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Optimization model for resource assignment problems of linear construction projects

Shu-Shun Liu [a,*], Chang-Jung Wang [b,1]

[a] Department of Construction Engineering, National Yunlin University of Science and Technology, No. 123, Section 3, University Road, Touliu, Taiwan, 640, ROC
[b] Graduate School of Engineering Science and Technology, National Yunlin University of Science and Technology, No. 123, Section 3, University Road, Touliu, Taiwan, 640, ROC

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Abstract

For linear construction projects, it has long been known that resource utilization is i]mportant in improving work efficiency. However, most existing scheduling techniques cannot satisfy the need for solving such issues. This study presents a flexible model for resolving linear scheduling problems involving different objectives and resource assignment tasks. The proposed model adopts constraint programming (CP) as the searching algorithm for model formulation, and CP program implemented in this study creates the flexibility for optimizing either total cost or project duration. Additionally, the concept of outsourcing resources is introduced here to improve project performance.

A bridge project from pertinent literature is selected, and model validation and two scenarios are conducted to demonstrate the model capability. According to the research results, consideration of outsourcing resources not only helps achieve diversity of resource supply but also creates a positive influence on the optimum solution by simultaneously reducing time and cost.

Keywords: Linear construction projects; Linear scheduling; Resource assignment; Constraint programming; Outsourcing resources

1. Introduction

Effective resource utilization for linear construction projects, such as high-rise buildings and bridges, has attracted growing attention in the construction industry. Linear construction projects typically have repetitive activities, and have the same operations repeated at each unit [1]. Linear construction projects generally have three principal activities – linear, block, and bar – that can be divided into repetitive and non-repetitive activities [2]. As a linear construction project primarily constitutes several similar activities in units or sections, the production rate for each activity is significantly influenced by major resources. Thus, arranging the usage of limited major resources is essential to achieving project goals.

Traditional scheduling methods such as CPM or PERT have been found to be inappropriate for application to resource assignment problems of linear construction projects. Due to the shortcomings of CPM, scheduling techniques, such as line of balance (LOB) and linear scheduling method (LSM), have been developed specifically for linear construction projects. This study develops a flexible model for linear scheduling problems that accommodates different optimization objectives such as minimizing project total cost or duration under the same model formulation. Furthermore, the concept of outsourcing resources derived from temporary subcontracting is also introduced, and integrated with the proposed model for implementing in current construction practices.

An example project from El-Rayes and Moselhi [3] is utilized to assess model capability and validate the proposed model. During model validation, project duration and total interruption minimizations are regarded as objectives. Moreover, the following two scenarios are analyzed: (1) outsourcing resources and related constraints are applied to model formulation, and the impact on project duration and total cost is discussed; and (2) given a desirable duration for the example project, optimum project total cost is determined by considering outsourcing resources. Through scenario analysis, the

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[*] Corresponding author. Tel.: +886 5 5342601x4724; fax: +886 5 5312049.
E-mail addresses: liuss@yuntech.edu.tw (S.-S. Liu), g9210817@yuntech.edu.tw (C.-J. Wang).
[1] Tel.: +886 5 5342601x4701; fax: +886 5 5312049.

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importance of outsourcing resources regarding linear construction projects is then recognized.

2. Literature review

Numerous techniques for handling linear scheduling problems have been developed in recent decades. Several studies have adopted mathematical programming, such as linear programming and integer programming. Huang and Halpin [4] propose a graphical-based approach called POLO System to assist in the linear programming (LP) modeling of linear scheduling problems. Mattila [5] presented an integer programming model for leveling the resources of activities in a linear construction project that is scheduled using the linear scheduling method.

Furthermore, some significant studies have been conducted using dynamic programming. Selinger [6] employed dynamic programming approach to minimize project duration, and it is the first work to adopt dynamic programming approach for linear construction projects. Moreover, Russell and Caselton [7] extended the work of Selinger in developing a two-state variable, N-stage dynamic programming formulation that minimizes the duration of linear construction projects. Additional research has utilized dynamic programming in minimizing total cost or project duration by integrating cost, time categories, or heuristic rules [3,8,9,10].

Given the rapid development of computer-based techniques, researchers have used artificial intelligence techniques, such as genetic algorithms, to solve the increasing complexity of construction projects. For example, Kang et al. [11] developed a construction scheduling model using a conceptual approach for improving the efficiency of construction resources for a multiple, repetitive construction process (MRCP). Leu and Hwang [12] addressed a GA-based resource-constrained linear scheduling model. Hyari and El-Rayes [13] constructed a multi-objective optimization model that includes genetic algorithms for planning and scheduling repetitive construction projects, and helps planners in evaluating optimum construction plans by minimizing project duration and maximizing work continuity, simultaneously.

Constraint programming (CP) was recently applied as a new technique for handling combinatorial problems. Brailsford [14] compared the advantages of CP techniques for solving numerous constraint satisfaction problems (CSPs) such as rostering and time-tabling problems. Lottaz et al. [15] constructed a constraint-based support approach for collaboration in design and construction, and provided a simple implementation of least-commitment decision strategies. Regarding construction applications, Chan and Hu [16] applied the CP approach to production scheduling for precast plants, and implemented several problem-derived constraints to improve resource utilization. However, literature review demonstrated that the CP method has not been adapted to linear scheduling problems for optimization purposes.

This study presents a novel model with the flexible structure of CP techniques for optimizing linear scheduling problems. Given the constraints arising from project contracting in current construction scheduling practices, the concept of outsourcing resources is introduced, and two relevant scenarios are examined to demonstrate model capability.

3. Constraint programming

Constraint programming (CP) is the computer implementation for solving constraint satisfaction problems (CSPs) and incorporates techniques from mathematics, artificial intelligence and operations research [16,17]. CSPs are generally treated as combinatorial problems [18], in which CP techniques recognize such subject by declaring clean problem statements, resolution of problems, and propagation of the effects of decisions. The principal advantage of CP is that constraints define feasible solution domains, and that it can be utilized by the solution procedure to locate optimum solutions [16]. Based on the declarative language in CP techniques, constraints can be added and rewritten simply; moreover, using consistency techniques and systematic search methods, solution domains are identified and feasible solutions are determined.

Basic CP formulation consists of problem specification, consistency techniques and systematic search algorithms for problem solving. In addition to recognizing a model objective, the problem definitions are constructed as follows: (1) a set of variables $X=\{x_1, \ldots, x_n\}$; (2) each variable, x_i, has a finite set D_i of possible values (its domain); and (3) a set of constraints restricting the values that the variables can simultaneously take, and then can be recognized [19]. Furthermore, CP provides users with different consistency techniques, such as node consistency, arc consistency, and path consistency, for variable domain reduction [20,21]. To improve the computational efficiency when solving problems, several CP algorithms can be utilized. The search strategies include generate and test (GT), backtracking (BT), forward checking (FC), etc. [21]. Additionally, appropriate ordering of initializing variable values also reduces computational efforts and promotes search ability.

