Random Variables

Random Variable: *Definition*

A Random Variable is a ***numerical*** description of the ***outcome*** of an experiment

Experiment	**Random Variable (x)**	**Possible Values for the RV**
Roll a die 10 times	Number of times even numbers is obtained	0, 1,2,…,10
Inspect a shipment of 100 parts	Number of defective parts	0, 1, 2,…100
Open a gas station	Number of customers entering in one hour	0, 1,2,…
Toss a coin	Upward side	0,1

Random Variables

- Experiment: Check light bulbs *until* a bad one (event=B) is obtained. Let G=event bulb is good.
- S = {*B, GB, GGB, GGGB, GGGGB,* …..}
- Define RV, X as:
  X =Number of light bulbs checked before a bad bulb is found.
  X = {1, 2, 3, 4,……}
- Notice how the RV is a rule to assign numbers to experimental outcomes
- Assignment is meaningless without the definition.

RV: Discrete or Continous

- Random variables can be discrete or continuous
- A discrete RV can only take a finite set of values or an infinite sequence of values where there is a first element, a second element, and so on.
- An RV is continuous if the values it can take consist of an interval on the number line (for example, the time it takes for a bulb to fail)

Probability Distribution of Discrete RV

Sales volume of a car dealer over three hundred working days in a year Define RV: **X = number of cars sold in one day**							
Sales Volume	No sale X=0	One car X=1	Two cars X=2	Three cars X=3	Four cars X=4	Five cars X=5	Total
Number of days	54	117	72	42	12	3	300

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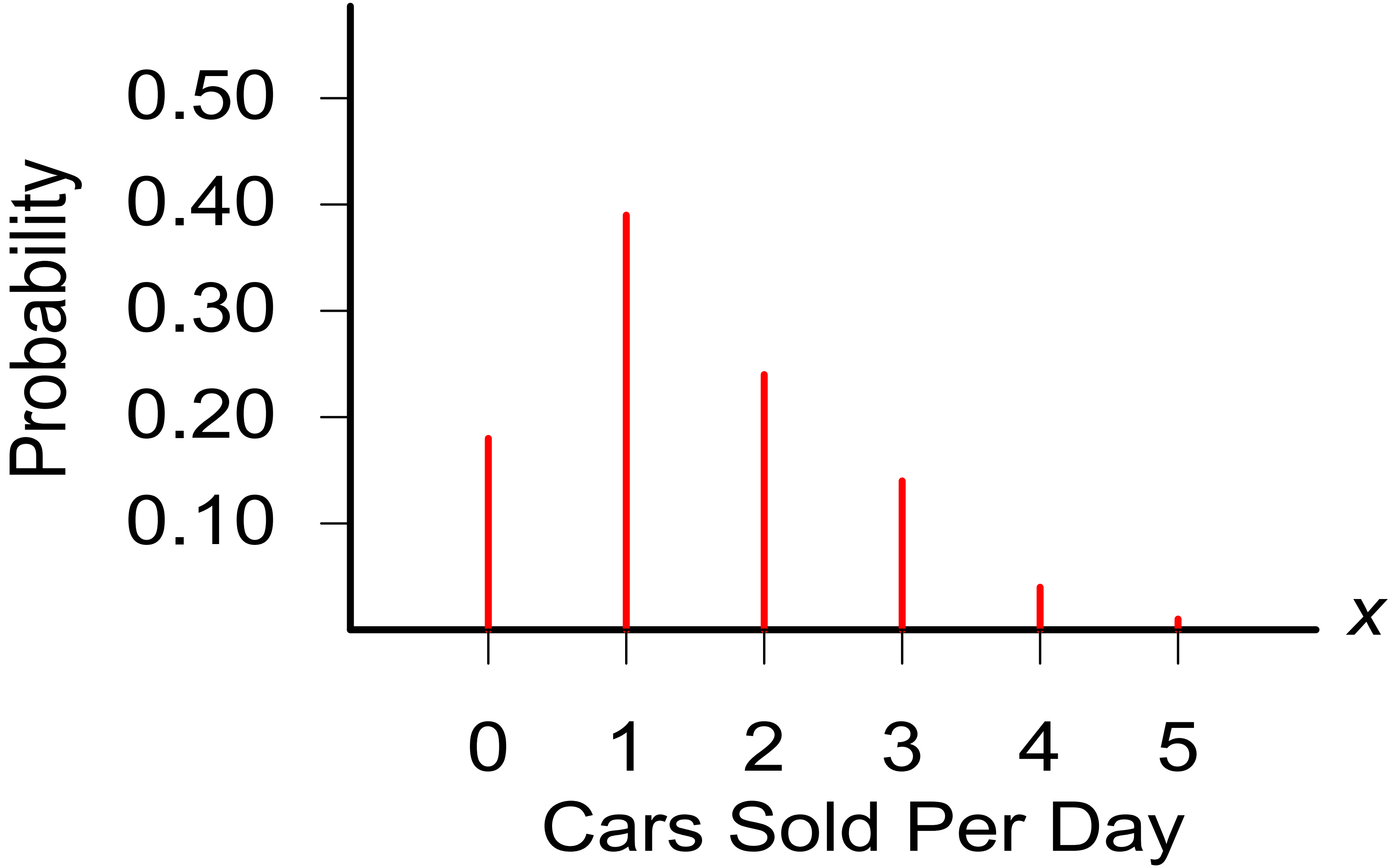
Sales Volume	No sale X=0	One car X=1	Two cars X=2	Three cars X=3	Four cars X=4	Five cars X=5	Total
$f(x)$	0.18	0.39	0.24	0.14	0.04	0.01	1.00

