# Decision Making with Utilities

- In some situations, there maybe factors that make decision making using expected value unacceptable.
- This usually happens when
  - the amount of loss is so big that it will not be acceptable even at low probability (e.g. insurance)
  - the amount of profit is so big that it will be sought even at low probability (e.g. lottery)

# Utility

Utility is a measure of the worth of an outcome to the decision maker. It reflects the decision maker’s attitude towards risk

- Consider the 3 decision alternatives
  $d_1$ = make investment A
  $d_2$ = make investment B
  $d_3$ = do not invest
- The states of nature are
  $s_1$ = prices go up ➔ $P(s_1)=0.30$
  $s_2$ = prices remain stable ➔ $P(s_1)=0.50$
  $s_3$ = prices go down ➔ $P(s_3)=0.20$

# Payoff Table

| | States of nature | | |
|---|---|---|---|
| Decision | Prices up $s_1$ | Prices stable $s_2$ | Prices down $s_3$ |
| $d_1$ | 30,000 | 20,000 | -50,000 |
| $d_2$ | 50,000 | -20,000 | -30,000 |
| $d_3$ | 0 | 0 | 0 |

# Decision based on Expected Value

$EV(d_1) = 0.3(30,000) + 0.5(20,000) + 0.2(-50,000) = 9,000$

$EV(d_2) = 0.3(50,000) + 0.5(-20,000) + 0.2(-30,000) = -1,000$

$EV(d_3) = 0.3(0) + 0.5(0) + 0.2(0) = 0$

Therefore, SELECT $d_1$ with EV = 9,000

- But, suppose that this company is in financial difficulty and the loss of 50,000 would be detrimental to it.
- In such a case, $d_1$ is not desirable
- To deal with this situation, we need to determine the **utilities** of the monetary values in the decision

# Determining Utilities

- Let U(M) = Utility of M [M=Monetary Amount]
- Assign (arbitrarily) utility to the highest monetary value in payoff table
  ➔ U(50,000) = 10
- Assign (arbitrarily) utility to the lowest monetary value in payoff table
  ➔ U(-50,000) = 0
  *Note: U(50,000) > U(-50,000)*
- Assign utilities to the other values as follows

# Assigning Utilities to Middle Values

Consider the payoff of 30,000:

Decision maker is asked to select one of two choices:
1. a guaranteed amount of 30,000, or
2. participate in the following lottery
DM get a payoff of 50,000 with probability *p*, or a payoff of -50,000 with probability (1-*p*)

a. If *p* is close to 0, DM will prefer the 30,000
b. If *p* is close to 1, DM will prefer lottery
c. As we increase *p* from 0, at what value does DM changes preference to the lottery?
d. At this value of *p*, DM has no preference between the two choices (equivalent)

# Assigning Utility to the Value 30,000

- Suppose the DM chooses $p$=0.95
- Now we can determine the U(30,000) as follows:
  $U(30{,}000) = pU(50{,}000) + (1-p)U(-50{,}000)$
  $= 0.95\ (10) + 0.05\ (0)$
  $= 9.5$
- Note that at p=0.95 the EV(lottery) is:
  $EV(\text{lottery}) = 0.95(50{,}000)+0.05(-50{,}000)$
  $= 45{,}000$

- DM is willing to accept a guaranteed amount of 30,000 rather than risk anything more than 5% chance of a loss of 50,000
- The difference between 45,000 and 30,000 is referred to as the risk premium.
- The DM is willing to pay 15,000 risk premium to avoid the 5% chance of losing 50,000.

