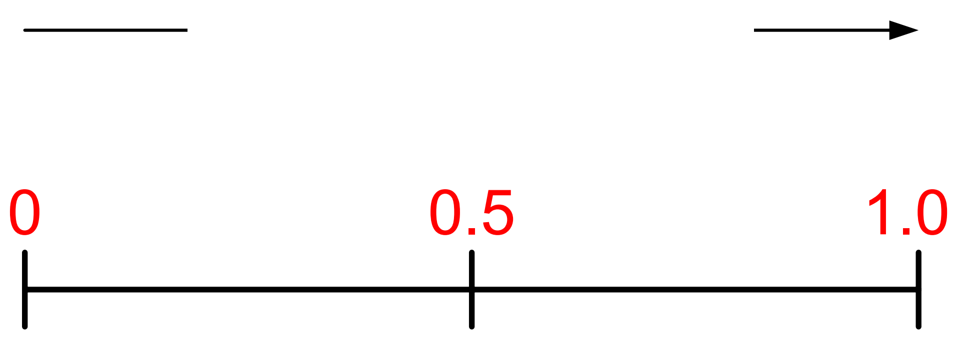
# Probability Basics

# Probability

- **A** **numerical measure** **of the likelihood of occurrence of an event**
- **Provides description of uncertainty**



# Experiments

- A process that generates well-defined outcomes
- At each repetition of experiment, *one and only* one outcome will occur

| Experiment | Experimental outcome |
| --- | --- |
| Flip a coin | Head, tail (side 1, side 2) |
| Play a game | Win, lose, tie |
| Roll a dice | 1, 2, 3, 4, 5, 6 |
| Inspect a part | Pass, fail |

# Sample Space

- Sample space is the set of all experimental outcomes
  - Flip a coin: S = {Head, Tail}
  - Roll a die: S = {1, 2, 3, 4, 5, 6}

# Assigning Probabilities

- Must adhere to two conditions:
  - The probability of an outcome:

    $0 \leq P(E_i) \geq 1$ for all i

    $E_i$ = *i* th Experimental outcome
  - The sum of the probabilities of all outcomes:

    $\sum P(E_i) = 1$

# Methods of assigning Probabilities

1. Classical Method
   - When it is reasonable to assume that all outcomes are equally likely:
     - $P(E_i) = 1/n$,   n= number of outcomes

# Methods of assigning Probabilities

2. Relative Frequency Method
   - Relative frequency based on limited repetition of the experiment
     - In a sample of 200 parts 10 are defective
       P (defective) = 10/200 = 0.05

