Resource-Activity Critical-Path Method for Construction Planning

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Abstract: In this paper, a practical method is developed in an attempt to address the fundamental matters and limitations of existing methods for critical-path method (CPM) based resource scheduling, which are identified by reviewing the prior research in resource-constrained CPM scheduling and repetitive scheduling. The proposed method is called the resource-activity critical-path method (RACPM), in which (1) the dimension of resource in addition to activity and time is highlighted in project scheduling to seamlessly synchronize activity planning and resource planning; (2) the start/finish times and the floats are defined as resource-activity attributes based on the resource-technology combined precedence relationships; and (3) the ''resource critical'' issue that has long baffled the construction industry is clarified. The RACPM is applied to an example problem taken from the literature for illustrating the algorithm and comparing it with the existing method. A sample application of the proposed RACPM for planning a footbridge construction project is also given to demonstrate that practitioners can readily interpret and utilize a RACPM schedule by relating the RACPM to the classic CPM. The RACPM provides schedulers with a convenient vehicle for seamlessly integrating the technology/process perspective with the resource use perspective in construction planning. The effect on the project duration and activity floats of varied resource availability can be studied through running RACPM on different scenarios of resources. This potentially leads to an integrated scheduling and cost estimating process that will produce realistic schedules, estimates, and control budgets for construction.

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Introduction

Due to the resource-driven nature of construction management, the construction manager must develop a plan of action for directing and controlling resources of workers, machines, and materials in a coordinated and timely fashion in order to deliver a project within the limited funding and time available (Halpin and Woodhead 1998). Hence, aside from a technology and process focus (i.e., what is to be done and how), a resource-use focus (i.e., who is to do it with what) must be adequately considered in describing a construction method or operation in a project plan (Halpin and Riggs 1992). Nevertheless, the most popular project-planning methods—the critical-path method (CPM) and the related network diagramming techniques (PERT, Node Diagramming, and Precedence Diagramming)—fail to seamlessly synchronize activity planning and resource planning, the two integral functions in project planning. CPM assumes limitless availability of resources (Waugh and Froese 1990). This assumption is not valid in most practical situations, in which there exist definite limits on the amount of resources available and these resources are shared by a number of activities or even projects. To overcome this recognized drawback, which brings about unrealistic or impossible CPM schedules, analytical or heuristic techniques for resources allocating/leveling on CPM network plans have also been developed since the early 1960s. These techniques generally consist of two stages. First, the project is broken down into distinct activities that are logically or technologically related to one another according to the construction process/method without imposing resource constraints (e.g., the superstructure follows the substructure; the concrete pouring succeeds the formwork and reinforcement). Second, basic CPM scheduling calculations are made for early and late start and finish dates and total and free float times, based on which (1) the project is rescheduled so that a limited number of resources can be efficiently utilized while minimizing the unavoidable extension of project duration (also known as resource allocation); or (2) the start times of certain activities are adjusted within the float limits for a leveled resource profile (also known as resource leveling).

In this paper, the prior research in resource-constrained CPM scheduling and repetitive scheduling is first reviewed to identify some fundamental matters and limitations of the existing methods. The aim of finding a better approach to address the identified problems has driven the research endeavor leading to the development of a practical method called the resource-activity critical-path method (RACPM).

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Literature Review

Resource-Constrained Critical-Path Method Scheduling

Limited-resource allocation algorithms aim to find the CPM schedule duration that is shortest as well as consistent with specified resource limits and essentially deal with the notorious ‘‘combinatorial explosion’’ problem in mathematics (Hegazy 1999). Various conventional approaches have been formulated to solve the problem optimally, including integer programming (IP), branch-and-bound, and dynamic programming and the implicit enumeration approaches (Lee and Gatton 1994; Nudtasomboon and Randhawa 1996; Sung and Lim 1996; Demeulemeester and Herroelen 1997). But none of these is computationally tractable for any real-life problem size, thus rendering them impractical (Hegazy 1999). For instance, Lee and Gatton (1994) presented complete IP formulations combining construction scheduling and resource utilization planning, but drew on a sequential suboptimization approach by prioritizing resources and an alternative heuristic procedure to solve the IP formulations. Latest developments have resorted to emerging AI techniques. For instance, Chan et al. (1996) and Hegazy (1999) used genetic algorithms (GAs) to achieve optimization of resource allocation and leveling concurrently and produced a shorter project duration and a better-leveled resource profile. Leu and Yang (1999) also employed GAs in a multicriteria optimal scheduling model to integrate the objectives of time/cost tradeoff, allocation of limited resources, and unlimited resource leveling. Leu et al. (1999) further incorporated the fuzzy set theory into a GA-based resource-scheduling model to accommodate the uncertainties of activity duration and minimize the project duration. Senouci and Adeli (2001) applied a patented nonlinear constrained optimization model to minimize the total project cost while performing the resource allocation and resource leveling simultaneously.

An alternative approach to resource-constrained CPM scheduling is the use of heuristic methods that apply priority rules based on activity characteristics, such as the ‘‘minimum total slack’’ rule, to prioritize activities that compete for limited resources. The resulting schedule satisfies the technological constraints and the resource constraints but is not optimal in terms of achieving the shortest project duration. The total floats (TF), earliest start times (EST), and latest start times (LST) as calculated from the CPM analysis usually serve as part of the criteria in the heuristic priority rules. Abeyasinghe et al. (2001) presented a new heuristic approach that does not require CPM calculations but used Gantt charts combined with an intermediate tool called ancillary networks to facilitate the process of resource-loading CPM, instead of using priority rules. Their method also attempted to define a critical path in the sense of the classic CPM by identifying the path with the longest activity duration.