# Assigning Utility to the Value -20,000

- As before, DM is asked to select one of two choices:
  1. a guaranteed amount of -20,000, or
  2. participate in the lottery
     DM get a payoff of 50,000 with probability *p,* or a payoff of -50,000 with probability (1-*p*)
- For example, we ask DM if *p*=0.90, would he choose lottery or an assured loss of 20,000 (➔lottery)
- We lower *p* until point of indifference is reached.
- Suppose at this point *p*=0.55
- Now we can determine the U(-20,000) as follows:
  U(-20,000) = *pU*(50,000) + (*1-p)U*(-50,000)
  = 0.55 (10) + 0.45 (0)
  = 5.5

# Assigning Utility in General

- We follow the same procedure for all monetary values in the payoff table.
- DM is asked to select one of two choices:
  1. a guaranteed amount M, or
  2. participate in the lottery
     DM get a payoff of 50,000 with probability *p*, or a payoff of -50,000 with probability (1-*p*)
- We find the probability *p* of indifference.
- Then, we can determine the U(M) as follows:

$$U(M) = pU(50{,}000) + (1-p)U(-50{,}000)$$
$$= p\,(10) + (1-p)(0)$$
$$= 10p$$

# Utility of Payoff Values

| Monetary Value | Indifference Value of p | Utility Value |
|---|---|---|
| 50,000 | NA | 10.0 |
| 30,000 | 0.95 | 9.5 |
| 20,000 | 0.90 | 9.0 |
| 0 | 0.75 | 7.5 |
| -20,000 | 0.55 | 5.5 |
| -30,000 | 0.40 | 4.0 |
| -50,000 | NA | 0 |

# Utility Table

| | States of nature | | |
|---|---|---|---|
| Decision | Prices up $s_1$ | Prices stable $s_2$ | Prices down $s_3$ |
| $d_1$ | 9.5 | 9.0 | 0 |
| $d_2$ | 10 | 5.5 | 4.0 |
| $d_3$ | 7.5 | 7.5 | 7.5 |

# Decision based on Expected Utility

$EU(d_1) = 0.3(9.5) + 0.5(9.0) + 0.2(0) = 7.35$

$EU(d_2) = 0.3(10) + 0.5(5.5) + 0.2(4.0) = 6.55$

$EU(d_3) = 0.3(7.5) + 0.5(7.5) + 0.2(7.5) = 7.50$

Therefore, SELECT $d_3$ [Do Not Invest] with EU = 7.50

# Ranking of Alternative with Expected Utility

| Rank of Alternative | Expected Utility | Expected Monetary Value |
|---|---|---|
| Do not Invest | 7.50 | 0 |
| Investment A | 7.35 | 9,000 |
| Investment B | 6.55 | -1,000 |

Investment A is rejected because the 20% of 50,000 loss was too RISKY for the DM.

DM, in this case, is a Risk Avoider

# Decision Maker is a Risk Taker

- Suppose the DM was feeling comfortable about the financial status of his company
- DM was seeking investments that may lead to high payoff and was willing to take risk
- The DM will follow the same procedure for determination of utilities of the payoffs
- But he will take the choice of the Lottery at lower probabilities

# Utility of Payoff Values for a Risk Taker

| Monetary Value | Indifference Value of p | Utility Value |
|---|---|---|
| 50,000 | NA | 10.0 |
| 30,000 | 0.50 | 5.0 |
| 20,000 | 0.40 | 4.0 |
| 0 | 0.25 | 2.5 |
| -20,000 | 0.15 | 1.5 |
| -30,000 | 0.10 | 1.0 |
| -50,000 | NA | 0 |

# Utility Table

| | States of nature | | |
|---|---|---|---|
| Decision | Prices up $s_1$ | Prices stable $s_2$ | Prices down $s_3$ |
| $d_1$ | 5.0 | 4.0 | 0 |
| $d_2$ | 10 | 1.5 | 1.0 |
| $d_3$ | 2.5 | 2.5 | 2.5 |