- Note that the sum of the probabilities equal 1.

Probability Distribution for *x*



Expected Value

- The expected value (mean) of a discrete random variable is the weighted average of all the possible values of the RV. The weights are the probabilities of the values.

$$E(x) = \mu = \sum x \, f(x)$$

The Expected Value

x	*f(x)*	*xf(x)*
0	0.18	0(0.18) = 0.00
1	0.39	1(0.39) = 0.39
2	0.24	2(0.24) = 0.48
3	0.14	3(0.14) = 0.42
4	0.04	4(0.04) = 0.16
5	0.01	5(0.01) = 0.05
		$E(x) = 1.5$

The Variance

- Variance is a measure of the variability of the values of the RV
- For a discrete RV:

$$\mathrm{Var}\ (x) = \sigma^2 = \sum(x-\mu)^2\ \mathrm{f}(x)$$

Calculation of Variance

x	$(x-\mu)$	$(x-\mu)^2$	$f(x)$	$(x-\mu)^2 f(x)$
0	0 - 1.5 = -1.5	2.25	0.18	(2.25)(0.18) = 0.4050
1	1 -1.5 = -0.5	0.25	0,39	(0.25)(0,39) = 0.0975
2	2 - 1.5 = 0.5	0.25	0.24	(0.25)(0.24) = 0.0600
3	3 – 1.5 = 1.5	2.25	0.14	(2.25)(0.14) = 0.3150
4	4 - 1.5 = 2.5	6.25	0.04	(6.25)(0.04) = 0.2500
5	5 – 1.5 = 3.5	12.25	0.01	(12.25)(0.01) = 0.1225
				σ^2 =1.2500

The Binomial Probability Distribution

Assumptions:

1. A fixed number of trials, say n.
2. Each trial results in a “Success” or “Failure”
3. Each Trial has the same probability of success p.
4. Different Trials are independent.

Define a RV *x* as:

***x* = Number of Successes in n trials**

The probability distribution of *x* is known as the Binomial Probability distribution.