Several approaches can be employed to handle CSPs, such as mathematical programming, genetic algorithms, and neural network. Mathematical programming and CP techniques are applied to find an exact solution. Compared with mathematical programming, CP can search for solutions more simply depending on the algorithm chosen by users, and it is not restricted by any particular model formulation, such as linear equations [22]. Genetic algorithms and neural network provide approximate solutions and aim to solve problems in a short period. Such techniques, which are applied in different fields, can be imprecise and varied due to different user experiences.

CP is a special-purpose technique that is widely employed for solving CSPs and combinatorial problems in which constraints are naturally incorporated into the problem description [16]. The CP techniques are appropriate for CSPs for the following four reasons [14]: (1) ease of implementation; (2) flexibility in handling a variety of constraints; (3) short computation time; and (4) good solution quality. Due to the rapid development of computers, CP has been widely and successfully applied to complex combinatorial problems, including rostering, and scheduling problems, in various fields [17,23], however, CP has not been widely applied to civil engineering research [16].

This study employs CP techniques to optimize the allocation of resources required by linear construction projects. In this study, CP techniques are employed during model construction for the following three reasons: (1) CP is proper for solving combinatorial problems such as resource assignment; (2) due to the logical and sequential constraints in CP, prioritizing activities when handling linear scheduling problems becomes clear; and (3) the model constructed via CP is flexible, and constraints and objection functions can be simply modified to meet various requirements without rebuilding the model. Based on these advantages, the proposed model can allocate resources, and identify the optimum solution that minimizes total cost or project duration. In this study, ILOG OPL language [24] is adopted as the model formulation language.

4. Outsourcing resources

It is essential to prepare feasible resource plans based on schedule plans and budgetary limitations for construction projects. In practice, additional temporary resources are typically purchased to support resource plans, shorten project duration or possibly decrease total cost. That is, this practice is sometimes referred to as “project duration compression”. Therefore, outsourcing actions such as temporary subcontracting are generally considered and executed to eliminate deficiencies in the initial resource plan. For example, adding crews and construction equipment can be regarded as outsourcing actions for improving project performance.

This study presents an optimization-based view that provides information for making outsourcing decisions. Outsourcing resource in this study is defined as the temporary addition of resource required to fulfill managerial goals and shorten the duration of specific activities. These resources enhance flexibility in linear construction projects during planning phases. In the proposed model, outsourcing resources are typically purchased at a higher-than-normal unit price, and, once the resources are assigned to specific activities, cannot be released until those activities finish. Regarding the applicability, outsourcing resources are frequently used for specific activities and improve project performance. Generally, utilizing outsourcing resource results in high production rate. However, the primary issues for planners when making outsourcing decisions are as follows: (1) which activities require outsourcing resources; (2) how many of outsourcing resources are needed; and (3) when outsourcing resources are needed and when they can be dismissed. Thus, the concept of outsourcing resources is implemented in the proposed model via a set of decision variables for each activity that determine the timing of assigning outsourcing resource assignment and the quantity of resources needed.

Essentially, quantities of outsourcing resources are constrained by the construction project environment. For example, the quantity of construction equipment is limited by the construction site and the project budget. Thus, establishing an appropriate range for outsourcing resources that comply with such constraints is central to successful resource planning. This study defines the ranges of outsourcing resources as parameters, which, in practice, should be based on planner experience.

5. Model formulation

The primary aim of this study is to identify the optimum solution that minimizes project duration or total cost for a linear construction project by considering outsourcing resources. To fit the characteristics of linear scheduling problems, the constraints in CP formulation are described in several parts, including activity and resource constraints, and the calculation of total cost and CP optimization algorithm are also illustrated.

However, the following assumptions must be made before deriving the model formulation:

1. The influence of learning behavior on crew formation for repetitive sections is ignored.
2. The production rate for outsourcing resources, which is defined in this study, is assumed to be identical for the same type of activities.
3. Given an upper limit for quantities of outsourcing resources, outsourcing resources remain available until projects’ end.

The constraints comprising activity precedence relationships and outsourcing resources are described as follows – the cost components are also listed.

5.1. Activity constraints

Four typical scheduling relationships and job continuity logic of repetitive activities are shown as the following equations. Additionally, the production rate associated with outsourcing resources is introduced in this section. The constraints of each type of activity relationship are, respectively, as follows:

Finish to Start (FS)

$$S_j^i \geqq F_j^{i-1} \tag{1a}$$

Start to Start (SS)

$$S_j^i \geqq S_j^{i-1} \tag{1b}$$

Finish to Finish (FF)

$$F_j^i \geqq F_j^{i-1} \tag{1c}$$

Start to Finish (SF)

$$F_j^i \geqq S_j^{i-1} \tag{1d}$$

where:

S_j^i	Start date of repetitive activity type i in section j
F_j^i	Finish date of repetitive activity type i in section j

To maintain job continuity for the same repetitive activities, a successor activity can start only once its predecessors finish. Moreover, for each crew formation, queuing time between two activities of the same type is defined as an interruption, and total interruption days are shown in Eq. (3). The principal constraint for linear scheduling problems is as follows:

$$S_j^i = F_{j-1}^i + I_{j-1}^i;\ j \geqq 2 \tag{2}$$

$$\mathrm{TID} = \sum_1^i \sum_2^j I_{j-1}^i \tag{3}$$

where:

F_{j-1}^i	Finish date of repetitive activity type i in section $j-1$
I_{j-1}^i	Interruption days of repetitive activity type i between section $j-1$ and j
TID	Total interruption days through the whole project

For each activity, the following precedence logic is used

$$F_j^i = S_j^i + D_j^i \tag{4}$$

where:

D_j^i	Duration of repetitive activity type i in section j

Depending on work quantity and production rate associated with outsourcing resource utilization, the duration for each activity is calculated as:

$$D_j^i = Q_j^i / \left[P^i + \left(\mathrm{OR}_j^i \times \mathrm{EP}^i\right)\right] \tag{5}$$

where:

Q_j^i	Quantity of work for repetitive activity type i in section j
P^i	Production rate of crew formation for repetitive activity type i
OR_j^i	Quantity of outsourcing resource per day added to crew formation for repetitive activity type i in section j, if needed
EP^i	Unit production rate for outsourcing resource added to crew formation for repetitive activity type i