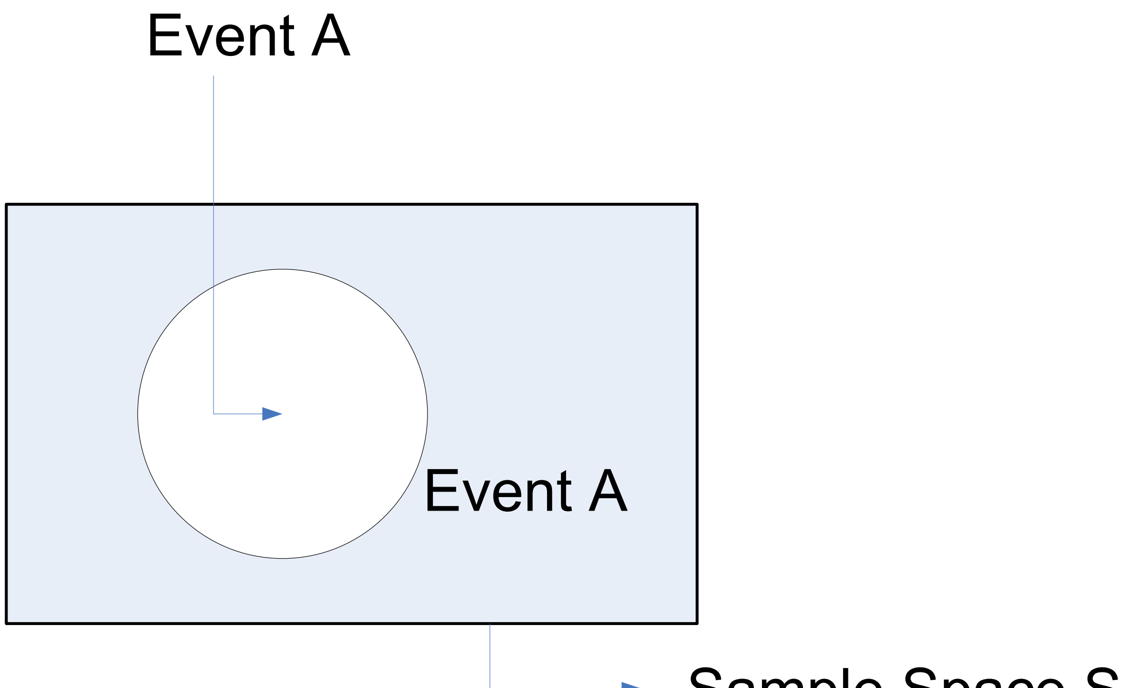
# Methods of assigning Probabilities

3. Subjective Method
   - Used when the classical or the relative frequency method are not possible, or as a support for these methods
   - Probability is determined as the *degree of belief* that an outcome will occur based on available information and past experience
   - Assignment must ensure the two conditions of assignment are applicable

# Events and Their Probabilities

- An event is a collection of outcomes (sample points)
  - Roll a die experiment: define event A as outcome is an even number
- P (event) = sum of probabilities of sample points
  - ➢ P (A) = P(2) + P (4) + P (6)
    = 1/6 + 1/6 + 1/6 = ½