Resource-Constrained Repetitive Scheduling

Network techniques are inefficient when applied to schedule repetitive projects, mainly because (1) using a large number of activities to represent repetitive activities in a CPM format makes the resulting CPM schedule difficult to visualize and analyze; and (2) the resource-leveled CPM networks do not guarantee work continuity of resources (Hegazy and Wassef 2001). Special resource-constrained repetitive scheduling models have been developed based on the line of balance (LOB) technique, which accounts for precedence relationships, crew availability, crew work continuity constraints, etc. The resulting schedule is a time-space chart with the space dimension representing a number of identical units (e.g., floor/road section) going through a series of repetitive activities (Halpin and Riggs 1992). Latest developments have coupled the resource-driven scheduling algorithm with the dynamic programming formulations and an automated interruption mechanism to optimize resource utilization and minimize the project cost and duration in scheduling repetitive serial activities (El-Rayes and Moselhi 1997, 2001). By adding the space constraint to a repetitive scheduling system specially developed for multistory building projects, Thabet and Beliveau (1997) considered the limited space availability at the workface and the effect of space congestion on crew productivity during the generation of the schedule. Hegazy and Wassef (2001) integrated the CPM and LOB methodologies and used GAs to minimize the total construction cost in projects with repetitive nonserial activities, achieving the optimum combination of construction methods, number of crews, and interruptions for each repetitive activity.

Observations on Existing Methods

The existing methods for resource-loading CPM have failed to address and clarify the ‘‘resource critical’’ issue brought up about 40 years ago (Fondahl 1961): noncritical activities in the sense of having positive float can still be ‘‘resource critical,’’ since project duration will be delayed if resource-critical activities fail to release resources that are required by critical activities on time. Fondahl’s solution was to manually adjust the theoretical activity floats of CPM using a heuristic method and calculate the ‘‘actual total floats,’’ which are no greater than the theoretical floats. Fondahl (1991) further pointed out that ‘‘the conventional concepts of float time and critical path in CPM break down in a resource-constrained project schedule’’ in that ‘‘the originally calculated network data (including TF/EST/LST), which are activity attributes and useful as a basis for establishing and applying priority rules in heuristic allocating/leveling procedures, may have little meaning once resource allocation or leveling has been performed.’’ This has caused unrealistic specifications, disputes, and dispute resolutions based on incorrect data in the construction industry (Fondahl 1991). The proposed RACPM defines the start/finish times and the floats as resource-activity attributes with similar implications to their counterparts in the conventional CPM, and it determines such attributes as the end results of the resource-constrained CPM network analysis.

It is also noted that, in the existing methods for resource-constrained CPM scheduling, the resource constraints are defined in terms of the resources utilized not exceeding the resources available, which are in the form of inequality conditions for the optimization problems or an external resource pool whose inventory should always remain positive as in the heuristic methods. The resulting project schedule satisfies both the constraints of technological precedence relationships between activities specified in the CPM network and the constraints of resource availability, but it still features two dimensions—activity and time, as in the Gantt chart. In order to clearly reflect the allocation and utilization of resources in the project schedule, the proposed RACPM highlights the dimension of resource explicitly in addition to activity and time in project scheduling and produces a detailed and feasible schedule for individual resources working on activities.

Although latest repetitive scheduling methods have advanced in terms of providing the flexibility to maintain or interrupt the work continuity of resources for optimizing project cost or duration, the repetitive scheduling methods have difficulty handling (1) multitasking resources (resources specializing in more than one type of activity at various units are shared by different types of activities at the same or different units); and (2) multiresource tasks (multiple resource requirements for an activity). For instance, the commencement of the multiresources concreting activity on one floor depends on the availability of both the concreting crew and a tower crane used for delivering concrete to the workface, where the tower crane is a multitasking resource shared by many specialty crews on the job. The proposed RACPM allows for both multitasking resources and multiresource tasks in order to accommodate the practical requirements in construction planning.

Overview of Resource-Activity Critical-Path Method Strategies

Waugh and Froese (1990) proposed a knowledge-based system that is independent of the network analysis techniques in order to extend the representation of schedule constraints beyond precedence relationships and resource availability. The project current state labeled as STATUS was continually tracked and updated in the scheduling process to reflect the timely changes in each activity's schedule constraints. In the forward pass calculations, the knowledge base, dynamically linking to the STATUS and consisting of if-then rules, guided each activity to undergo a sequence of states, namely, TODO, CANDO, DOING, ENDED, and DONE. However, the system did not feature a backward pass, due to the difficulty in representing lags associated with ‘‘STATUS’’ constraints such as weather, labor conditions, etc. The proposed RACPM follows a forward pass strategy similar to that in Waugh and Froese (1990), but it only considers three states for each activity: TODO (unscheduled), CANDO (predecessor activities finished and eligible to request resources), and DONE. Following the forward pass, a backward pass processing is invoked as in the conventional CPM to determine the resource-activity floats.

