KING FAHAD UNIVERSITYOF

PERTROLEUM AND MINERALS

Civil Engineering Department

CE 411-Senior Project

Pipe-Line Block Optimization

Prepared for

Senior Project Community

Under Guidance of the Project Advisor

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Abstract

An anchor block was fully analyzed and optimized and the optimization provided ~50% saving in concrete. Several tools with high flexibility were provided to supply the engineer with the proper and optimum dimensions of an anchor block for a given thrust force.

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Introduction

Thrust blocks, that we are going to optimize, are the most common restraints in use for pressurized pipes. They are usually built from concrete. The earth pressures play an important role in soil-structure interaction. They resist lateral movement of structures and provide stabilizing forces for anchor blocks, retaining walls, and laterally loaded pile caps.

Passive pressures induce large loads in integral bridges. When rising temperatures cause an integral bridge to expand in length, pushing its abutments against the approach fills, passive resistance applies a compressive load to the bridge through the abutments. The superstructure must be designed to resist these loads.

Maximum passive pressures can be computed using the well-known Rankine, Coulomb, and Log Spiral earth pressure theories. However, each of these theories has limitations and none provides information on the relationship between resistance and movement.

The passive resistance that develops when a structure moves against soil depends on (1) the amount and direction of the movement; (2) strength and stiffness of the soil; (3) friction and/or adhesion between the structure and soil; and (4) shape of the structure. Each of these factors has an important influence on the magnitude of passive earth pressure.

Both soil strength and soil stiffness are important in determining the amount of passive resistance that develops in a given circumstance. The greater the strength of the soil, the larger is the maximum possible passive pressure. The stiffer the soil, the greater is the passive pressure induced by a given amount of movement.

Since, as we have stated before, the shape of the structure has a mean influence on the magnitude of passive earth pressure, in this project we are going to construct a model for pipe-line anchor blocks that give us the minimum volume of a block for a given pipe force. Nowadays the method used in investigating the dimensions of such a block is by the trial and error methods. In our project we aim to obtain the proper solution or the block that gives us the minimum cost and we would like also to set out a model that enables the user (the engineer) to design or analyze his block effectively and efficiently.

Problem Description

The block that we aim to analyze and optimize in this project is a regular one with the dimensions shown in Figure 1 where

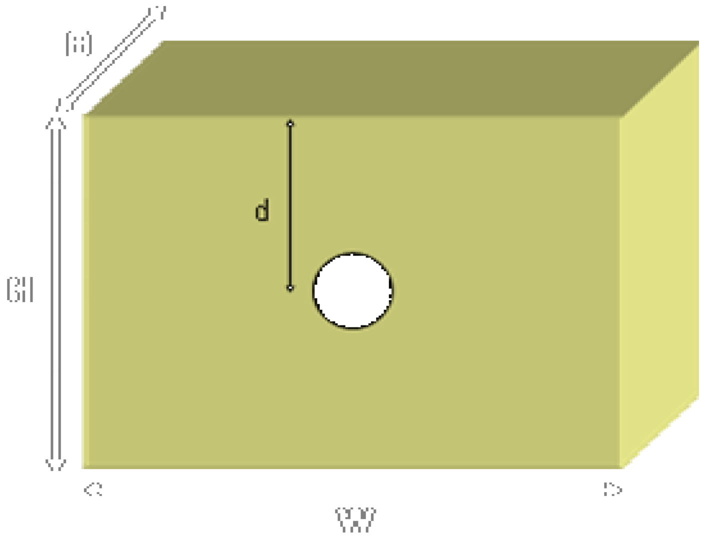


Figure 1: Typical Anchor Block

H: is the height of the block

W: is the block's width

B: is the block's depth

d: is the distance where the pipe should be installed measured from the top of the block

According to their orientation, the forces that act on the block can be classified into two categories: forces that are normal to the phases of the block and those are parallel to them. Since the block is inserted in the soil some phases are exposed to active soil forces and other are exposed to passive forces. To begin our analysis let us first identify the orientation (i.e. magnitudes, directions and locations) of these forces. Since it is difficult and not an efficient manner to show all the forces on that block, side view pictures is used in the analysis.

