}^{k-1} d_j \right]$$

Example 7.21

The fixed capital investment (excluding the cost of land) of a new project is estimated to be $150.0 million, and the salvage value of the plant is $10.0 million. Assuming a seven-year equipment life, estimate the yearly depreciation allowances using:

a. The straight line method
b. The sum of the years digits method
c. The double declining balance method.

Example 7.21

$$FCI_L = \$150 \times 10^6$$

$$S = \$10 \times 10^6$$

$$n = 7$$

$$1^{st}\ Year$$

$$d_{SL} = \frac{150 - 10}{7} = 20$$

Same for Years 1-7

Example 7.21 (cont'd)

$$d_{SOYD_1} = \frac{(7+1-1)}{\frac{1}{2}(7)(8)}[150-10] = \frac{7}{28}[150-10] = 35$$

$$d_{SOYD_2} = \frac{(7+1-2)}{\frac{1}{2}(7)(8)}[150-10] = \frac{6}{28}[150-10] = 30$$

$$d_{DDB_1} = \frac{2}{7}(150) = 42.9$$

$$d_{DDB_2} = \frac{2}{7}(150-42.9) = 30.6$$

Table E7.21 Calculations and Results for Example 7.21: The Depreciation of Capital Investment for a New Chemical Plant (all values in $\$10^7$).

Year (k)	d_k^{SL}	d_k^{SOYD}	d_k^{DDB}	Book Value $FCI_L - Sd_k^{DDB}$
0				$(15-0)=15$
1	$\frac{(15-1)}{7}=2$	$\frac{(7+1-1)(15-1)}{28^a}=3.5$	$\frac{(2)(15)}{7}=4.29$	$(15-4.29)=10.71$
2	$\frac{(15-1)}{7}=2$	$\frac{(7+1-2)(15-1)}{28^a}=3.0$	$\frac{(2)(10.71)}{7}=3.06$	$(10.71-3.06)=7.65$
3	$\frac{(15-1)}{7}=2$	$\frac{(7+1-3)(15-1)}{28^a}=2.5$	$\frac{(2)(7.65)}{7}=2.19$	$(7.65-2.19)=5.46$
4	$\frac{(15-1)}{7}=2$	$\frac{(7+1-4)(15-1)}{28^a}=2.0$	$\frac{(2)(5.46)}{7}=1.56$	$(5.46-1.56)=3.90$
5	$\frac{(15-1)}{7}=2$	$\frac{(7+1-5)(15-1)}{28^a}=1.5$	$\frac{(2)(3.90)}{7}=1.11$	$(3.90-1.11)=2.79$
6	$\frac{(15-1)}{7}=2$	$\frac{(7+1-6)(15-1)}{28^a}=1.0$	$\frac{(2)(2.79)}{7}=0.80$	$(2.79-0.80)=1.99$
7	$\frac{(15-1)}{7}=2$	$\frac{(7+1-7)(15-1)}{28^a}=0.5$	$1.99-1.0=0.99^b$	$(1.99-0.99)=1.00$
Tot.	14.0	14.0	14.0	1.0 = Salvage Value[b]

Taxation, Cash Flow, and Profit

- Tables 7.3 – 7.4
- Expenses = $COM_d + d_k$
- Income Tax = $(R - COM_d - d_k)t$
- After Tax (net)Profit = $(R - COM_d - d_k)(1 - t)$
- After Tax Cash Flow = $(R - COM_d - d_k)(1 - t) + d_k$

Inflation

- $ Now Net Worth vs. $ Next Year

$$CEPCI\left(j+n\right)=\left(1+f\right)^{n}CEPCI\left(j\right)$$

- *f* = Average inflation rate between Years *j* and *n*

$$i' \cong i-f=\frac{i-f}{i+f}$$