Algorithm of Resource-Activity Critical-Path Method

The RACPM is essentially a heuristic method for resource-constrained CPM scheduling. The RACPM can handle (1) reusable work resources that perform an activity (such as a worker/crew or a piece of equipment) or facilitate an activity (such as discrete space blocks required by an activity to accommodate work resources or store materials); and (2) nonreusable material resources that do not work but get used by the activity (such as the prefabricated concrete blocks or pallets of bricks). To present the method clearly, two working tables are custom designed to organize the scheduling data and run the RACPM analysis. The following subsections will discuss the new method in detail in terms of the serial fashion, the heuristic rules, the network format, the algorithm, and the definitions of floats, along with an example for demonstrating how the method works.

Serial Fashion

The majority of existing heuristic methods for resource-constrained CPM scheduling prioritize activities that compete for limited resources in either a serial fashion or a parallel fashion (Ahuja et al. 1994). The main difference of two fashions is that, in a serial method, resources are freed from one activity and replenished to the resource pool for reallocation only when the activity is completed; in contrast, a parallel method frees and reallocates resources at the start of each time unit of scheduling (e.g., one day) based on the updated priorities for each ready activity, thus causing activity interruptions from time to time. Considering the apparent advantages of maintaining resource work continuity (such as maximized learning curve effect, minimized costs for transportation, and mobilization/demobilization of resources), the serial fashion is preferred over the parallel fashion. In cases where splitting one activity is technically and economically justifiable, the activity can be broken down into subactivities representing the physical work sections in order to apply a serial method. The RACPM maintains the resource work continuity in conducting each activity and hence is a serial method.

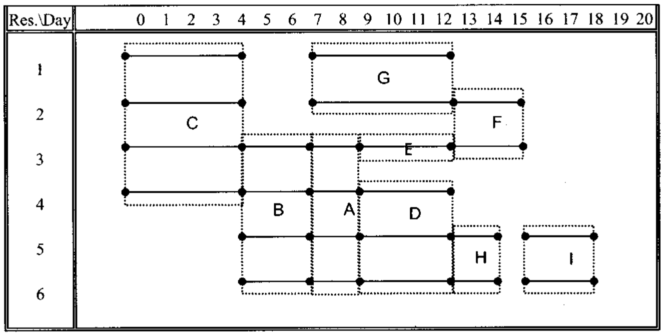


Fig. 1. Resource-activity critical-path method schedule highlighting resources, time, and activity

Heuristic Rules

In the proposed RACPM, the work content (expressed as the number of resources required times the duration of the activity) will be the primary criteria for assigning priority to competing activities in acquiring resources: The larger the work content, the higher priority an activity has to acquire resources and be executed; in cases where there is a tie with respect to the work content among activities, the higher priority is given to the activity with a larger number of resources required or a longer duration. In fact, any established heuristic rule for prioritizing activities can be embedded in the RACPM such as the ‘‘minimum total slack’’ or the ‘‘latest start prioritization’’ rule, since the objective of proposing the RACPM is not to find better priority rules that result in shorter project duration consistently but to obtain a working schedule and clearly resolve the ‘‘resource critical’’ issue. The work-content based priority rule is used in the RACPM mainly for two considerations. First, the work content reflects the relative weight of an activity in a schedule with both the time dimension and the resource dimension highlighted. Such a schedule is shown in Fig. 1, in which the horizontal axis is the timeline, the vertical axis is individual resources, the two end points of a straight-line section represent the earliest start time (EST) and earliest finish time (EFT) for one resource working on one activity, the gap between line sections represents the idle time (IDT) of the corresponding resource, and activities are symbolized using dashed rectangles. Visually, the work content is actually the area an activity (a dashed rectangle in Fig. 1) occupies in the schedule. The greater the work content, the greater impact an activity has on holding up project resources and extending the project duration. Hence, in order to bring the whole project to an early completion, limited resources are first committed to an activity with higher work content. As later demonstrated in the example problem, the simple work-content priority rule is comparable to

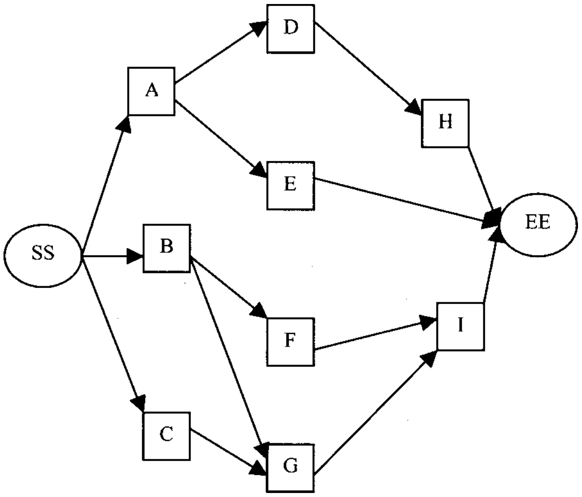


Fig. 2. Converted activity-on-node critical-path method network for example problem

the popular "minimum total slack" rule in terms of the project duration obtained. The second consideration of using the work-content priority rule lies in its dependability on duration and resource requirements of activities only, thus skipping the CPM calculations, as often required by other rules. For a project involving multiresource activities (the later case study is one example), the project manager may give a higher priority to activities that require a certain type of resource in acquiring resources and being executed, depending on the actual conditions. For instance, activities with a higher work content of using a rented crane are given higher priority to be executed so as to return the crane as soon as possible and minimize the rental cost.