The Binomial Probability Distribution

$$f(x) = \frac{n!}{x!(n-x)!} p \ (1-p) \qquad x = 0,\ 1,\ 2,\ \ldots,n$$

n = number of trials

p =probabiltiy of success on one trial

x = number of successes in n trials

f(x) = probability of x successes in n trials

Example

- Consider customers visiting a store. Suppose you want to know the probability of exactly 2 people of the next five customers will make a purchase. It was previously estimated that the probability of a customer making a purchase is 0.30
- The situation meets the assumptions of the binomial experiment
  1. Five identical trials
  2. Two outcomes (purchase/no purchase)
  3. Probability of purchase is same for all customers
  4. Purchase of a customer is independent of others

$n!$

Expected Value & Variance of the Binomial Distribution

$\mu = np$

$\sigma^2 = np\,(1-p)$

In the previous example:

$n = 5$

$p = 0.3$

$\mu = np = 1.5$

$\sigma^2 = np\,(1-p) = (5)(0.3)\,(0.7) = 1.05$

$\sigma = 1.02$

The Poisson Probability Distribution

- A discrete Probability Distribution
- The RV describes the number of occurrences of an event over a specified interval of space or time
- Examples of possible RV's:
  - Number of customers arriving at a bank in one hour period
  - Number of defects in 1 km distance of pipeline
  - Number of camels met in 10 km distance of the road to Riyadh

Poisson Distribution: Assumptions

- Two assumptions must hold for the Poisson Distribution to be use
  1. The probability of occurrence of the event is the same for any two intervals of equal length
  2. The occurrence of the event, in any interval, is independent of the occurrence in any other interval

Poisson Distribution

$$f(x) = \frac{\lambda^x e^{-\lambda}}{x!} \qquad \textbf{for } x = 0, 1, 2, ...$$

where

λ = **mean or average number of occurrences in an interval**

e = **2.71828**

x = **number of occurrences in the interval**

$f(x)$ = **probability o x occurrences in the interval**

Example: (problem 15)

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a. $f(x=3)$ in 5-minute interval

Average number of calls per 5 minutes = 4

$f(3) = 4^3 e^{-4} / 3! = 0.1954$

b. $f(x=10)$ in 15-minute interval

Average number of calls per 10 minutes = 12

$f(10) = 12^{10} e^{-12} / 10! = 0.1048$

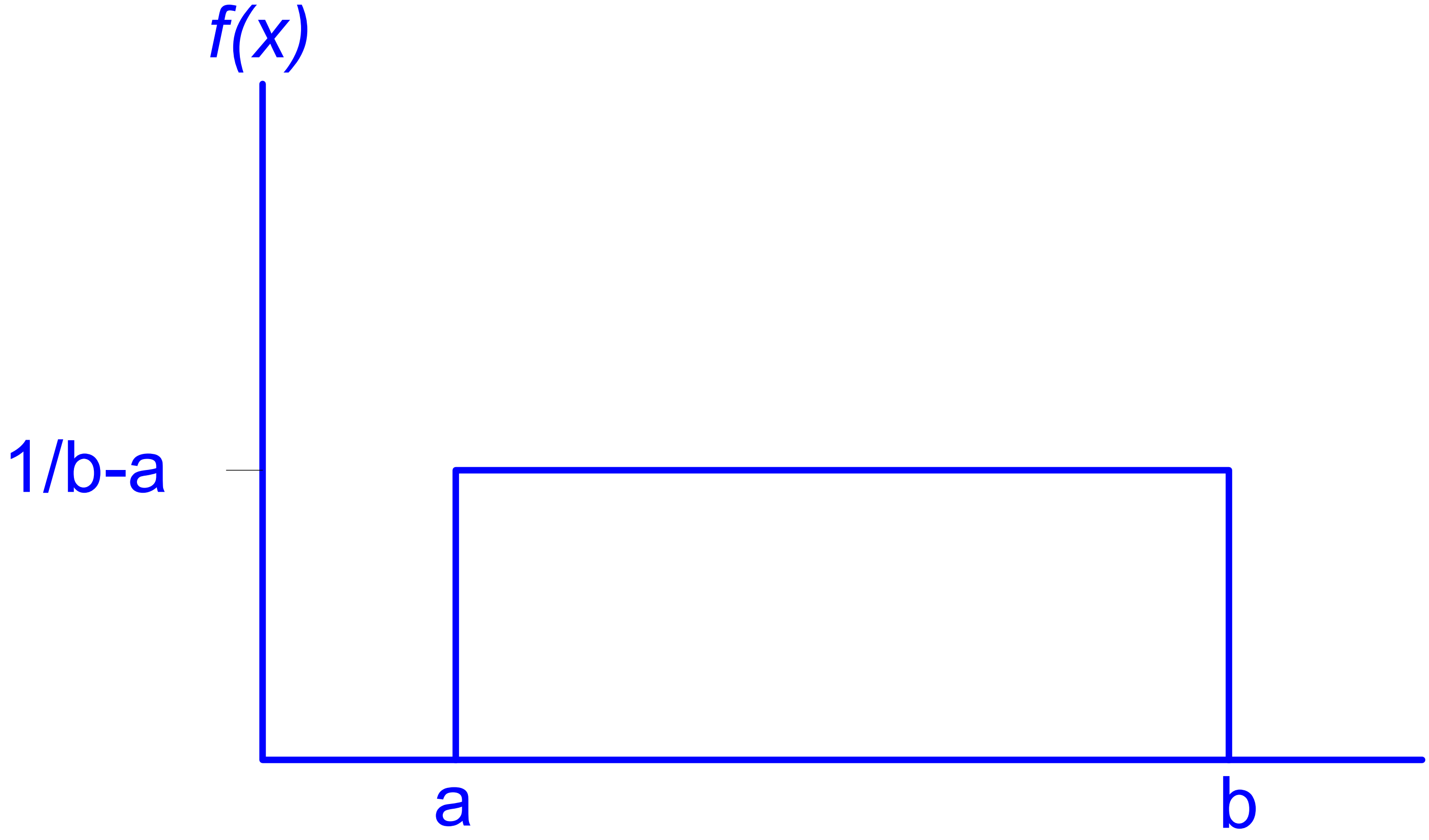
c. Average number of calls in 5 minutes = 4