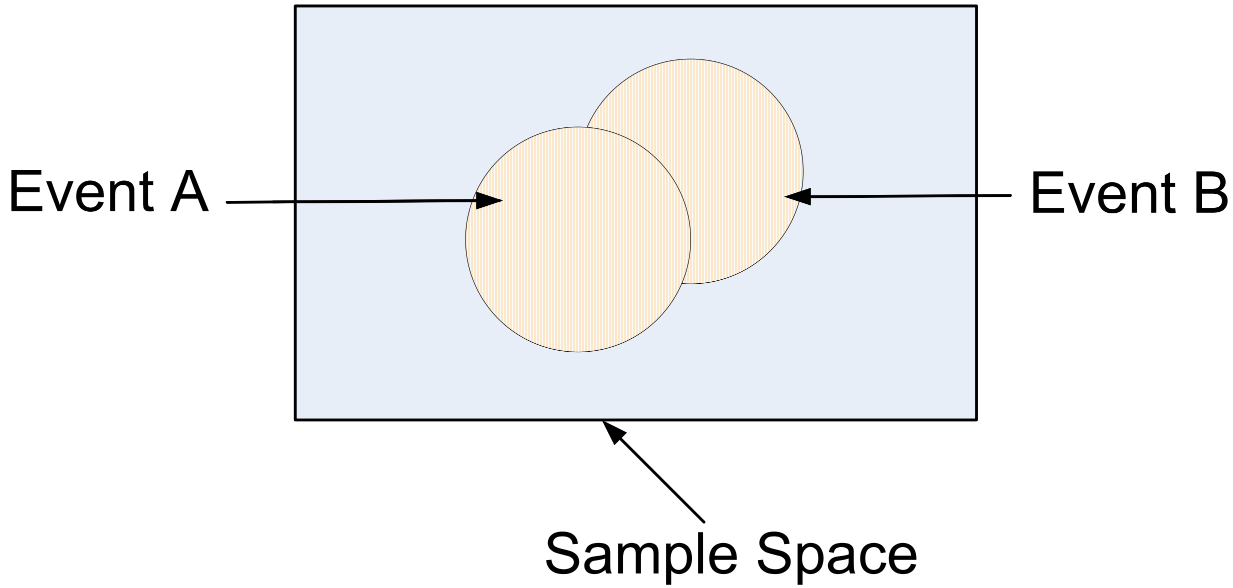
# Complement of an Event




$P(A) + P(A\ ) = 1$

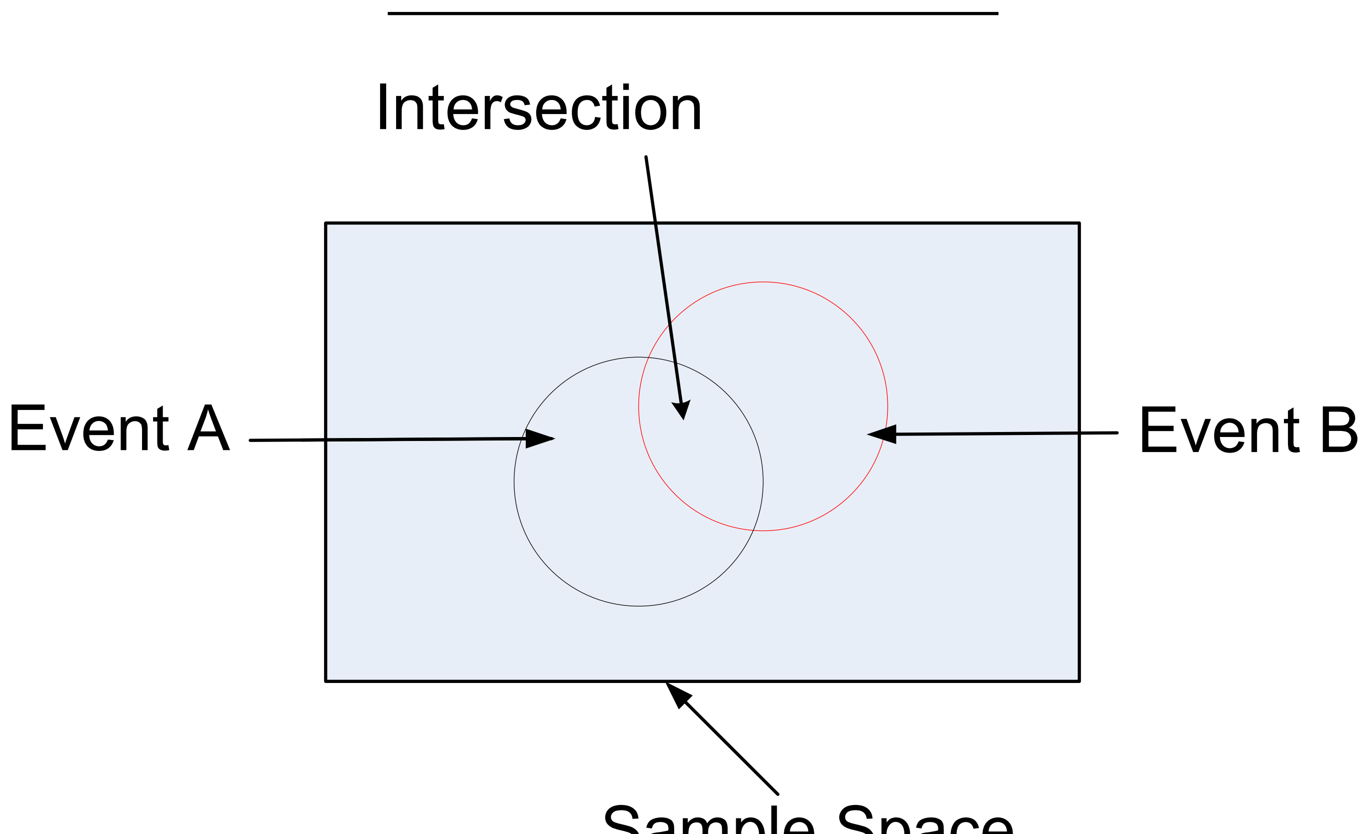
$P(A) = 1 - P(A\ )$

# Union of Two Events

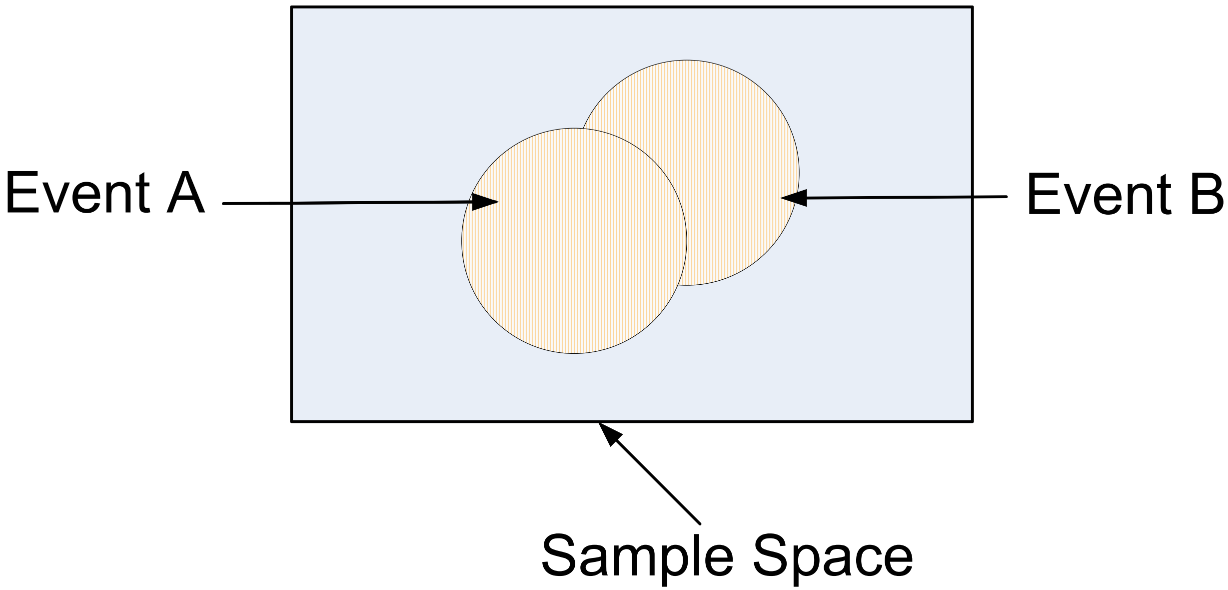



Union of A & B is event containing all sample points in A or B or both

# Intersection of Two Events



# Addition Law



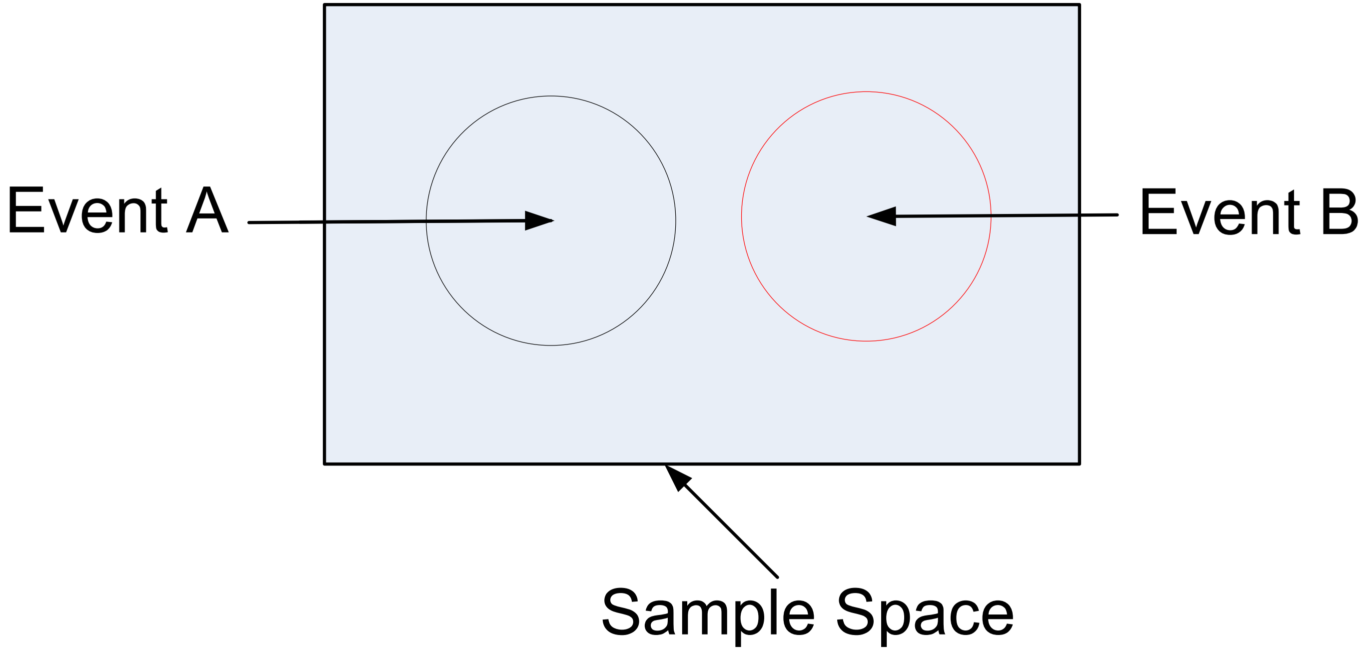

$$\overline{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

# Mutually Exclusive Events

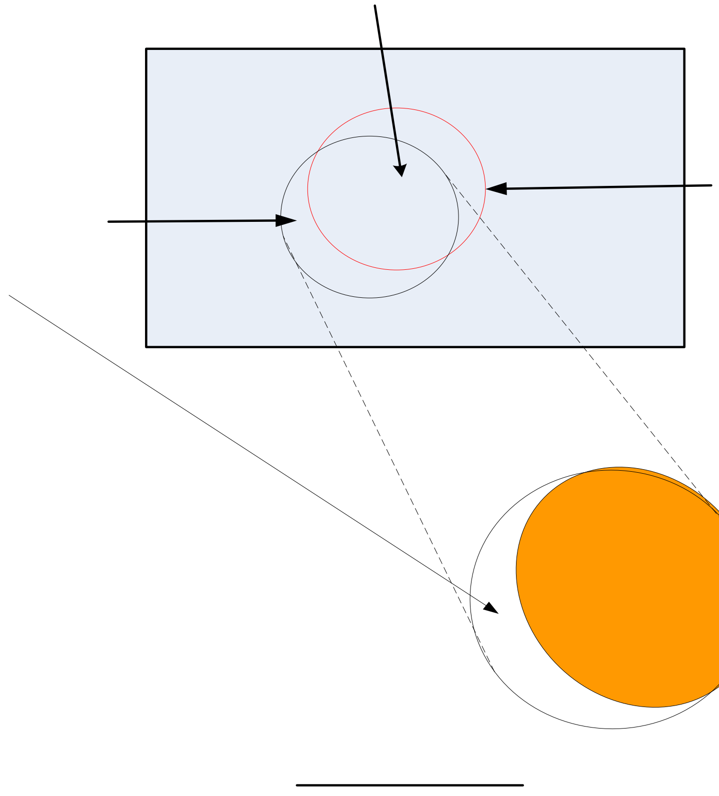
- Events that are mutually exclusive do not share any sample points in common (probability of the intersection is zero)