1. Block Design

1.1 Formulations

First of all let us establish our coordinate system by choosing the point O as our origin and numbering the phases of the block as shown in Figure 2. The direction of the pipe force Fp is assumed to be acting in the positive z-direction. Due to this assumption the soil will exert active stresses on first phase and passive on the second. The fifth and sixth phases both undergo active stresses.

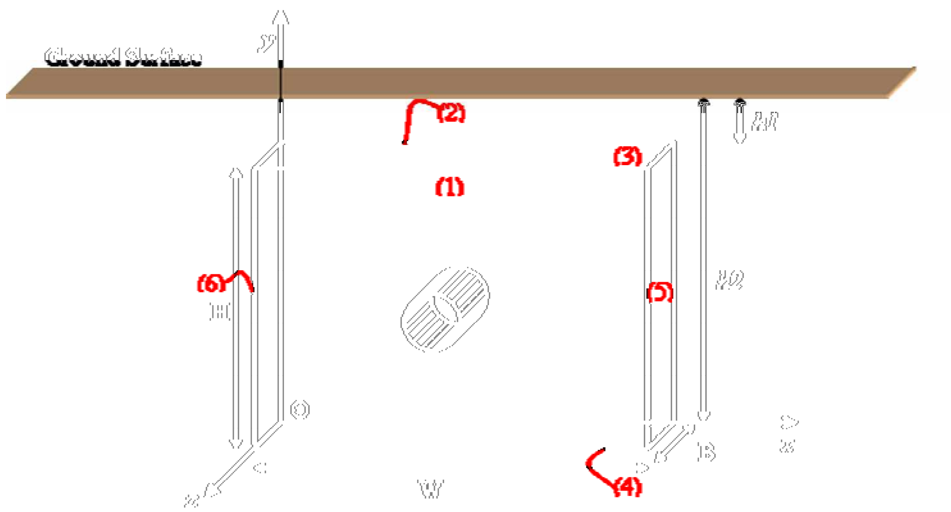


Figure 2: Anchor Block's Design Dimensions

Form soil concept it is known that $\sigma_v = \gamma_s h$ where

σ_v: Vertical stress due to soil

γ_s: Soil unit weight, or in other words the unit weight of the soil

h: Depth in soil at which the stress is investigated

The vertical stress σ_v is linearly related to the horizontal stress by a proportionality constant and the relation as follows

$$\sigma_h = k\,\sigma_v$$

Where

k: coefficient of earth pressure

σ_h: horizontal stress due to soil

The value of *k* is either k_a in case of active soil or k_p is case of passive soil. The relation between the two constants is given by the relation

$$k_a = \frac{1}{k_p}$$

The two following figures, Figure 3 and Figure 4, give a front and side view of the stresses distribution respectively.