Unit production rate for outsourcing resource (EP^i) is determined by the following equation:

$$\mathrm{EP}^i = \left(P^i / \mathrm{CS}^i\right) \times \mathrm{IFP}^i \tag{6}$$

where:

CS^i	Crew size of crew formation for repetitive activity type i
IFP^i	Influence factor of unit production rate for outsourcing resource added to repetitive activity type i, which may be higher or lower than 1, depending on the evaluation for the productivity of outsourcing resource

5.2. Resource constraints

One topic investigated in this study is maintaining or accelerating project progress by allocating outsourcing resources to the crew formation for specific activities in order to adjust the production rate. However, the upper limit of the outsourcing resource should be reasonable and practical

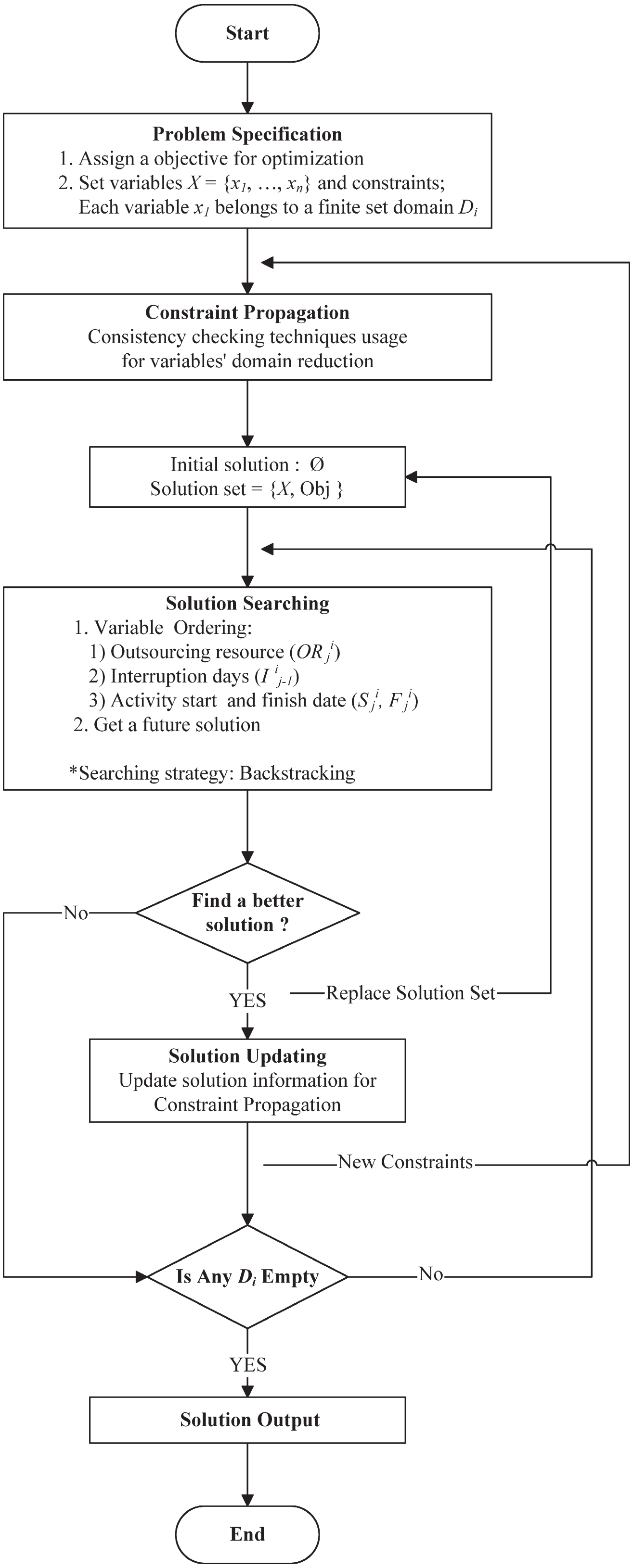

Fig. 1. CP optimization algorithm.

Table 1
Example data for validation (El-Rayes and Moselhi [3])

Repetitive activity	Quantity (m^3)				Possible crew formations' data					
	Section 1	Section 2	Section 3	Section 4	Crew formation no.	Crew size	Output (m^3/day)	Material cost (\$/m^3)	Labor cost (\$/day)	Equipment cost (\$/day)
Excavation	1147	1434	994	1529	1	6	91.75	0	340	566
Foundations	1032	1077	943	898	1	10	89.77	92	3804	874
					2	8	71.81	92	2853	655
					3	6	53.86	92	1902	436
Columns	104	86	129	100	1	10	5.73	479	1875	285
					2	12	6.88	479	2435	371
					3	14	8.03	479	3000	456
Beams	85	92	104	80	1	7	9.90	195	3934	315
					2	6	8.49	195	3238	259
					3	5	7.07	195	2544	204
					4	4	5.66	195	1850	148
Slabs	0	138	114	145	1	9	8.73	186	2230	177
					2	8	7.76	186	1878	149

No outsourcing resources involved; daily indirect cost=\$1000.

Table 2
Comparison of optimum solutions of project duration minimization

Section (1)	Activity									
	Excavation		Foundations		Columns		Beams		Slabs	
	Start (2)	Finish (3)	Start (3)	Finish (4)	Start (5)	Finish (6)	Start (7)	Finish (8)	Start (9)	Finish (10)
(a) Two-state variable formulation (El-Rayes and Moselhi [3])										
1	0	12.5	15.6	27.1	27.2	40.1	43.0	51.6		
2	12.5	28.1	28.1	40.1	40.1	50.8	51.6	60.9	61.3	77.1
3	28.1	38.9	40.1	50.6	50.8	66.9	66.9	77.1	77.1	90.1
4	38.9	55.6	55.6	65.6	66.9	79.4	80.1	88.1	90.1	106.8
Interruption	0		6		0		9		0	
Project duration	106.8 days									
Optimum crew formation	1, 1, 3, 1, 1 (sorted by activity precedence)									
Total interruption	15 days									
(b) Proposed constraint programming formulation										
1	0	12.5	15.7	27.2	27.2	40.1	43.4	52.0		
2	12.5	28.1	28.1	40.1	40.1	50.8	52.0	61.3	61.3	77.1
3	28.1	38.9	40.1	50.6	50.8	66.9	66.9	77.1	77.1	90.1
4	38.9	55.6	55.6	65.6	66.9	79.4	79.4	87.4	90.1	106.8
Interruption	0		5.9		0		7.9		0	
Project duration	106.8 days									
Optimum crew formation	1, 1, 3, 1, 1 (sorted by activity precedence)									
Total interruption	13.8 days									

based on considerations of budget and outsourcing resource availability. Eq. (7) shows the outsourcing resource constraints:

$$0 \leqq \mathrm{OR}_j^i \leqq \mathrm{ORA}^i \tag{7}$$

where:

ORA^i	Upper limit per day of the quantity of outsourcing resource utilized for repetitive activity type *i*

5.3. Project total cost

In this study, project total cost equals the sum of the direct and indirect costs, as illustrated in Eq. (8). Direct cost comprises material, equipment, labor, and outsourcing resource costs, as shown in Eq. (9), and indirect cost which is calculated on a daily basis is defined as the expression of Eq. (10).

$$\mathrm{TC} = \mathrm{DC} + \mathrm{IC} \tag{8}$$

$$\mathrm{DC} = (\mathrm{MC} + \mathrm{EC} + \mathrm{LC} + \mathrm{ORC}) \tag{9}$$

$$\mathrm{MC} = \sum_1^i \sum_1^j Q_j^i \times \mathrm{MC}^i \tag{9a}$$

$$\mathrm{LC} = \sum_1^i \sum_1^j D_j^i \times \mathrm{LC}^i \tag{9b}$$

$$\mathrm{EC} = \sum_1^i \sum_1^j D_j^i \times \mathrm{EC}^i \tag{9c}$$

$$\mathrm{IC} = \mathrm{ICP} \times T \tag{10}$$

where:

TC	Total cost
IC	Indirect cost
DC	Total direct cost
MC	Total material cost
EC	Total equipment cost
LC	Total labor cost
ORC	The sum of outsourcing resource cost utilized through a linear project
MC^i	Unit material cost per cubic meter for repetitive activity type i
LC^i	Unit labor cost per day with crew formation for repetitive activity type i
EC^i	Unit equipment cost per day with crew formation for repetitive activity type i
ICP	Indirect cost per day
T	Project duration

The calculation of outsourcing resource cost is shown as the following equations:

$$\mathrm{ORC} = \sum_{1}^{i}\sum_{1}^{j}\mathrm{OR}_j^i \times \mathrm{ORC}^i \times D_j^i \tag{11}$$

$$\mathrm{ORC}^i = \left[\sum_{1}^{j}\left(\mathrm{LC}^i + \mathrm{EC}^i\right)/\mathrm{CS}^i\right] \times \mathrm{IFC}^i \tag{12}$$

where:

ORC^i	Unit cost for outsourcing resource utilized for repetitive activity type i, which may be higher than the normal contract price as it is a temporary supplementary resource
IFC^i	Influence factor of unit cost for outsourcing resource added to repetitive activity type i, which may be higher or lower than 1, depending on the evaluation on the expense of outsourcing resource

5.4. CP optimization algorithm

Fig. 1 illustrates the CP optimization algorithm in the proposed model. The objective and variables are determined in the problem specification stage. The consistency checking technique is then applied to a constraint propagation mechanism to reduce variables' domains and locate feasible solutions. Next, a set of initial solution, including variables and the objective, is assigned as an empty set. Furthermore, three elements involved in solution searching – search strategy, variable ordering, and future solution acquisition – must be determined in the algorithm. This study adopts backtracking (BT) search that is utilized for most constraint satisfaction problems (CSPs). The key variables' values are initialized in the following order: (1) Outsourcing resource (OR_j^i); (2) Interruption days (I_{j-1}^i); and (3) Activity start and finish date (S_j^i, F_j^i). The solution searching in the optimization algorithm begins from the smallest value in each variable's domain. When a better solution is obtained, the original solution set is replaced, and solution information is updated and becomes a new constraint for constraint propagation and variables' domain reduction. However, the current solution is retained when a poorer solution is found, and further searching is executed when necessary. The algorithms for solution updating and re-searching are repeated until any variable's domain (D_i) is empty, thereby ensuring algorithm efficiency of

Table 3
Comparison of solutions for duration minimization and interruption minimization

Section (1)	Activity									
	Excavation		Foundations		Columns		Beams		Slabs	
	Start (2)	Finish (3)	Start (3)	Finish (4)	Start (5)	Finish (6)	Start (7)	Finish (8)	Start (9)	Finish (10)
(a) Project duration minimization										
1	0	12.5	15.7	27.2	27.2	40.1	43.4	52.0		
2	12.5	28.1	28.1	40.1	40.1	50.8	52.0	61.3	61.3	77.1
3	28.1	38.9	40.1	50.6	50.8	66.9	66.9	77.1	77.1	90.1
4	38.9	55.6	55.6	65.6	66.9	79.4	79.4	87.4	90.1	106.8
Interruption	0		5.9		0		7.9		0	
Project duration	106.8 days									
Optimum crew formation	1, 1, 3, 1, 1 (sorted by activity precedence)									
Total interruption	13.8 days									
(b) Total interruption minimization										
1	0	12.5	13.7	28.1	32.6	45.5	47.3	59.3		
2	12.5	28.1	28.1	43.1	45.5	56.2	59.3	72.3	72.3	88.1
3	28.1	38.9	43.1	56.2	56.2	72.3	72.3	86.5	88.1	101.1
4	38.9	55.6	56.2	68.7	72.3	84.8	86.5	97.7	101.1	117.8
Interruption	0		0		0		0		0	
Project duration	117.8 days									
Optimum crew formation	1, 2, 3, 3, 1 (sorted by activity precedence)									
Total interruption	0 days									

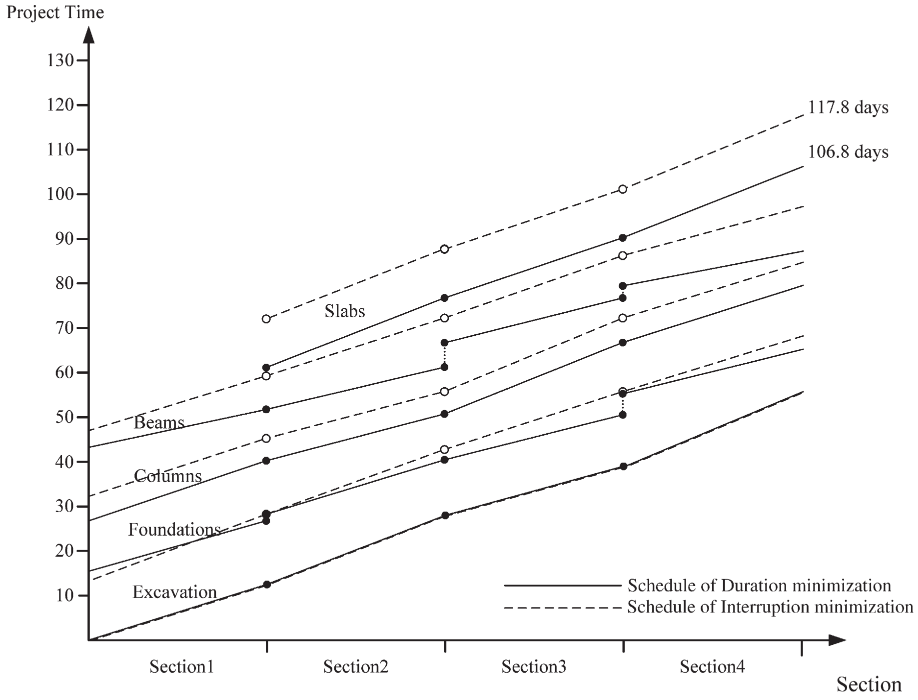

Fig. 2. LSM diagram of model validation.

solution searching. Consequently, an optimum solution for the objective can be guaranteed by this algorithm.