- Therefore:
  - $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Becomes ➔

$P(A \cup B) = P(A) + P(B)$


Event A
Event B
Sample Space

# Conditional Probability



# Independent Events

- Two events are independent if
  
  $P(B/A) = P(B)$ or
  
  $P(A/B) = P(A)$
  
  [The probability of one does not affect the probability of the other]
- Are mutually exclusive events independent events?

# Example

Suppose we have the following data for why students consider joining a college (Question 13)

| | | Reason for Joining | | | Totals |
|---|---|---|---|---|---|
| | | quality | Cost | Other | |
| | Full time | 421 | 393 | 76 | 890 |
| | Part time | 400 | 593 | 46 | 1039 |
| | Totals | 821 | 896 | 122 | 1929 |

# Events

| | |
|---|---|
| Event student is full time | F |
| Event student is part time | T |
| Event reason is Quality | Q |
| Event reason is Cost | C |
| Event reason is Other | O |

# Joint Probability Table

| | | Reason for Joining | | | Totals |
|---|---|---|---|---|---|
| | | Quality (Q) | Cost (C) | Other (O) | |
| Status | Full time (F) | 0.22 | 0.20 | 0.04 | 0.46 |
| | Part time (T) | 0.21 | 0.31 | 0.02 | 0.54 |
| | Totals | 0.43 | 0.46 | 0.06 | 1.00 |