d. $f(x=0)$ in 3-minute interval

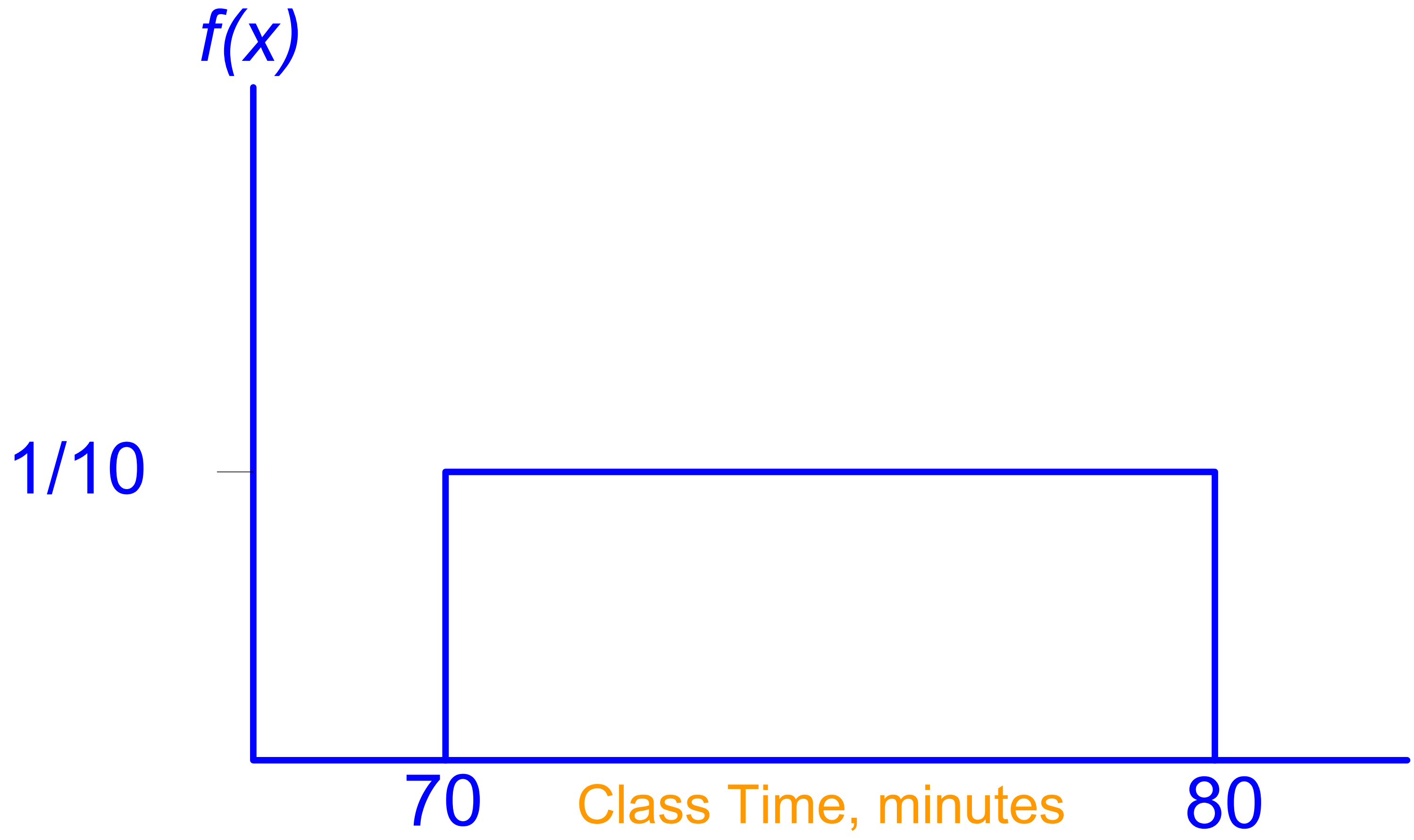
Average number of calls per 3 minutes = 2.4