6. Model validation

The bridge example originally introduced by El-Rayes and Moselhi [3] is adopted to validate the proposed model, and Hyari and El-Rayes [13] also employed the example to consider the balance between minimizing project duration and maintaining crew work continuity. To maintain work continuity, total interruption time (the summary of all queuing times between two activities of the same type) is introduced as a variable. Therefore, two situations, project duration and total interruption minimization, are discussed in assessing model accuracy, and Table 1 presents the bridge project data.

6.1. Project duration minimization

The objective of previous research [3] was to optimize project duration with minimum interruption days using dynamic programming. However, the concept of outsourcing resources is not utilized in this section, and thus the amount for each

Table 4
Modified example data

Repetitive activity	Quantity (m^3)				Base crew size	Output (m^3/day)	Material cost (\$/m^3)	Labor cost (\$/day)	Equipment cost (\$/day)	Limit of outsourcing recourse	Unit cost of outsourcing resource	Production rate of outsourcing resource
	Section 1	Section 2	Section 3	Section 4								
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)[a]	(13)[b]
Excavation	1147	1434	994	1529	6	91.75	0	340	566	3	181	9.18
Foundations	1032	1077	943	898	6	53.86	92	1902	436	3	468	5.39
Columns	104	86	129	100	10	5.73	479	1875	285	5	259	0.34
Beams	85	92	104	80	4	5.66	195	1850	148	2	599	0.85
Slabs	0	138	114	145	8	7.76	186	1878	149	4	304	0.58

Non-repetitive activity

Activity	Total cost (\$)	Duration (day)	Note
Ground improvement_1	12,000	10.5	No outsourcing resources involved
Ground improvement_2	10,000	8.5	Indirect cost: \$1000 per day

[a] Col. (12) = Eq. (12) = $\{[\text{Col. (9)} + \text{Col. (10)}]/\text{Col. (6)}\} \times IFC^i$; $IFC^i = 1.2$.

[b] Col. (13) = Eq. (6) = $[\text{Col. (7)} \div \text{Col. (6)}] \times IFP^i$; $IFP^i = 0.6$.

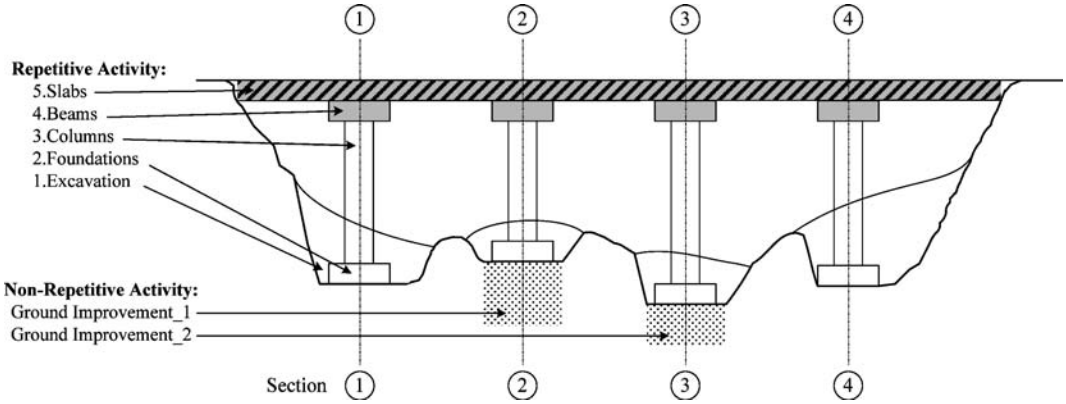

Fig. 3. Modified bridge example.

outsourcing resource is set at zero. The objective function is established as follows:

Objective: Minimize T

The proposed model optimizes project duration, and Table 2 presents a comparison between the analytical results form different formulations. The optimum project durations obtained are identical (106.8 days), however, the details of the activity schedules vary as shown in Table 2. Additionally, the total interruption days (13.8 days) calculated by the proposed model are less than the original result (15 days), because the parameters for interruption are set as integers in the previous study. Therefore, due to the similar number of interruption days, crew work continuity is maintained with the same objective function in this case. However, as the algorithm for optimizing total interruption days is not utilized in this study, the minimum number of interruption days is not guaranteed.

6.2. Total interruption minimization

The importance of striking a balance between minimizing project duration and maintaining work continuity was discussed in previous research [13]. Maintaining work continuity is also treated as a critical issue in this research. Therefore, to demonstrate the capability of proposed model to maintain work continuity, a second validation operation is performed to minimize total interruption. The same example and data (Table 1) are adopted and minimizing total interruption days is the objective.

Objective: Minimize TID

Table 3 shows the comparison of optimum solutions and schedules using the objectives of minimizing project duration and total interruption. Fig. 2 presents the project schedules obtained using these two objectives. Compared with project duration minimization, the project duration with interruption minimization extends from 106.8 days to 117.8 days owing to no interruption allowed (0 day). The results present the possibility of shortening project duration by permitting interruption days. Nevertheless, allowing interruption implicates that construction planners can schedule projects with increased flexibility for shortening project duration. Therefore, planners can locate a balance between minimizing project duration and maintaining work continuity.