In selecting resources to execute activities, the earliest-ready, first-serving rule is applied in the RACPM in order to assure that the work is uniformly distributed to all resources assigned such that each resource can be utilized as fully as possible. As a result, overworked and underworked resources can be avoided to a certain extent.

Network Format

Three network formats are commonly available for CPM scheduling: (1) I-J (also known as activity-on-arrow or AOA); (2) activity-on-node (AON); and (3) the precedence diagramming method (PDM) (Callahan et al. 1992). AON is similar to AOA in terms of only allowing straightforward finish-start relationships among activities, but AON is more streamlined than AOA in that it eliminates the dummy activities that are used in AOA for specifying precedence relationships among activities. PDM, as a more complicated form of AON featuring four logical relationships with lags (FS/SS/FF/SF), can significantly reduce the number of activities in a project schedule as compared with AON. Because of PDM's compactness in presenting a schedule and its availability in popular commercial scheduling packages (such as P3), PDM is gaining acceptance and popularity in the construction industry. Nonetheless, PDM is not as structured as the AON or I-J format and "as a result differences in diagrams, mathematical calculations, and project completion dates abound" (Callahan et al. 1992). Oberlender (2000) further elaborated the complexities and problems with PDM and maintained that "a PDM network can be converted to an AON by simply adding additional activities, which provides a clear understanding of the sequence of work, thus preventing confusions and misunderstandings of the project schedule." Therefore, AON is the network format selected for the proposed RACPM. A more involved PDM-based RACPM algorithm could be a future enhancement.

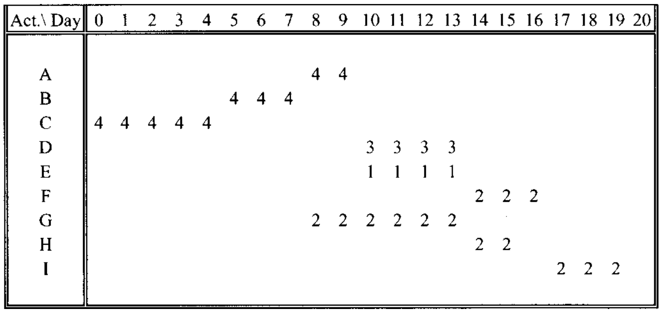
Table 1. Structure of Activity Table

Status (1)	Act. (2)	Dur. (3)	Res. (4)	ResDur (5)	EST (6)	EFT (7)
1	A	2	4L	8L		
1	B	3	4L	12L		
1	C	5	4L	20L		
0	D	4	3L	12L		
0	E	4	1L	4L		
0	F	3	2L	6L		
0	G	6	2L	12L		
0	H	2	2L	4L		
0	I	3	2L	6L		

Next, the algorithm of the RACPM will be presented based on an example problem taken from the text of Ahuja et al. (1994), which originally took the format of AOA and was used to illustrate the series method for resource-loading CPM. The converted AON network containing nine activities is shown in Fig. 2, with the ovals "SS" and "EE" symbolizing the start and finish of the total project. The duration and resource requirements for each activity can be found in Table 1 (Columns 3 and 4). The CPM analysis without imposing resource constraints gives a project duration of 14 days. Given six laborers available, the project duration is determined to be 20 days by applying a series method based on the "minimum total slack" rule, and the derived solution (activity-time Gantt chart) is plotted in Fig. 3 with the numbers in the plot area denoting the resource requirements. Next, the proposed RACPM is applied to the same problem.

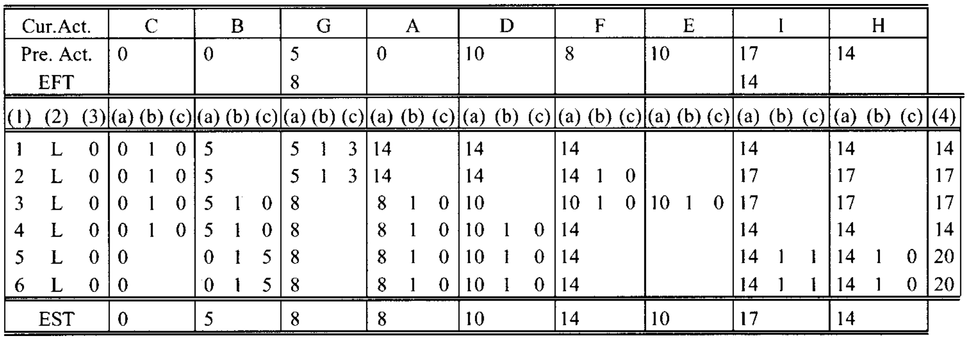
Initialization of Resource-Activity Critical-Path Method

In the activity table (Table 1), the "ResDur" column (Column 5) represents the work content of an activity, obtained by multiplying the duration of an activity (Column 3) with the number of resources required (Column 4). "L" in columns 4 and 5 represents the resource type of laborer. The Status column (Column 1) is updateable with three options for each activity: status 0, standing for TODO; status 1, for "CANDO"; and status 2 for DONE. Columns 6 and 7 are the EST and EFT of an activity and will be filled in during the forward pass processing. At the beginning, the status column in the example problem should be initialized with



Act.\ Day	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
A									4	4											
B						4	4	4													
C	4	4	4	4	4																
D											3	3	3	3							
E											1	1	1	1							
F															2	2	2				
G									2	2	2	2	2	2							
H															2	2					
I																		2	2	2	

Fig. 3. Schedule solution to example problem [source: Ahuja et al. (1994)]