# Decision based on Expected Utility

$EU(d_1) = 0.3(5.0) + 0.5(4.0) + 0.2(0) = 3.50$

$EU(d_2) = 0.3(10) + 0.5(1.5) + 0.2(1.0) = 3.95$

$EU(d_3) = 0.3(2.5) + 0.5(2.5) + 0.2(2.5) = 2.50$

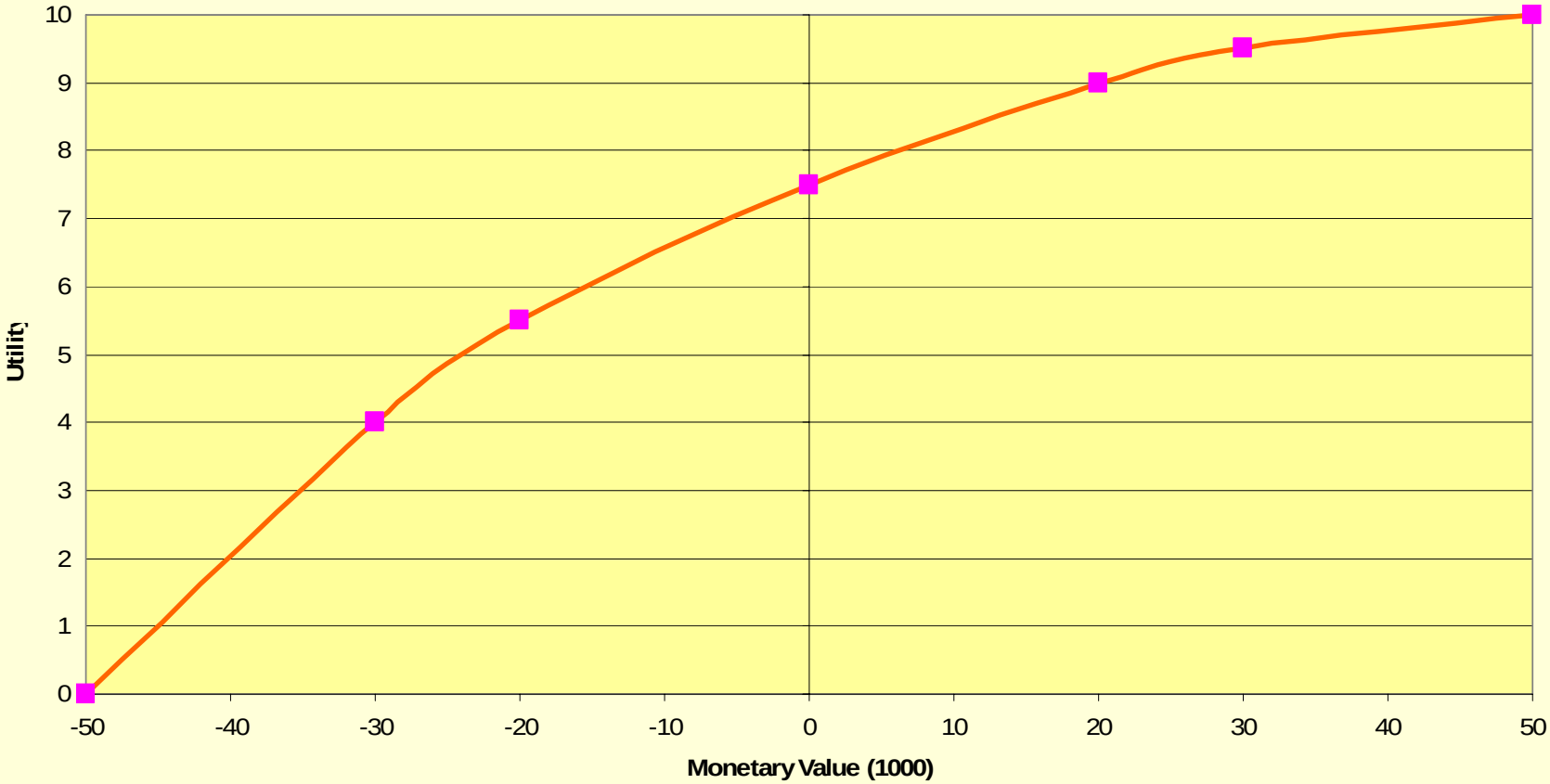
Therefore, SELECT $d_2$ [Investment B] with EU = 3.95