$f(0) = 2.4^0 e^{-2.4} / 0! = 0.0907$

The Uniform Distribution



The Uniform Distribution



Determining Probabilities

- What is the probability that the class time will be 75 minutes?
- What is the probability class time will be between 74 to 76 minutes, less than 75?
  - Measured by the area under the distribution curve between the points 74 and 76
  - Probability $(74 \leq x \leq 76) = 1/10\ (76-74) = 0.2$
  - Probability $(x \leq 75) = 1/10\ (75-70) = 0.5$

The Normal Probability Distribution

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2} \text{ for } -\infty < x < \infty$$

where

μ = mean of the RV x

σ^2 = variance x

σ = standard deviation of x

$\pi = 3.14159$

$e = 2.71828$

The Normal Probability Distribution

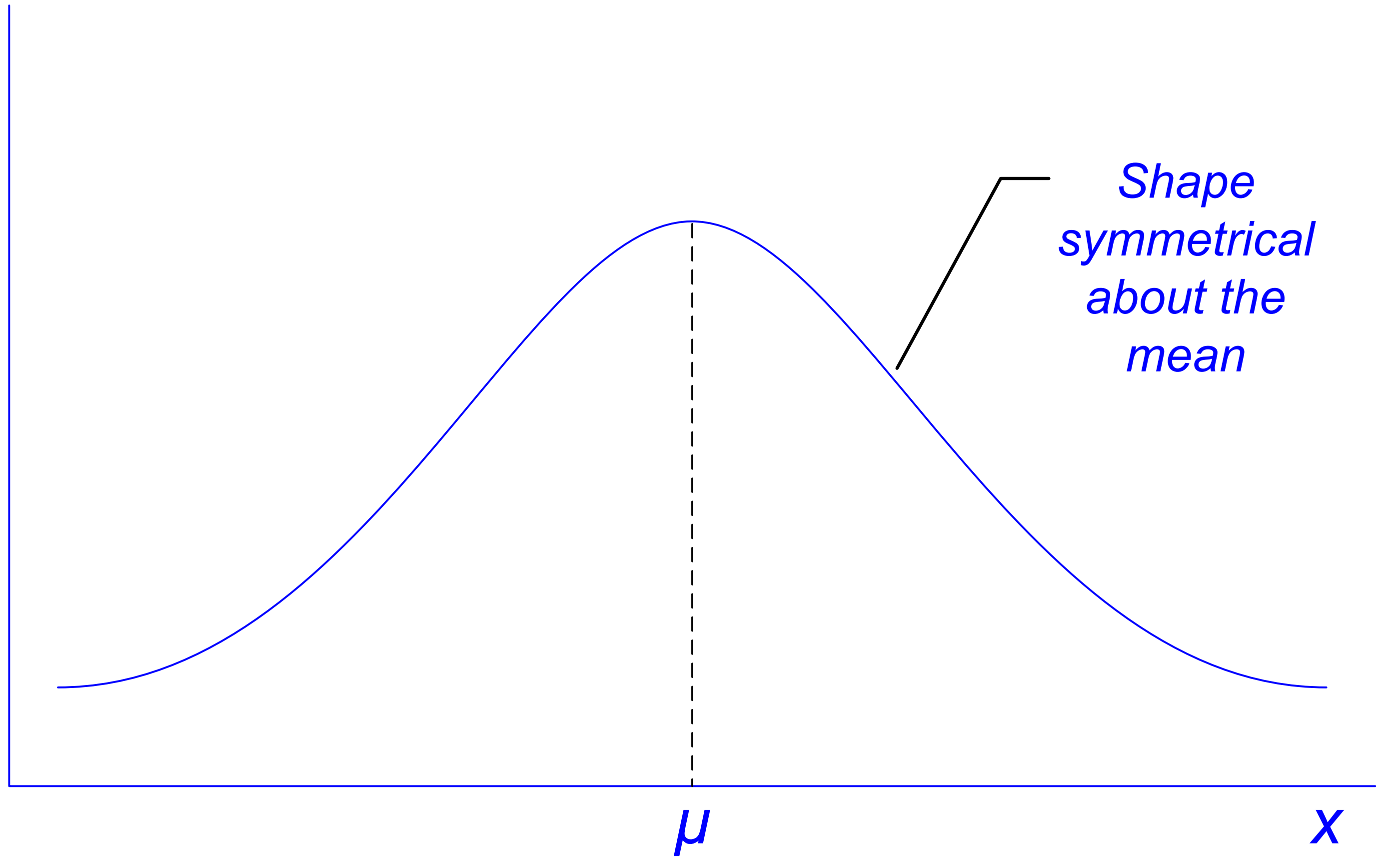
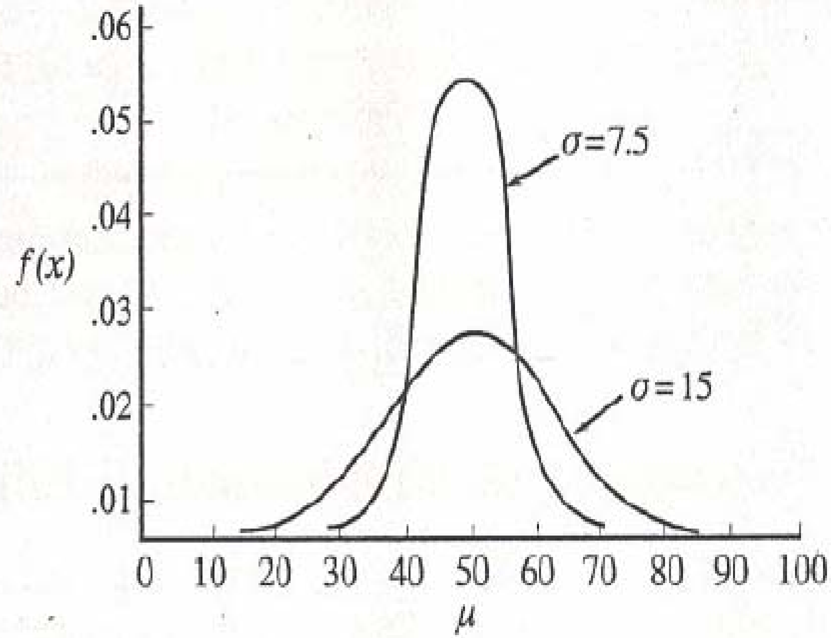