Cur.Act.			C			B			G			A			D			F			E			I			H			
Pre. Act. EFT			0			0			5 8			0			10			8			10			17 14			14			
(1)	(2)	(3)	(a)	(b)	(c)	(a)	(b)	(c)	(a)	(b)	(c)	(a)	(b)	(c)	(a)	(b)	(c)	(a)	(b)	(c)	(a)	(b)	(c)	(a)	(b)	(c)	(a)	(b)	(c)	(4)
1	L	0	0	1	0	5			5	1	3	14			14			14						14			14			14
2	L	0	0	1	0	5			5	1	3	14			14			14	1	0				17			17			17
3	L	0	0	1	0	5	1	0	8			8	1	0	10			10	1	0	10	1	0	17			17			17
4	L	0	0	1	0	5	1	0	8			8	1	0	10	1	0	14						14			14			14
5	L	0	0			0	1	5	8			8	1	0	10	1	0	14						14	1	1	14	1	0	20
6	L	0	0			0	1	5	8			8	1	0	10	1	0	14						14	1	1	14	1	0	20
EST			0			5			8			8			10			14			10			17			14			

Fig. 4. Resource-activity interaction working table

status "0" except for activities A, B, and C, which don't have predecessor activities and hence are flagged with status " 1".

In the resource-activity interaction table (Fig. 4), the current activity being processed is entered in the "Cur. Act." line and the EFT of its corresponding predecessor activities are entered in the "Pre. Act. EFT" line. Note that the number of entries in the "Pre. Act. EFT" line of the current activity must be equal to the number of its predecessor activities in the CPM network. The EST of a current activity is entered in the "EST" line at the bottom. The four columns between the "Pre. Act. EFT" line and the "EST" line are designated to hold the attributes of the resources employed in the project. These are Column (1) for the resource ID, Column (2) for resource type (e.g., L stands for laborer), Column (3) for the initial ready-to-serve time (RST) (when the resources are ready to work; e.g., all the resources are assumed to be ready at day 0 as in the example), and Column (4) at the rightmost for the final end-of-service time. As shown in Fig. 4, three columns are specially designated to track the information of resource-activity interaction for the current activity. These are Column (a) for the RST time of the resources, Column (b) for participation of the resources ("l" is used to mark the resources that are involved in the current activity; a blank means "not involved"), and Column (c) for the IDT of the participating resources prior to the start of the current activity.

Forward Pass Processing

Running the forward pass of the RACPM on the working tables involves the following steps:

1. Fill in Column (a) under the first current activity with the initial RST times of the resources as in Column (3) (shown in Fig. 4).
2. In Table 1, select the Current Activity from those activities with status "1" following the work-content based priority rule. For example, in Table 1, activities A, B, and C all have a status of 1, but C will be chosen as the current activity due to its largest "ResDur" value (i.e., 20).
3. Enter the current activity in the "Cur. Act." line in Fig. 4, and mark the participating resources in Column (b) with "1" according to the resource requirement (Column 4 in Table 1) and the earliest-ready-first-serve rule—i.e., the smaller the number in Column (a), the higher priority to be chosen to participate in the current activity. Choose one resource randomly if more than one resource of the same type have identical RST times. Enter the "Pre. Act. EFT" line for the current activity in Fig. 4 by reading the EFT of its predecessor activities from Table 1.
4. From Fig. 4, determine the EST for the current activity by taking the maximum from the EFTs of predecessor activities in the "Pre. Act. EFT" line combined with the RST times of the participating resources in Column (a). Note that both the precedence relationship constraints and the resource constraints are accommodated in obtaining EST as such. Enter the EST of the current activity in the "EST" line.
5. Determine the IDTs of the participating resources for the current activity, i.e., IDT$=$EST$-$RST [as in Column (a)], or the EST of the current activity minus the RST time of the resource in Column (a). IDT is then entered in Column (c).
6. Check whether the resources that participate in the current activity and have idle times can be utilized to perform any other activity with Status "1" in Table 1. Note that it is only feasible to utilize the idle resources if two conditions are met: (1) such resources meet the resource requirements of the other CANDO activity; and (2) the determined EFT of the other CANDO activity is earlier than the EST of the

Table 2. Final Results of Activity Table for Example Problem

Status (1)	Act. (2)	Dur. (3)	Res. (4)	ResDur (5)	EST (6)	EFT (7)
2	A	2	4L	8L	8	10
2	B	3	4L	12L	5	8
2	C	5	4L	20L	0	5
2	D	4	3L	12L	10	14
2	E	4	1L	4L	10	14
2	F	3	2L	6L	14	17
2	G	6	2L	12L	8	14
2	H	2	2L	4L	14	16
2	I	3	2L	6L	17	20

Table 3. Resource-Constrained Precedence Relationships

Act. (1)	Suc.Act. (2)
A	E, D
B	A
C	B, G
D	H
E	F
F	EE
G	F
H	I
I	EE

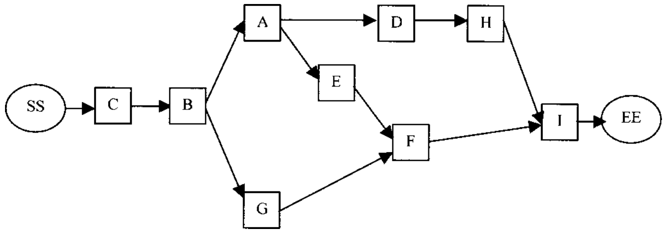


Fig. 5. Activity-on-node network combining resource-technology precedence relationships

current activity. For example, in Fig. 4, prior to the start of Activity F, resource 3 originally has a 4-day IDT, which can be utilized to schedule Activity E without affecting the start of Activity F. Similarly, two of the three idle days for resources 5 and 6 prior to the start of Activity I can be efficiently utilized by conducting Activity H. Note that the IDT in Column (c) for Activities F and I should be accordingly adjusted to reflect the utilization of such resources, as shown in Fig. 4. Also note that the EST and EFT of Activities E and H along with their own statuses and the statuses of their ready succeeding activities should be updated in time in Table 1.