7. Scenario analysis

This study investigates two scenarios to illustrate the feasibility of the proposed model. The bridge example employed in the previous study [3] is used in both scenarios. Table 4 shows the

Table 5
Original project schedule

Section (1)	Activity											
	Excavation		Ground improvements		Foundations		Columns		Beams		Slabs	
	Start (2)	Finish (3)	Start (3)	Finish (4)	Start (5)	Finish (6)	Start (7)	Finish (8)	Start (9)	Finish (10)	Start (11)	Finish (12)
1	0	12.5			12.5	31.6	31.6	49.7	49.7	64.7		
2	12.5	28.1	28.1	38.6	38.6	58.5	58.5	73.5	73.5	89.7	89.7	107.4
3	28.1	38.9	38.9	47.4	58.5	76.0	76.0	98.5	98.5	116.3	116.3	130.9
4	38.9	55.6			76.0	92.6	98.5	115.9	116.3	130.4	130.9	149.5
Interruption	0				7		11.3		17.6		8.9	
Project duration	149.5 days											
Project total cost	$1,487,370											
Total interruption	44.8 days											

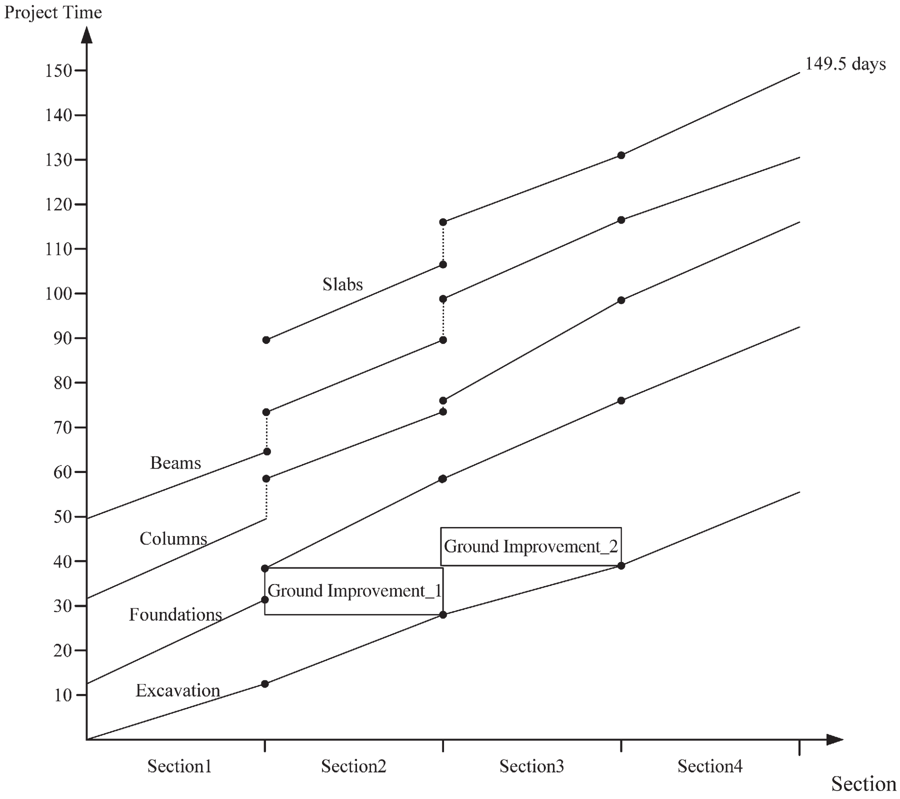

Fig. 4. LSM diagram of original project schedule.

modified example data, which includes five repetitive activities, two non-repetitive activities (ground improvement activities), and relevant crew information. Fig. 3 presents the layout of modified bridge project with the two ground improvement activities. These ground improvement activities can only start after excavation finish. Furthermore, both the cost and duration of non-repetitive activities are assigned, and no outsourcing resources are involved.

Related variables and parameters are listed in Table 4. The constraints are established in each scenario for the specific purpose, and the results from two scenarios are observed and analyzed. Several parameters based on planner assessments for each repetitive activity are as follows:

1. Production rate regarding outsourcing resource (EP^i) should be reasonable; that is, the values must be derived from planner evaluations. Referring Eq. (5), EP^i is calculated based on the production rate (IFP^i), which is set to 0.6 in this study.

Table 6
Optimum project duration with outsourcing resources

Section (1)	Activity											
	Excavation		Ground Improvements		Foundations		Columns		Beams		Slabs	
	Start (2)	Finish (3)	Start (3)	Finish (4)	Start (5)	Finish (6)	Start (7)	Finish (8)	Start (9)	Finish (10)	Start (11)	Finish (12)
1	0	9.6			9.7	28.8	28.8	46.9	46.9	61.9		
2	9.6	21.6	21.6	32.1	32.1	47.4	47.4	60.8	61.9	78.1	78.2	91.8
3	21.6	32.4	32.4	40.9	47.4	60.8	60.8	78.1	78.1	91.8	91.8	103.1
4	32.4	49.0			60.8	77.4	78.1	92.2	92.2	103.0	103.1	117.4
Interruption	0				3.3		0.5		0.4		0	
Project duration	117.4 days (Original: 149.5 days)											
Project total cost	$1,541,712 (Original: $1,487,370)											
Total interruption	4.2 days											
Model information												
Variables	161											
Constraints	94 for project duration minimization; 95 for interruption minimization											
Objective	Minimize the project duration											

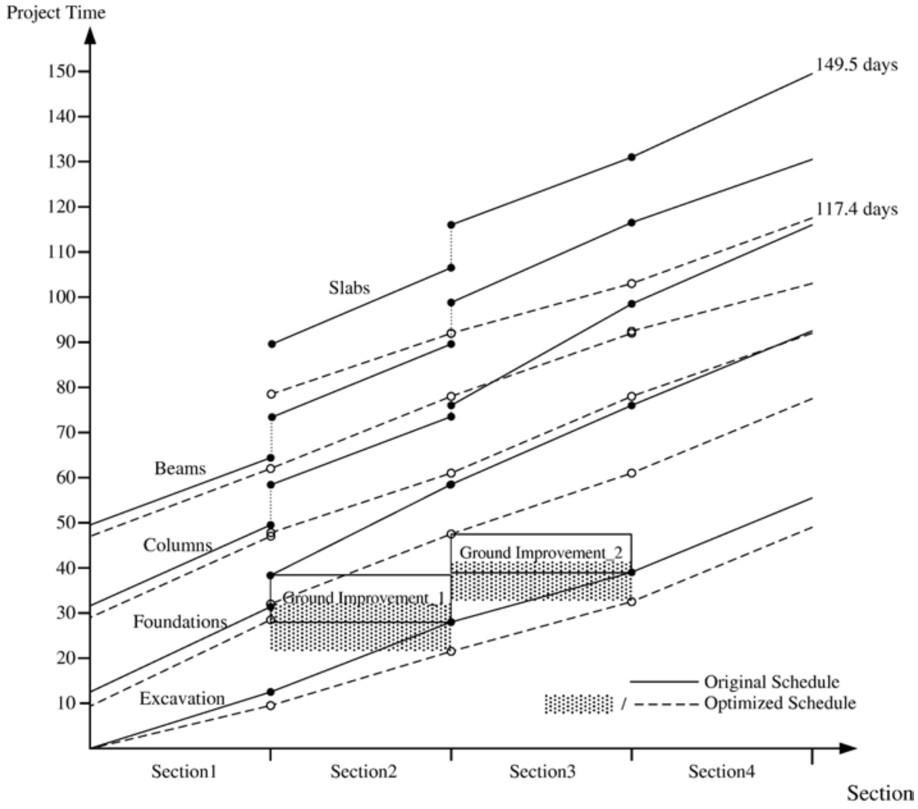

Fig. 5. Optimized LSM diagram (scenario 1).