# Ranking of Alternative with Expected Utility

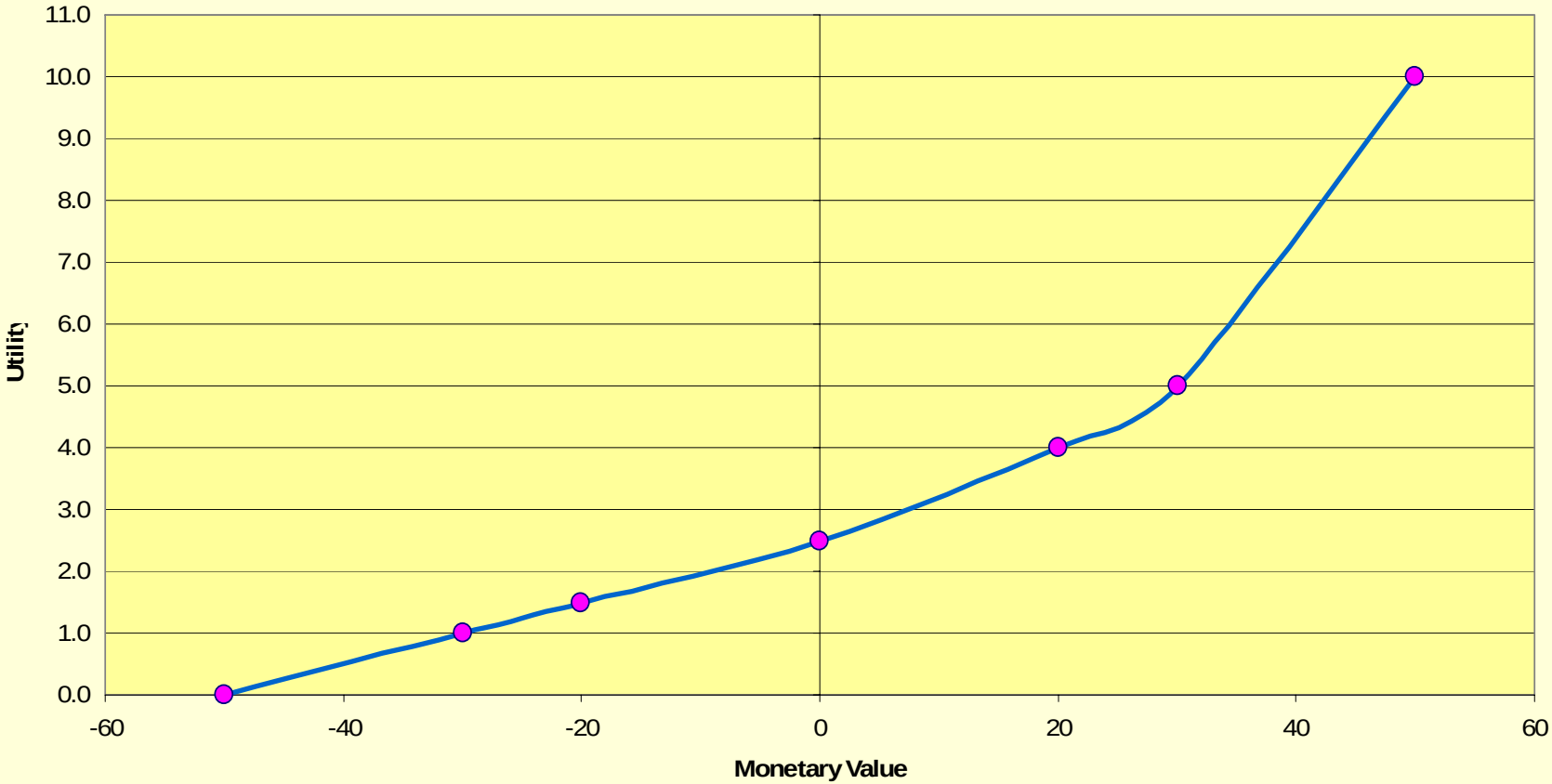
| Rank of Alternative | Expected Utility | Expected Monetary Value |
|---|---|---|
| Investment B | 3.95 | -1000 |
| Investment A | 3.50 | 9,000 |
| Do not Invest | 2.50 | 0 |

- Investment B is selected even though EV is negative.
- DM, in this case, is a Risk Taker and willing to seek the opportunity of 50,000 payoff in Investment B.