7. Determine the EFT of the current activity as EFT=EST+DUR, in which EST is from the "EST" line corresponding to the current activity in Fig. 4 and DUR is the duration of the current activity as read from Table 1. The determined value of EFT is also the updated RST time for those resources participating in the current activity. Thus, copy the values of Column (a) for the current activity to the next Column (a) in Fig. 4, and update the RSTs in the new Column (a) only for the resources participating in the current activity with the determined value of EFT.
8. Enter the EST and EFT of the current activity in Table 1 and update its status (from 1 to 2) and the statuses of any ready succeeding activities (from 0 to 1).
9. Move back to Step 2 and repeat the above operations until the status turns to "2" for all the activities in Table 1, marking the end of the project. The final RST times of resources are updated and entered in Column (4) in Fig. 4, representing the earliest end-of-service times of the resources. For instance, the earliest end-of-service time for resource 1 is the end of day 13 (or the beginning of day 14), while that for resource 6 is the end of day 19 (or the beginning of day 20).

The final results of the RACPM forward pass processing for the example problem are presented in Table 2 and Fig. 4. Intermediate tables for each step, actually part of the final table, are not included within the paper due to size limitations, but can be readily reconstructed by the reader based on Table 2 and Fig. 4. The schedule obtained from the RACPM forward pass calculations has activity EST/EFT values and project durations (20 days) identical to the solution in Ahuja et al. (1994) (shown in Fig. 3). However, the schedule derived from the RACPM is more informative and detailed about the utilization of individual resources (Fig. 1).

Table 4. Backward Pass Calculations and Float Determinations

Act. (1)	Suc.Act. (2)	LFT (3)	LST (4)	EFT (5)	EST (6)	TF (7)	FF (8)
I	EE	20	17	20	17	0	0
H	I	17	15	16	14	1	1
F	I	17	14	17	14	0	0
E	F	14	10	14	10	0	0
G	I, F	14	8	14	8	0	0
D	H	15	11	14	10	1	0
A	E, D	10	8	10	8	0	0
B	A, F, G	8	5	8	5	0	0
C	G, B	5	0	5	0	0	0

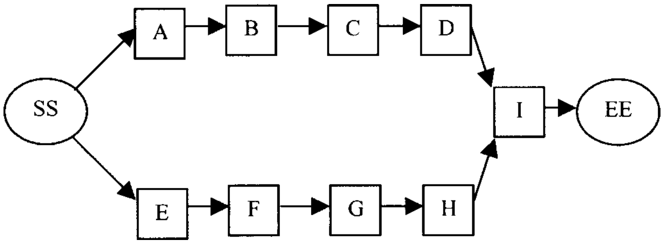


Fig. 6. Critical-path method network for footbridge construction project

Backward Pass Processing

The backward pass processing of the RACPM requires the definition of the resource-activity combined precedence relationships, based on which the latest finish time (LFT) and the latest start time (LST) for resources working on each activity are determined and floats (TF/FF) subsequently computed.

The resource-constrained precedence relationships between activities, differing from the technology-constrained ones in the original CPM, can be defined as follows: For one current activity, its resource-constrained successor activities include the immediately following activities that in part or in total involve the resources used in the current activity. For instance, in Fig. 1, resources 1, 2, 3, and 4 participate in Activity C; Activity G is the immediately following activity that resources 1 and 2 work on, and Activity B is what resources 3 and 4 do after finishing Activ-

Table 5. Activities' Duration and Resource Requirements for Footbridge Project

Act. (1)	Description (2)	Dur. (3)	Res. (4)	ResDur (5)
A	Excavation stage 1	2	2LB, 1EX	4LB, 2EX
B	Formwork stage 1	3	4LB, 1FM, 1MC	12LB, 3FM, 3MC
C	Concrete stage 1	5	4LB	20LB
D	Backfill stage 1	4	2LB, 1EX	8LB, 4EX
E	Excavation stage 2	3	2LB, 1EX	6LB, 3EX
F	Formwork stage 2	3	4LB, 1FM, 1MC	12LB, 3FM, 3MC
G	Concrete stage 2	6	4LB	24LB
H	Backfill stage 2	2	2LB, 1EX	4LB, 2EX
I	Erect steel work	3	3LB, 2MC, 1ST	9LB, 6MC, 3ST