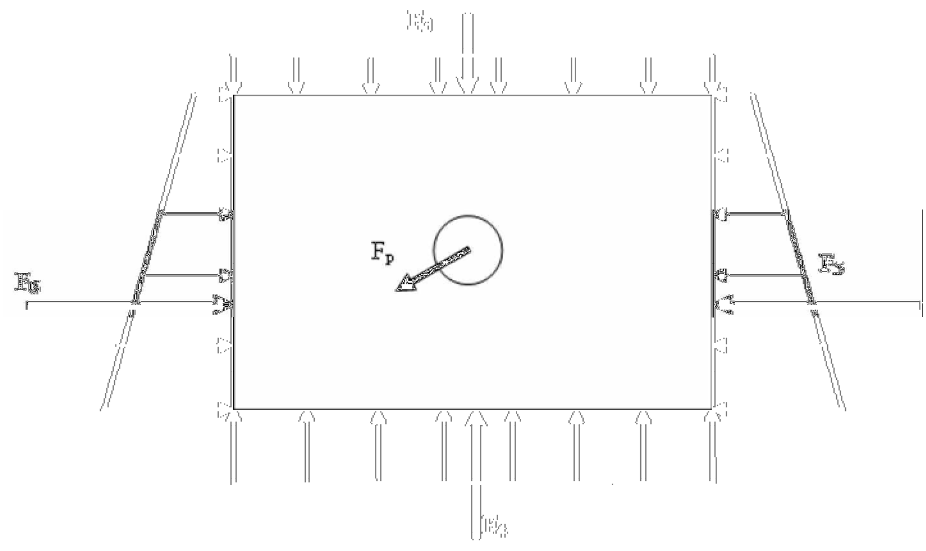


Figure 3: Front view of the anchor block stresses distribution

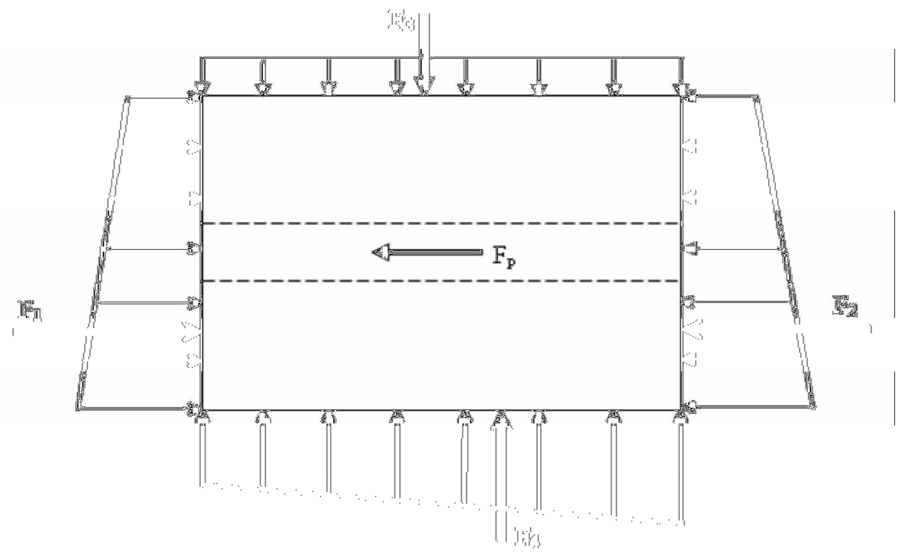


Figure 4: Side view of the anchor block stresses distribution

Now we are ready to analyze the forces on the block.

Referring to the side view in Figure 5, for the horizontal stresses acts on phase 1 at a distance $y=h_1$ is given by $\sigma_{h1} = k_p \sigma_v = k_p \gamma_s h_1$ and the stress at y=h_2 is $\sigma_{h2} = k_p \gamma_s h_2$.

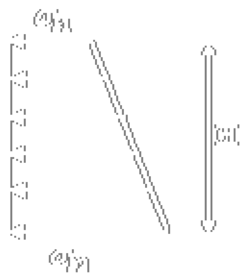


Figure 5: Sectional Stress Distribution

For a unit width the stress distribution execute a trapezoidal shape as shown in Figure. The area of this tarpozoidal, or the force per unit width, is given by

$$\frac{F_1}{W} = [\frac{1}{2}(\sigma_{h2} - \sigma_{h1})(h_2 - h_1) + \sigma_{h_1}(h_2 - h_1)] = [\frac{(h_2 - h_1)}{2}(\sigma_{h_1} + \sigma_{h_2})]$$

$$= k_p \gamma [\frac{h_2 - h_1}{2}.(h_2 + h_1)] = \frac{k_p \gamma}{2}(h_2^2 - h_1^2)$$

$\therefore$ The force acts on phase one is $F_1 = \frac{k_p \gamma}{2}(h_2^2 - h_1^2)W$ in the direction of –*z*-axis.