FIGURE 3.6 TWO NORMAL DISTRIBUTIONS WITH $\mu = 50$



The Normal Distribution

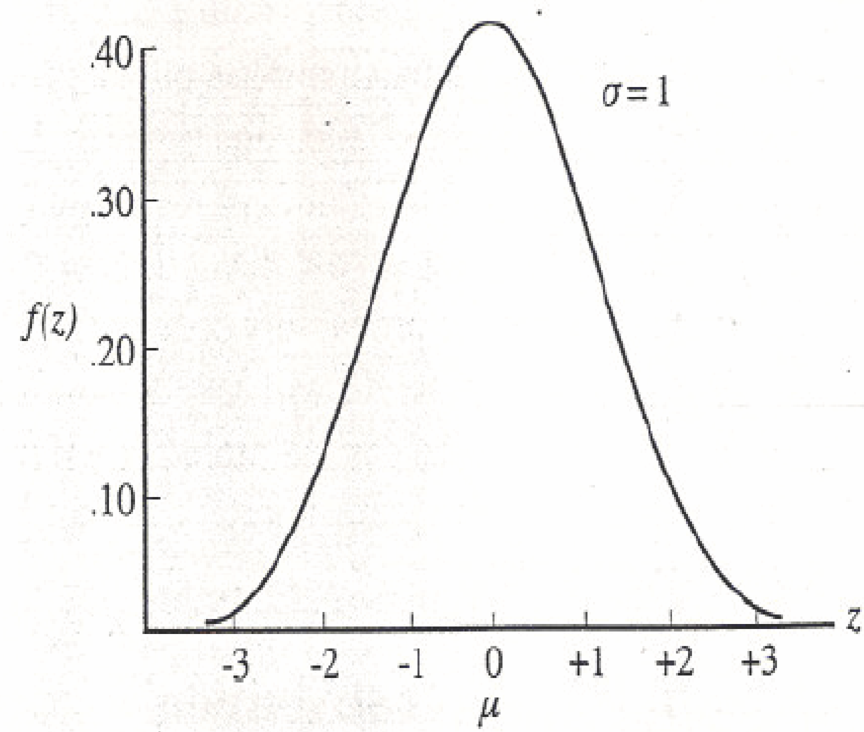
- Some handy rules of thumb:

  $\Pr(\mu - \sigma < x > \mu + \sigma) \approx 0.68$

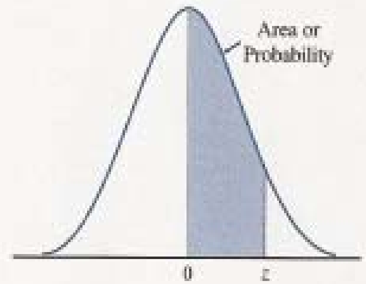
  $\Pr(\mu - 2\sigma < x > \mu + 2\sigma) \approx 0.95$

  $\Pr(\mu - 3\sigma < x > \mu + 3\sigma) \approx 0.99$

FIGURE 3.7 STANDARD NORMAL DISTRIBUTION



ABLE 3.9 AREAS OR PROBABILITIES FOR THE STANDARD NORMAL DISTRIBUTION



Entries in the table give the area under the curve between the mean and z standard deviations above the mean. For example, for $z = 1.25$, the area under the curve between the mean and z is 0.3944.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2518	0.2549
0.7	0.2580	0.2612	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	**0.3944**	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4986	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

The Exponential Distribution

$$f(x) = \frac{1}{\mu} e^{-x/\mu} \text{ for } x \geq 0, \mu > 0$$

$$F(x \leq x_0) = 1 - e^{-x_0/\mu}$$

The Exponential Distribution

FIGURE 3.15 EXPONENTIAL DISTRIBUTION FOR THE SCHIPS LOADING DOCK EXAMPLE