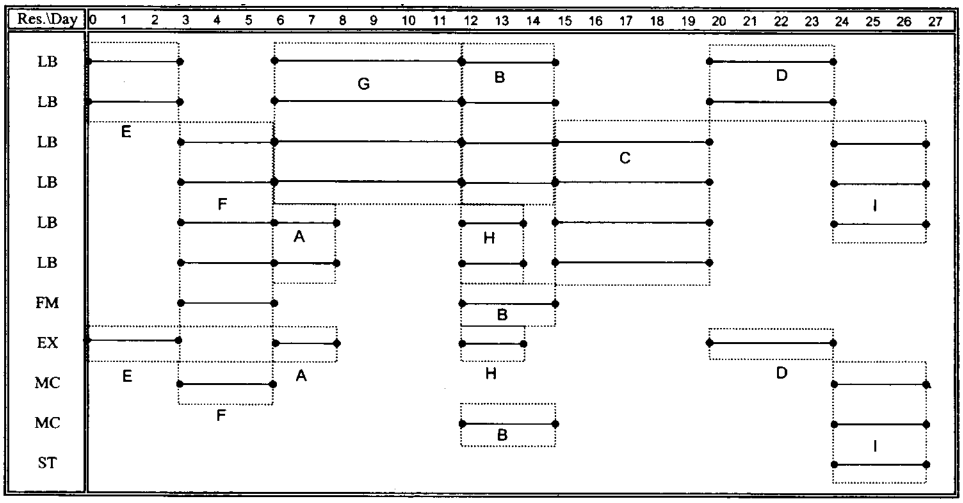


Fig. 7. Resource-activity critical-path method sample application: resource-activity interaction scheme

ity C. Hence, Activity C precedes Activities B and G in the resource-constrained sense. Table 3 summarizes the resource-constrained precedence relationships for the example problem. In order to facilitate the determination of LST/LFT/TF/FF in the backward pass calculations, the resource-constrained precedence relationships (Table 3) are combined with the technology-constrained ones (Fig. 2) and presented in the AON network format (Fig. 5). Reminiscent of constructing an AON network for the classic CPM, redundant relationships among activities should be removed in plotting the combined AON network to show the immediate preceding activities only. For instance, the relationship of G preceding C, sufficiently defined by the relationships of G preceding B and B preceding C, is deemed redundant and hence is eliminated. From Fig. 5, the backward pass calculations are made in the following two steps, analogous to the classic CPM:

1. Let the LFT of the last activity on the network be equal to its EFT (i.e., the project completion time), and LST=LFT −DUR. In Fig. 5, activity I is the last one, so $\mathrm{LFT}_{\mathrm{I}}=\mathrm{EFT}_{\mathrm{I}}=20$, and $\mathrm{LST}_{\mathrm{I}}=\mathrm{LFT}_{\mathrm{I}}-\mathrm{DUR}_{\mathrm{I}}=20-3=17$.
2. Moving in the backward direction along the network, determine the LFT and LST for each activity by Eqs. (1) and (2) until the "project start" SS is reached:

$$\mathrm{LFT}_{\mathrm{cur}}=\mathrm{Min}(\mathrm{LST}_{\mathrm{suc}})_N \tag{1}$$

$$\mathrm{LST}_{\mathrm{cur}}=\mathrm{LFT}_{\mathrm{cur}}-\mathrm{DUR}_{\mathrm{cur}} \tag{2}$$

where $\mathrm{LFT}_{\mathrm{cur}}$ = LFT of the current activity; $\mathrm{LST}_{\mathrm{suc}}$ = LST of an activity succeeding the current activity; N = total number of activities succeeding the current activity; and $\mathrm{Min}(\,)_N$ = function of taking the minimum value among N data items.

The process and results of calculating LFT and LST for the example are listed in the first four columns in Table 4. To determine the TF and FF, the EFT/EST for each activity are simply read from Columns (5) and (6) in Table 2. The TF/FF values of the RACPM are defined in a similar sense to those in the original CPM, with TF referenced to the end of the project and FF referenced to the early start schedule of succeeding activities, as shown in Eqs. (3) and (4):

$$\mathrm{TF}_{\mathrm{cur}}=\mathrm{LFT}_{\mathrm{cur}}-\mathrm{EFT}_{\mathrm{cur}}=\mathrm{LST}_{\mathrm{cur}}-\mathrm{EST}_{\mathrm{cur}} \tag{3}$$

$$\mathrm{FF}_{\mathrm{cur}}=\mathrm{Min}(\mathrm{EST}_{\mathrm{suc}})_N-\mathrm{EFT}_{\mathrm{cur}} \tag{4}$$

where $\mathrm{TF}_{\mathrm{cur}}$ = TF of the current activity; $\mathrm{LST}_{\mathrm{cur}}$, $\mathrm{LFT}_{\mathrm{cur}}$, $\mathrm{EST}_{\mathrm{cur}}$, and $\mathrm{EFT}_{\mathrm{cur}}$ = LST, LFT, EST, and EFT of the current activity; $\mathrm{FF}_{\mathrm{cur}}$ = FF of the current activity; $\mathrm{EST}_{\mathrm{suc}}$ = EST of an activity succeeding the current activity; and N = total number of activities succeeding the current activity. Hence, the TF is a nonnegative