The second 2nd phase is subjected to active soil stresses and in this case with the same analysis followed with F_1 except that k_p is replaced by k_a in this situation; we can express the force as the following:

$$\frac{F_2}{W} = [\frac{1}{2}(\sigma_{h2} - \sigma_{h1})(h_2 - h_1) + \sigma_{h_1}(h_2 - h_1)] = [\frac{(h_2 - h_1)}{2}(\sigma_{h_1} + \sigma_{h_2})]$$

$$= k_a \gamma [\frac{h_2 - h_1}{2}.(h_2 + h_1)] = \frac{k_a \gamma}{2}(h_2^2 - h_1^2)$$

The force acts on phase two is $F_2 = \frac{k_a \gamma}{2}(h_2^2 - h_1^2)W$ in the direction of +*z*-axis.

To find the force acts on the top of the block, on phase 3, that force is equivalent to the weight of soil above the block and consequently

$F_3 = \rho_s V_s.g = \gamma_s.WB.h_1$ acting in the direction of –*y*-axis.

The force for which the 4th phase is subjected is equivalent to the block's weight in addition to the weight of the sand above the block. In other words

$$F_4 = F_3 + \textit{weight of the block} = F_3 + \rho_c.HWB.g = F_3 + \gamma_c.HWB$$

Where

γ_c: is the unit weight of concrete

This force acts in the –*y*-axis direction.

The forces on the 5th and 6th phases are equal in magnitude but opposite in direction ($F_5 = F_6$) and the two forces are due to active stresses. To compute the resultant force due to these stress as shown in Figure 6:

$$F_5 = (\sigma_{h2} + \sigma_{h1})\frac{H}{2}B = k_a \gamma_s \frac{H}{2}B(h_1 + h_2)$$

acting in the –*x*-direction and

$$F_6 = (\sigma_{ha2} + \sigma_{ha1})\frac{H}{2}B = k_a \gamma_s \frac{H}{2}B(h_1 + h_2)$$

acting in the +*x*-direction and

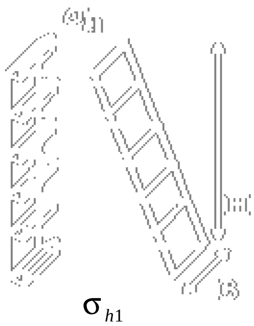


Figure 6: Lateral Stress Distribution

All the forces that are normal to the block phases are found right now. Since the material assumed to have the same characteristics around the block, the friction coefficient of the soil is assumed to be a constant μ. In this case, the frictional forces can be expressed in the form $fr = \mu N$ where

fr: is the frictional force

N: normal force

μ: coefficient of static friction

and this force *fr* acts in a perpendicular position with respect to the normal forces with a direction toward the –*z*-axis.

To investigate the friction; $f = \mu N$ as stated before, so the friction due to F_i is $f_i = \mu F_i$ for *i* = 3, 4, 5 and 6.

The frictional forces then can be expressed in vector notation which obviously includes besides their magnitude the direction also as follows:

$$\vec{fr}_3 = <0, 0, -\mu \gamma_s WBh_1> \quad , \quad \vec{fr}_5 = <0, 0, -\mu \frac{H}{2} B k_a \gamma_s (2h_1 + H)>$$

$$\vec{fr}_4 = <0, 0, -\mu (\gamma_s WBh_1 + \gamma_c HWB)> \quad , \quad \vec{fr}_6 = <0, 0, -\mu \frac{H}{2} B k_a \gamma_s (2h_1 + H)>$$

For a block to be under equilibrium:

$$\sum F_z = 0 \Rightarrow F_p + F_2 - F_1 - \mu(F_3 + F_4 + F_5 + F_6) = 0 \quad \text{and} \quad \sum \vec{M} = \vec{0}$$

Now we are in a position to identify the locations of these forces.

We begin with locating $\vec{F}_5$ since it needs some effort. For that purpose we will consider only a unit width of the stresses along phase 5 since the stresses are with the same uniformity along the whole width of that phase. In other words, the location (*y*-coordinate only) of the net force for a unit width of stresses distribution is the same as that for the whole stresses along that phase.