2. The limit for outsourcing resources for (ORA^i) is assigned depending on availability. Referring to Eq. (6), ORA^i is calculated and shown in column 11 of Table 4.
3. The unit cost of outsourcing resource for repetitive activity type i (ORC^i) is typically higher than the normal price since the procurement behavior is temporary. The calculation of ORC^i is shown as Eq. (11), and the influence factor of unit cost for outsourcing resource (IFC^i) is set to 1.2.

Table 5 presents the original project schedule based on the base crew formation, and Fig. 4 presents the corresponding LSM diagram.

7.1. Scenario 1: minimize project duration considering outsourcing resources

Scenario 1 attempts to minimize project duration (Minimize T) by outsourcing resources, and compares the results, such those for project duration and total cost, with the original project schedule. Due to the need for duration compression, the concept of outsourcing resources is introduced in the proposed model for solving such problems, and the influence of outsourcing resources on project duration is clearly demonstrated. To avoid unnecessary idleness, an optimization procedure for maintaining work continuity is executed after minimizing project

Table 7
Outsourcing resource assignment and project total cost allocation for scenario 1

Activity (1)	Quantity of outsourcing resource usage				Outsourcing resource cost (6)	Direct cost (7)
	Section 1 (2)	Section 2 (3)	Section 3 (4)	Section 4 (5)		
Excavation	3	3	0	0	$11,728	$ 56,122
Ground improvement_1						$12,000
Ground improvement_2						$10,000
Foundations	0	3	3	0	$40,294	$554,261
Columns	0	2	4	5	$43,951	$380,516
Beams	1	0	2	2	$29,350	$210,448
Slabs		4	4	4	$47,665	$200,965
Accumulation					$172,988	$1,424,312
Total direct cost	$1,424,312					
Total indirect cost	$117,400 ($1000/day)					
Project total cost	$1,541,712					

Table 8
Optimum project total cost under an assigned duration (145 days)

Section (1)	Activity											
	Excavation		Ground improvements		Foundation		Columns		Beams		Slabs	
	Start (2)	Finish (3)	Start (3)	Finish (4)	Start (5)	Finish (6)	Start (7)	Finish (8)	Start (9)	Finish (10)	Start (11)	Finish (12)
1	0	9.6			13.0	32.1	36.4	54.5	60.8	75.8		
2	9.6	21.6	21.6	32.1	32.1	52.0	54.5	69.5	75.8	92.0	92.1	109.8
3	21.6	32.4	32.4	40.9	52.0	69.5	69.5	92.0	92.0	109.8	109.8	124.4
4	32.4	49.0			69.5	86.1	92.0	109.4	109.8	123.9	124.4	143.0
Interruption	0		0		0		0		0		0	
Project duration	143.0 days (Original: 149.5 days)											
Project total cost	$1,486,709 (Original: $1,487,370)											
Total interruption	0 day											
Model information												
Variables	162											
Constraints	95 for project total cost minimization; 96 for interruption minimization											
Objective	Minimize the project total cost given an assigned date											

duration. During this procedure, minimization of total project interruption days is regarded as an objective, and the optimized project duration is treated as a new constraint. This procedure ensures the maximum work continuity and eliminates unnecessary idleness.

Table 6 presents the optimum solution and related information after engaging outsourcing resources, and Fig. 5 shows the optimized LSM diagram. The model formulation generated 161 variables and 94 constraints in achieving its objective. A constraint is then added to bind project duration for executing the optimization of total interruption minimization. A comparison of the optimized results (Table 6) with the original project schedule (Table 5) indicates that the project duration reduces by 32.1 days, or approximately 21.47% (from 149.5 days to 117.4 days); the total cost increases by $54,342 (from $1,487,370 to $1,541,712), or approximately 3.65%. Therefore,

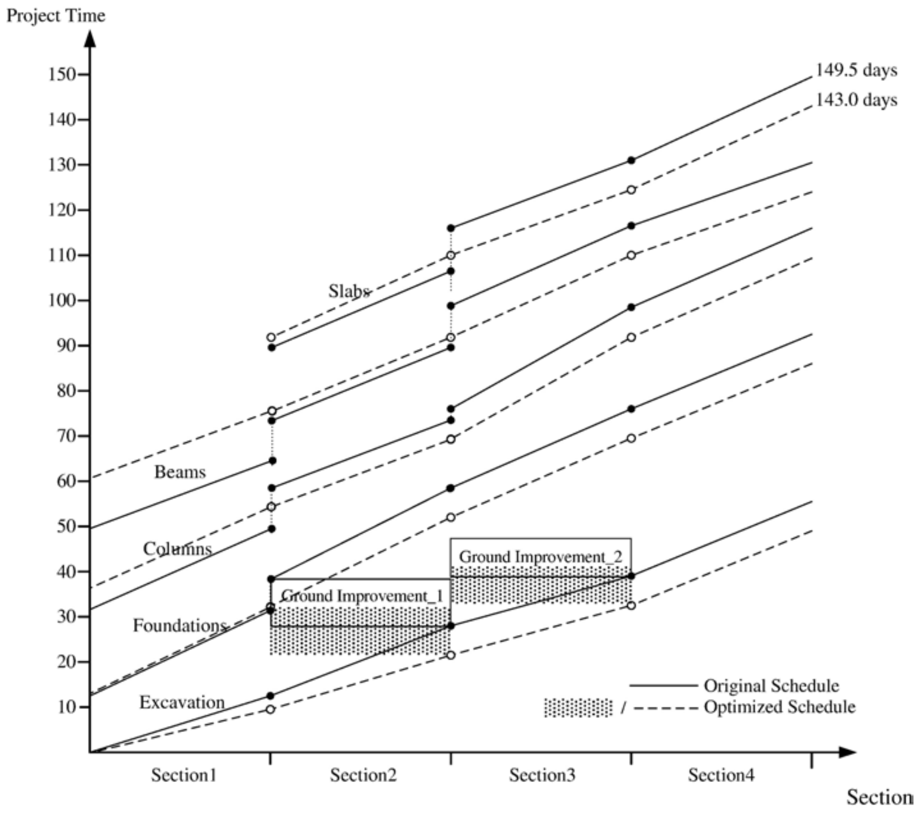

Fig. 6. Optimized LSM diagram (scenario 2).