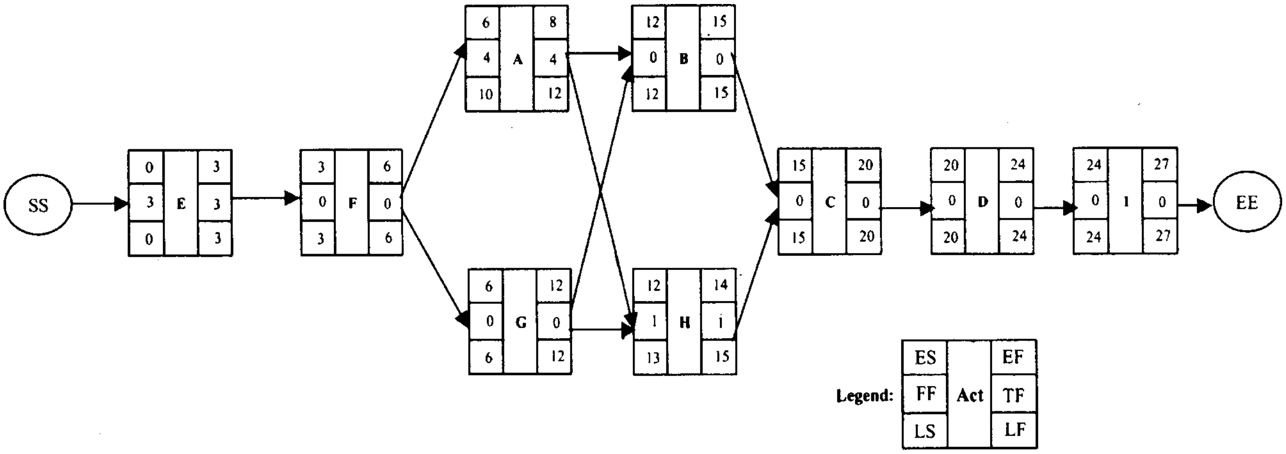


Fig. 8. Resource-activity critical-path method sample application: combined activity-on-node network with floats calculation

resource-activity attribute defined for the resources working on one activity such that any delay up to the time of TF on the current activity will not extend the total project duration. For example, from Table 4, all activities are critical except D and H, which have a positive TF of one day, meaning only resources working on either D or H (i.e., laborers 4, 5, and 6) may delay their work for one day for some reason, without extending the project duration. On the other hand, the FF is a nonnegative resource-activity attribute defined for the resources working on one activity such that any delay up to the FF of the current activity will not postpone the earliest start of all the succeeding activities according to the schedule. For example, from Table 4, only activity H has a positive FF of one day, meaning if resources 5 and 6 working on activity H delay the finish of the activity for one day, their early start schedule of working on activity I will not be affected.

The obtained TF/FF can be visually validated by moving the activity block in Fig. 1. Based on TF calculations, two critical paths originating at the project start ‘‘SS’’ and terminating at the project end ‘‘EE’’ can be identified in Fig. 5, namely, C-B-A-E-F-I and C-B-G-F-I. It should be emphasized once again that the TF is resource-activity attributes and the paths are resource-activity critical; both the activities and the resources involved in those activities are critical to completing the project within schedule.

The RACPM is not limited to single-type resources and work resources as in the example problem; the RACPM is also applicable to: (1) multiple resource types, e.g., different specialty crews or laborers with different skills, or different types of construction equipment; and (2) material resources, which are nonreusable construction resources and participate in activities once only, and thus for the corresponding material resource only a single ‘‘1’’ is allowed in Columns (b) across Fig. 4. Those situations are encountered in the following sample application project.

Sample Application

A sample application of the proposed RACPM is based on a local project of constructing a small footbridge, consisting of three stages of construction and requiring multiple types of resources. The two abutments (including footers and supports) are constructed in Stages 1 and 2, respectively, which are reinforced cast-in-place concrete structures. Stage 3 is to erect the superstructure, which is prefabricated in a remote steel plant and moved into the site for installation. The project network is shown in Fig. 6 with the duration and resource requirements for each activity listed in Table 5, in which LB stands for multitasking skilled laborers, FM stands for the formwork set for concreting footer and abutment, EX stands for an excavator, MC stands for a mobile crane, and ST stands for the steel superstructure prefabricated. Note that the work contents of identical activities on two stages are slightly different due to particular site conditions and slight design variations on each abutment. The resources available are six laborers, one set of formwork rented, one excavator, two mobile cranes, and one set of prefabricated superstructure scheduled to be moved to site on day 17. On this project, the labor work content of multitasking skilled laborers is the primary criteria in deciding the priority for assigning resources to competing activities.

The project schedule resulting from the RACPM analysis is presented in Figs. 7 and 8. Given the resources assigned, it takes 27 days to complete the footbridge construction and the prefabricated superstructure should be ready for erection by day 24. Resources working on activities H and A (i.e., laborers 5 and 6 and the excavator) own nonzero FF/TF values (four-day TF/FF for A, and one-day TF/FF for H), with the other activities making up a resource-activity critical path (i.e., E-F-G-B-C-D-I).

Conclusions

This paper has addressed the long-standing scheduling problem of how to consider resource capabilities and availability in CPM scheduling by proposing the RACPM, in which the dimension of resources are considered in addition to activity and time in construction planning. RACPM defines the start/finish times and the floats as resource-activity attributes based on the resource-technology combined precedence relationships. Hence, practitioners can readily relate the RACPM to the classic CPM to interpret and utilize an RACPM schedule in practice, as demonstrated in a sample application of constructing a footbridge. The RACPM provides the schedulers with a convenient vehicle for seamlessly integrating the technology/process perspective with the resource-use perspective in construction planning. The effect on the project duration and activity floats of varied resource availability can be studied through running RACPM on different scenarios of resources. This potentially leads to an integrated scheduling and cost-estimating process that will produce realistic schedules, estimates, and control budgets for construction.

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