Let T1 represents the net force for the rectangle shape of the stresses distribution and located at a distance $\frac{H}{2}$ from our reference point and let T2 represents the net force of the triangle located at a distance $\frac{2H}{3}$. The equivalent distance or the distance at which the net of the two forces T1 and T2 act is called d_5. With this in mind we can analyze the illustration graph given in Figure 7 as follows:

$$\frac{F_5}{B} = T_1 + T_2 = R$$

$$\sum M_S = 0 : T_1(\frac{H}{2}) + T_2(\frac{2H}{3}) = d_5 R$$

$$\Rightarrow d_5 = \frac{T_1(\frac{H}{2}) + \frac{4}{3}T_2(\frac{H}{2})}{R} = \frac{\frac{H}{2}(T_1 + \frac{4}{3}T_2)}{T_1 + T_2}$$

$$= \frac{H(3T_1 + 4T_2)}{6(T_1 + T_2)}$$

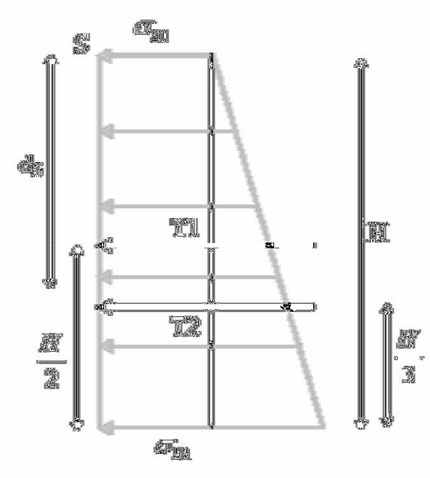


Figure 7: Location of F5

But we know that $T_1 = \sigma_{h1} H = k_a \gamma_s h_1 H$

and

$$T_2 = \frac{1}{2}(\sigma_{h2} - \sigma_{h1})H = \frac{k_a}{2}\gamma(h_2 - h_1)H, \qquad H = h_2 - h_1$$

$$\Rightarrow T_2 = \frac{k_a}{2}\gamma H^2$$

$$\therefore d_5 = \frac{H(2H + 3h_1)}{3(H + 2h_1)}$$

From this result we can observe that the location of the force depends only on h_1 and H where H is expressed in both h_1 and h_2.

Since $\vec{F}_5$ and $\vec{F}_6$ are equal in magnitude and opposite in direction only, the arm distance that $\vec{F}_6$ has is the same as that of $\vec{F}_5$ and consequently $d_5 = d_6$.

To find the arm distance of $\vec{F}_1$ we should neglect the effect of the pipe hole for simplicity. The force that acts on phase 1 is a passive force and consequently

$$T_1 = k_p \gamma_s h_1 H \qquad \text{and} \qquad T_2 = \frac{k_p}{2}\gamma_s H^2$$

Following the same approach to get d_5 we got

$$d_2 = \frac{H(3T1+4T2)}{6(T1+T2)} = \frac{H(2H+3h_1)}{3(H+2h_1)}$$

This result is equivalent with that obtained for d_5. By comparing the two results we see that both are independent of k_a or k_b. in other words, we can say that the line of action of the forces on those phases is independent of whether the soil is active or passive!.

Now we could conclude that F_1, F_2, F_5 and F_6 pass through a common point in space and they are at the same level!. In fact the whole points coincide in a single point.

Then we can say that the difference affects the moment is their magnitudes.

The locations of the forces F_1, F_2, F_5 and F_6 can be said to be done and found. We are left with two normal forces and the frictional forces.

For the forces F_3 and F_4, the locations at which they affect measured from our origin O, Figure 2, are $\vec{r}_3 =< \frac{W}{2}, H, \frac{B}{2} >$ and $\vec{r}_4 =< \frac{W}{2}, 0, \frac{B}{2} >$ respectively.