Joint probabilities

Marginal probabilities

# Probabilities of Events

| | |
|---|---|
| P (F) = | |
| P (T) = | |
| P (Q) = | |
| P (C) = | |
| P (O) = | |

# Probabilities of Events

| | |
|---|---|
| Student is FT → ? P (Reason is Quality) | |
| Student is PT → ? P (Reason is Quality | |
| Are T and Q independent? | |

# Multiplication Law

- Multiplication law is use to find the intersection between two events

$$P(A \cap B) = P(A/B)\,P(B)$$
$$P(A \cap B) = P(B/A)\,P(A)$$

- If A and B are independent then

$$P(A \cap B) = P(A)\,P(B)$$

# Bayes' Theorem

- Bayes' theorem is used to update the probabilities of events based on new information (a sample, a test, etc.)

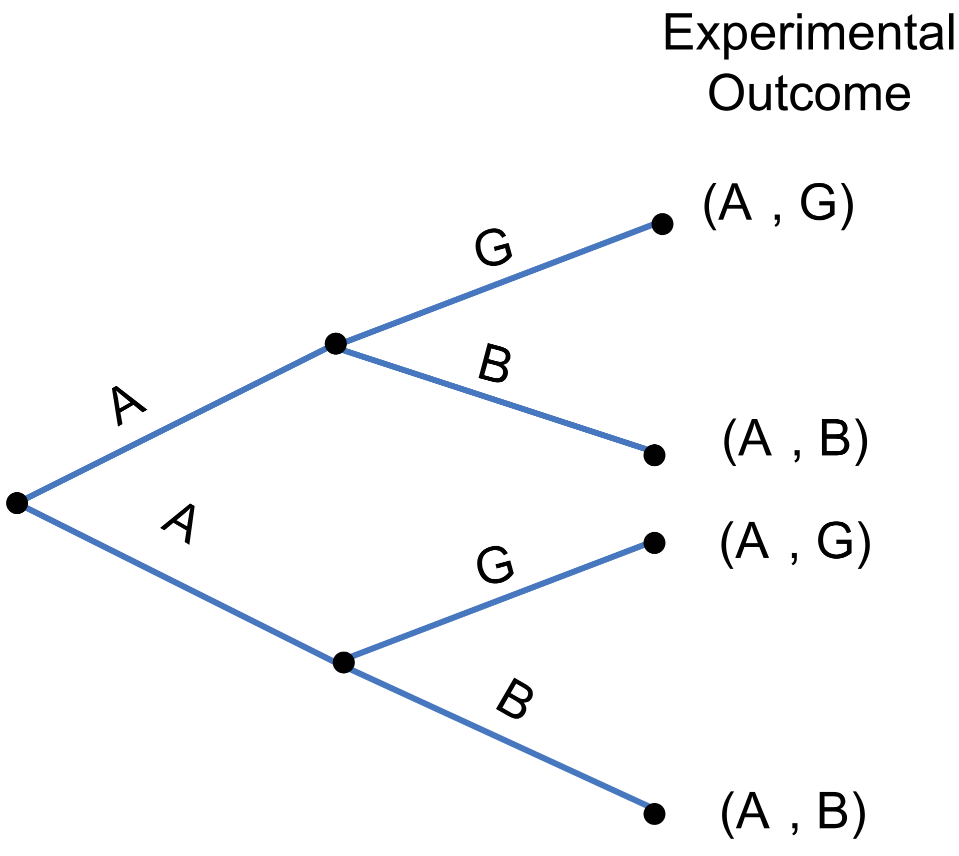
# Example

- A factory receives parts from two suppliers
  - $A_1$ = Event part is from supplier 1 ➔ $P(A_1) = 0.65$
  - $A_2$ = Event part is from supplier 2 ➔ $P(A_2) = 0.35$
  - G = Event part is good
  - B = Event part is bad