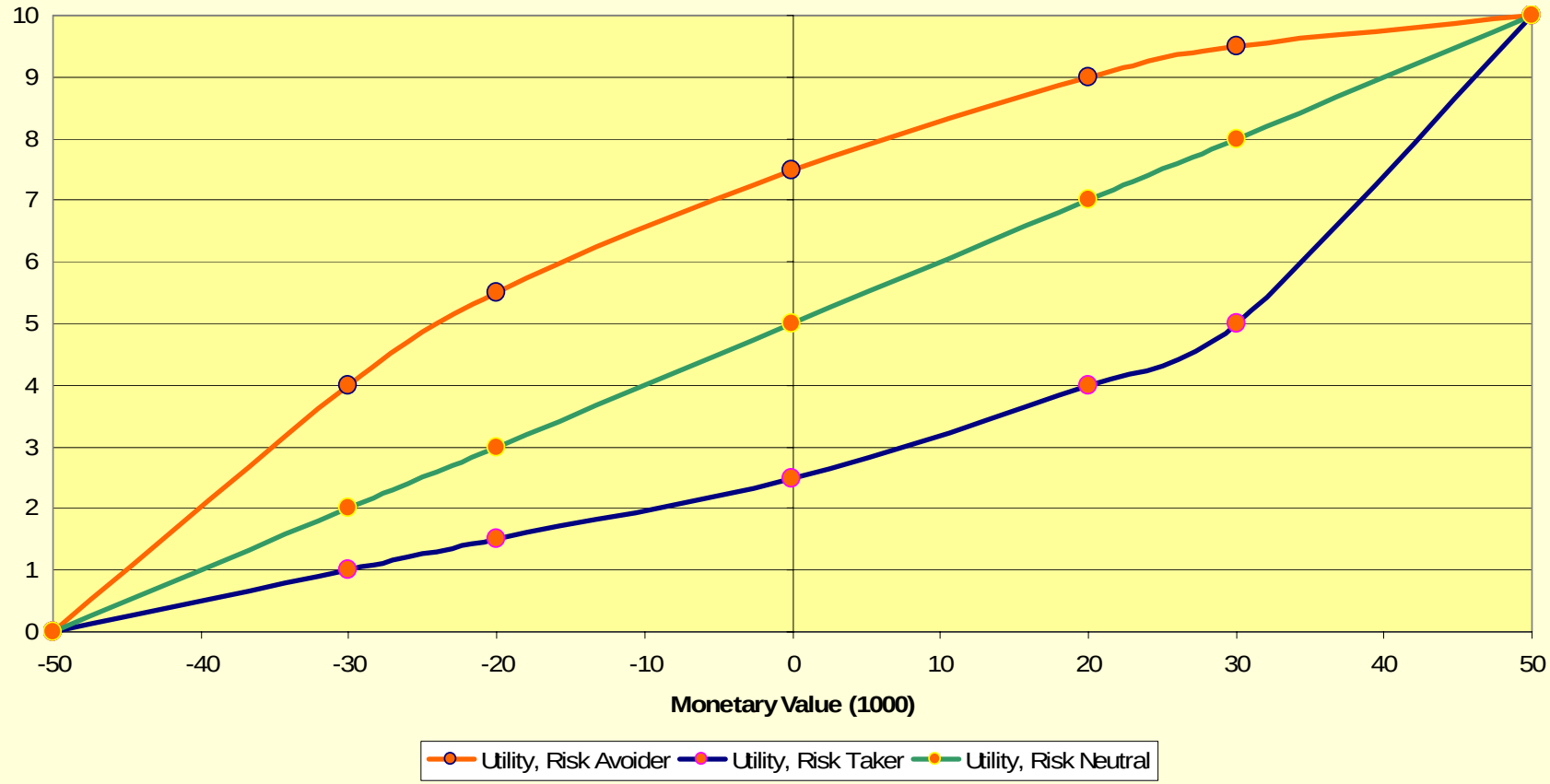
## Utility Function for Risk Avoider



## Utility of a Risk Taker



## Utility Function for Risk Avoider, Risk Taker, and Risk Neutral



# EV vs. EU

- As can be seen from the graph above, EV and EU will result in the same recommendation for a risk neutral DM.
- There is a range of monetary values where the DM is neutral ➔ the EV approach should be selected because EV & EU will be in agreement.
- The range is where the payoffs (profit or loss) are not considered great.