For the frictional forces:

The locations of the friction forces are as that for the normal forces which they come from. This can be expressed as follows:

$$\vec{r}_{fr1} = < \frac{W}{2}, H-d, B > \quad , \vec{r}_{fr2} = < \frac{W}{2}, H-d, 0 >$$
$$\vec{r}_{fr3} = < \frac{W}{2}, H, \frac{B}{2} > \quad , \vec{r}_{fr5} = < W, H-d, \frac{B}{2} >$$
$$\vec{r}_{fr4} = < \frac{W}{2}, 0, \frac{B}{2} > \quad , \vec{r}_{fr6} = < 0, H-d, \frac{B}{2} >$$

We may construct the following list as a summary for the forces that acts on the block expressed in vector notations:

$$\vec{F}_2 = < 0, 0, \frac{+k_a}{2}\gamma_s W H(2h_1 + H) > \quad \vec{fr}_2 = < 0, 0, \frac{k_a}{2}\mu\gamma_s H(2h_1 + H) >$$
$$\vec{F}_1 = < 0, 0, -\frac{\gamma_s}{2k_a} W H(2h_1 + H) > \quad \vec{fr}_1 = < 0, 0, \frac{-\gamma_s}{2k_a}\mu H(2h_1 + H) >$$
$$\vec{F}_3 = < 0, -\gamma_s WBh_1\gamma, 0 > \quad \vec{fr}_3 = < 0, -\gamma_s \mu WBh_1\gamma, 0 >$$
$$\vec{F}_4 = < 0, +\gamma_s WBh_1 + HWB\gamma_c, 0 > \quad \vec{fr}_4 = < 0, \mu(\gamma_s WBh_1 + HWB\gamma_c), 0 >$$
$$\vec{F}_5 = < -\frac{H}{2} Bk_a\gamma_s(2h_1 + H), 0, 0 > \quad \vec{fr}_5 = < -\frac{H}{2} Bk_a\gamma_s \mu (2h_1 + H), 0, 0 >$$
$$\vec{F}_6 = < \frac{H}{2} Bk_a\gamma_s(2h_1 + H), 0, 0 > \quad \vec{fr}_6 = < \frac{H}{2} Bk_a\gamma_s \mu (2h_1 + H), 0, 0 >$$

Finally we are left with a list of the orientation of the normal forces which are as follows:

$$\vec{r}_1 = < \frac{W}{2}, H-d, B > \quad , \vec{r}_2 = < \frac{W}{2}, H-d, 0 >$$
$$\vec{r}_3 = < \frac{W}{2}, H, \frac{B}{2} > \quad , \vec{r}_5 = < W, H-d, \frac{B}{2} >$$
$$\vec{r}_4 = < \frac{W}{2}, 0, \frac{B}{2} > \quad , \vec{r}_6 = < 0, H-d, \frac{B}{2} >$$

With these results in hand we have finished the analysis of our block.

1.2 Location of Resultant

To find the location of the resultant force, the magnitude (scalar) of each force is only considered in the calculation. By taking the moment summation about the *x*-axis for equilibrium condition:

$$\sum M_{x-axis} = 0$$

$$F_4(W/2) + F_2(H - d_5) - F_1(H - d_5) - fr_3H - fr_5(H - d_5) - fr_6(H - d_5) = M_R$$

Solving the previous equation for *d* we get:

$$d = \frac{H - (F_1 - F_2 + fr_3 + fr_5 + fr_6)H - d_5(f_1 - F_2 + fr_5 + fr_6)}{F_1 - F_2 + fr_3 + fr_5 + fr_6}$$

Where *d* is measured from point S as shown in Figure 7.

2. Design Constrains

According to field requirements there are some constrains that we should stick with in order to obtain a model that satisfy the field job.

These constrains are as follows:

1. factor of safety against the sliding is greater than or equal to 1.25
2. factor of safety against the overturning moment is greater than or equal to 1.50
3. bearing capacity is less than or equal to 3 ksf and greater than or equal to zero in order to maintain the soil under compression conditions (its allowable bearing capacity).

The third condition is taken with consideration of safety since the actual bearing capacity of our soil (sand) is about 12 ksf. In this case we assigned a limit of 3 ksf which is about one fourth (¼) of the soil bearing capacity. This provides more enhancements regarding the safety issues.