    $P(G/A_1) = 0.98$ $P(B/A_1) = 0.0.02$

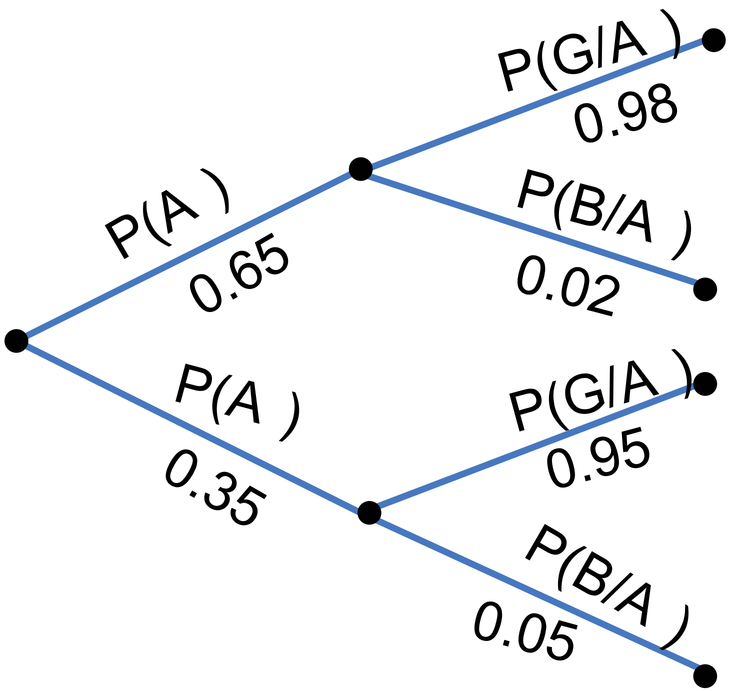
    $P(G/A_2) = 0.98$ $P(B/A_2) = 0.0.02$

- Suppose we select a bad part:
  What is the probability that it came from supplier 1? $P(A_1/B)$
  What is the probability that it came from supplier 2? $P(A_2/B)$


Experimental Outcome
A
A
G
B
G
B
(A , G)
(A , B)
(A , G)
(A , B)

# Probability Tree for Two-supplier Example

Probability Outcome




P (A ∩ G) = P(A ) P(G/A ) = 0.6370

P (A ∩ B) = P(A ) P(B/A ) = 0.0130

P (A ∩ G) = P(A ) P(G/A ) = 0.3325

P (A ∩ B) = P(A ) P(B/A ) = 0.0175

$$P\,(A\ \ /\,B) = \frac{P\,(A\ \ \cap B)}{P\,(B)}$$

$$P\,(A\ \ \cap B)\ =\ P\,(A\ )\,P(B/A\ )$$

$$P(B) = P\,(A\ \ \cap B) + P\,(A\ \ \cap B)\ = P\,(A\ )\,P(B/A\ ) + P\,(A\ )\,P(B/A\ )$$

$$P\,(A\ \ /\,B) = \frac{P\,(A\ )\,P(B/A\ )}{P\,(A\ )\,P(B/A\ ) + P\,(A\ )\,P(B/A\ )}$$

$$P\,(A\ \ /\,B) = \frac{(0.65)\,(0.02)}{(0.65)\,(0.02) + (0.35)\,(0.05)} = \frac{(0.013)}{(0.013) + (0.0175)}$$

$$= \frac{(0.013)}{(0.0305)} = 0.4262$$

Similarly

$$P\,(A\ \ /\,B) = \frac{P\,(A\ )\,P(B/A\ )}{P\,(A\ )\,P(B/A\ ) + P\,(A\ )\,P(B/A\ )}$$

$$= \frac{(0.0175)}{(0.0305)} = 0.5738$$

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# Bayes' Theorem: *Tabular Procedure*

| (1) Event | (2) Prior Prob | (3) Cond Prob | (4) Joint Prob | (5) Posterior Probability |
|---|---|---|---|---|
| $A_i$ | $P(A_i)$ | $P(B/A_i)$ | $P(A_i \cap B)$ | $P(A_i/B)$ |
| $A_1$ | 0.65 | 0.02 | 0.0130 | 0.0130/0.0305 = 0.4262 |
| $A_2$ | 0.35 | 0.05 | 0.0175 | 0.0175/0.0305 = 0.5738 |
| | 1.00 | | P(B) = 0.0305 | 1.0000 |