Table 9
Outsourcing resource assignment and project total cost allocation for scenario 2

Activity (1)	Quantity of outsourcing resource usage				Outsourcing resource cost (6)	Direct cost (7)
	Section 1 (2)	Section 2 (3)	Section 3 (4)	Section 4 (5)		
Excavation	3	3	0	0	$11,728	$56,122
Ground improvement_1						$12,000
Ground improvement_2						$10,000
Foundations	0	0	0	0	0	$534,307
Columns	0	0	0	0	0	$358,381
Beams	0	0	0	0	0	$195,883
Slabs		0	0	0	0	$177,016
Accumulation					$11,728	$1,343,709
Total direct cost	$1,343,709					
Total indirect cost	$143,000 ($1000/day)					
Project total cost	$1,486,709					

outsourcing resources to reduce project duration is financially sound.

Based on the outsourcing resource assignment scheme (Table 7), planners can identify the activities requiring outsourcing resources to reduce project duration. For instance, three of outsourcing resource for excavation are added, respectively, in Section 1 and Section 2 of the project, and column (7) in Table 7 shows the direct cost, including cost for outsourcing resources for each activity. Furthermore, the cost of each resource type is listed, and the total cost of outsourcing resources is $172,988. Planners can clarify the utilization of outsourcing resources, including the efficiency for shorting project duration, and evaluate the quantities and timing of outsourcing resources when arranging work plans for linear construction projects.

7.2. Scenario 2: minimize total cost under an assigned duration

In scenario 2, the total cost is optimized for a given assigned duration (T^*). Similar to the concept of project duration compression in construction projects, the proposed model allows planners to refine a schedule plan by considering outsourcing resources, for ensuring that a project finishes no later than a desirable duration. Compared with scenario 1, only the project duration constraint is added to the model. The desirable duration (T^*) is set to 145 days in this scenario, where T^* is subjected on user's judgment to general cases; the objective and related constraints are as follows.

Objective: Minimize TC

Project duration constraint: $T \leqq T^*$ (145 days)

Table 8 shows the detailed results and model information. The model formulation generated 162 variables and 95 constraints to achieve its objective, and similarly the optimization procedure for maximizing crew work continuity is then executed with a new constraint (TC). Based on the results (Table 8), the optimum total cost is $1,486,709, which is less than the total cost for the original project schedule ($1,487,370) in Table 5, and total duration in this scenario is 143.0 days, which is superior to the desirable duration (145 days). Fig. 6 presents the optimized LSM diagram.

The optimum total cost of scenario 2 ($1,486,709) shown in Table 8 is less than that of the original schedule ($1,487,370). Therefore, the benefits of outsourcing resources in this scenario lie not only in compressing the project duration, but also in simultaneously reducing the total cost. Even when the expense of outsourcing resources shown in Table 9 is added ($11,728), the total cost is reduced by around $661 (from $1,487,370 to $1,486,709) as a result of shortened project duration and reduced indirect costs. However, the cost reduction cannot be guaranteed as it depends on the difference between outsourcing resource costs and indirect cost.

The scenario analysis illustrates the abilities of proposed model to satisfying varied objectives for linear construction projects. Two different objectives – project duration and total cost minimizations – are achieved by employing the proposed model. Notably, the solution search time in scenarios 1 and 2 are both under 1 s, and the model efficiency is shown.

8. Conclusions

This study presents a flexible model for handling the optimization problems for linear construction projects, and two scenarios are conducted for model demonstration. The proposed model constructed by constraint programming (CP) techniques implements the concept of outsourcing resources for optimizing the total cost or project duration of linear construction projects, even when several non-repetitive activities are involved. Due to the effectiveness of the CP techniques, planners can solve such problems using simple declarations, and programs for different objective functions can be adapted using the same framework, without redesigning the algorithm and sacrificing solution searching time. The scenario analysis demonstrates that the model provides an efficient tool for solving linear scheduling problems, such as project duration compression and resource planning. As a result, the abilities of the presented model for meeting varied requirements are explored.

Moreover, examining the research results can identify the significance of outsourcing resources on objective functions for linear scheduling problems. The concept of outsourcing resources provides planners with an opportunity to review work plans and evaluate options for scheduling and resource utilization plans. Consequently, planners can make appropriate decisions when attempting to optimize project total cost or duration.

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Appendix A. Notation

The following symbols are used in this paper:

S_j^i	Start date of repetitive activity type *i* in section *j*
F_j^i	Finish date of repetitive activity type *i* in section *j*
F_{j-1}^i	Finish date of repetitive activity type *i* in section $j-1$
I_{j-1}^i	Interruption days of repetitive activity type *i* between section *j* and $j-1$
TID	Total interruption days through whole project
D_j^i	Duration of repetitive activity type *i* in section *j*
Q_j^i	Quantity of work for repetitive activity type *i* in section *j*
P^i	Production rate of crew formation for repetitive activity type *i*
OR_j^i	Quantity of outsourcing resources per day added to crew formation for repetitive activity type *i* in section *j*
EP^i	Unit production rate for outsourcing resource added to crew formation for repetitive activity type *i*
CS^i	Crew size of crew formation for repetitive activity type *i*
IFP^i	Influence factor of unit production rate for outsourcing resource added to repetitive activity type *i*, which may be higher or lower than 1, depending on the evaluation for the productivity of outsourcing resource
ORA^i	Upper limit per day of the quantity of outsourcing resource utilized for repetitive activity type *i*
TC	Total cost
IC	Indirect cost
DC	Total direct cost
MC	Total material cost
EC	Total equipment cost
LC	Total labor cost
ORC	The sum of outsourcing resource cost utilized through a linear project
MC^i	Unit material cost per cubic meter for repetitive activity type *i*
LC^i	Unit labor cost per day with crew formation for repetitive activity type *i*
EC^i	Unit equipment cost per day with crew formation for repetitive activity type *i*
ICP	Indirect cost per day
T	Project duration
ORC^i	Unit cost for outsourcing resource utilized for repetitive activity type *i*, which may be higher than the normal contract price as it is a temporary supplementary resource
IFC^i	Influence factor of unit cost for outsourcing resource added to repetitive activity type *i*, which may be higher or lower than 1, depending on the evaluation on the expense of outsourcing resource
T^*	Assigned/Desirable project duration

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