To assure these conditions are satisfied in our final results, the following analysis was done on those constrains:

1. *factor of safety:*

Since it is required that the factor of safety against the sliding is greater than or equal to 1.25, this condition can be expressed as:

$$\frac{R\ against\ F_p}{F_p} \in [1.25, x] \qquad \Rightarrow \qquad \frac{R}{F_p} \geq 1.25$$

The second condition that is the factor of safety against the overturning moment is greater than or equal to 1.50 can also be expressed as:

$$\frac{Moment\ due\ Do\ all\ forces\ but\ F_p}{Moment\ due\ to\ F_p} \in [1.5, x] \qquad \Rightarrow \qquad \frac{M_R}{M_p} \geq 1.5$$

Where x is any number that is greater than the lower boundary of the interval that contains x.

$$\therefore \text{Our } F_p \text{ allowable} = \frac{R}{F.S._F} \qquad \text{and} \qquad M_{p_{all}} = \frac{M_R}{F.S_m}$$

Where

$F.S._F$: is the factor of safety against sliding

$F.S._M$: is the factor of safety against overturning moment

The force Fp lies in a distance dp from the top of the block and our aim is to obtain $M_R = F.S._M\ M_p$ such that $R\ d_R = F.S._M\ M_p$.

The moment M_p is expressed in scalar notation by $\sum M_{x-axis} = 0$ or $M_p = F_p\ (H - d_p)$.

2. *bearing capacity:*

To account for the bearing, we should maintain the soil under the block in a compression status with a maximum value of 3 ksf as stated previously. To achieve this target stresses under the block should be investigated.

We have to types of stresses in this region: either stresses due to forces or stresses due to moments. Maximum moment's stresses are given by the formula:

$$\sigma_{\max} = \frac{Mc}{I}$$

Where

$\sigma_{\max}$: maximum stress due to moment

M: bending moment

c: maximum distance measured from the neutral axis

I: area moment of inertia

Our moment is taken about *x*-axis and formulated as

$$M = -M_p + R \times d_R,$$

The area moment of inertia is $I = \dfrac{WB^3}{12}$ and maximum distance measured from the neutral axis is $c = \dfrac{B}{2}$.

So, the maximum stress value due to moments (about *x*-axis) is

$$\sigma_{\max 1} = \pm \frac{R \times d_R - F.S._M M_p}{\frac{WB^3}{12}} \cdot \frac{B}{2}$$

The forces F_5 and F_6 have no effects in the stresses magnitude in the under block region since they are equal in magnitude and opposite in direction .The remain forces that cause stresses beneath the block are the F_3 force and the block's weight which are both equivalent to F_4 in their magnitude .

$$\therefore \qquad \sigma_2 = \frac{\left|\vec{F}_4\right|}{A} = -\frac{(\gamma_s WBh_1 + \gamma_c HWB)}{BW} = \gamma_s h_1 + \gamma_c H = \sigma_{\max 2}$$

This stress is purely compressive on the block under beneath.

The total maximum stress under beneath the block is $\sigma_{\max 1} + \sigma_{\max 2}$; and to preserve the soil from bearing failure with a conservative value of 3ksf and maintain a whole compressive stresses area eliminate the tension part of stresses related to σ_1.

The maximum stress that results from the given stresses distribution is

$$\sigma_{\max 1} + \sigma_{\max 2} = \pm \frac{6(Rd_R - M_p)}{WB^2} - (\gamma_s h_1 + \gamma_c H)$$

In other words, the requirement is to maintain $0 \le \sigma \le 3$ or:

$$(\frac{6(M_p - FS_M M_p)}{WB^2}) - (\gamma_s h_1 + \gamma_c H) \leq \text{ max (3ksf) and}$$

$$\frac{-6(M_p - FS_M M_p)}{WB^2} - (\gamma_s h_1 + \gamma_c H) \geq 0\,\text{ksf.}$$

By this we end up the design constrains formulation part.

3. Objective Function

The objective right now is to optimize our block by reflecting the results obtained from Design Constrains part on that from the Analysis part. The optimization main parameter is the minimization of the block volume by changing the values of *W, H, B,* and the location of the pipe *d*. Also we are interested in finding a direct relationship between the force applied on the block and the block's parameters.

In order to attain this goal Microsoft Excel Package is used in the optimization process.

4. Excel Part

In this section we try to model our problem using Microsoft Excel in order to optimize the block with satisfying all constrains mentioned in the Design Constrains part.

To start our calculations certain properties should be firstly specified to work with. In this project the soil properties we are dealing with are as stated in the next section.

4.1 Soil Properties Identification

Calculations are carried on assuming the following properties about the soil:

1. soil specific weight = 100 pcf

2. allowable bearing capacity = 3 ksf

3. static friction coefficient = 0.4

4. concrete specific weight = 150 pcf

5. angle of internal friction of the soil $\phi = 30^\circ$

By knowing the angle of internal friction of soil ϕ, the soil coefficients can be computed as follows:

$$k_a = \frac{1-\sin(\phi)}{1+\sin(\phi)} = \frac{1-\sin(30^\circ)}{1+\sin(30^\circ)} = 0.333; \qquad \text{so} \qquad k_p = \frac{1}{k_a} = 3$$

These parameters are usually provided by the geotechnical engineer.

4.2 Excel Model Construction

In order to construct our model we should first identify our input and the required output and their corresponding Excel Cells. The inputs, the information that we have the ability to specify or provide, are the force that the pipe exerts on the block, concrete specific weight, block position from the ground surface h_1 and soil properties (i.e. k_a or k_p μ, and γ_s). Our desired outputs are the block dimensions (i.e. *W*, *B*, and *H*) and the location of the pipe, Figure 8.

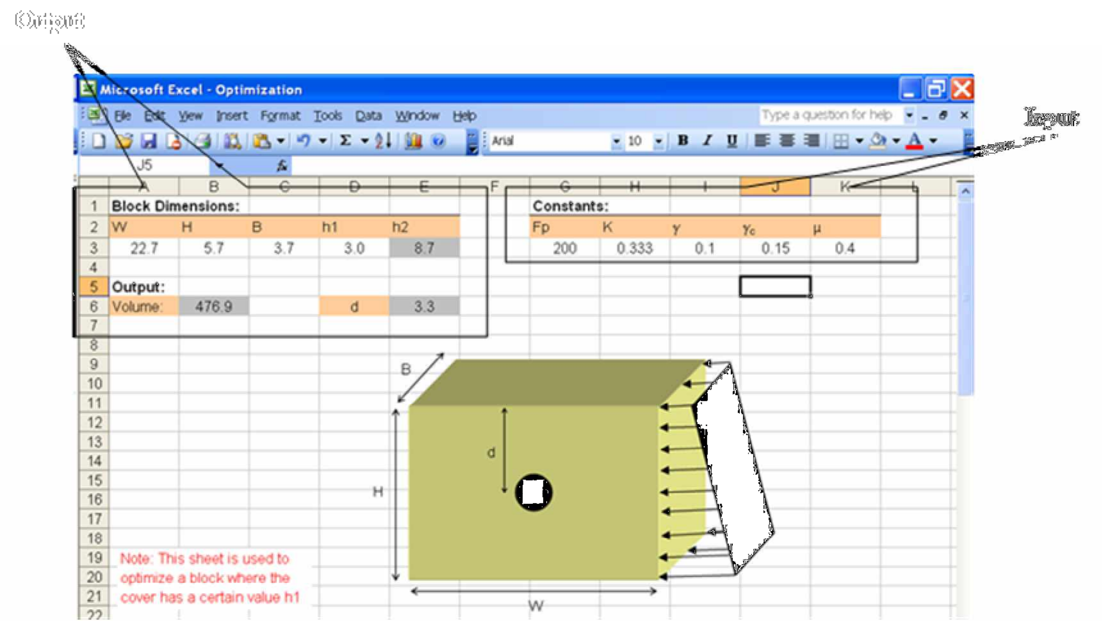
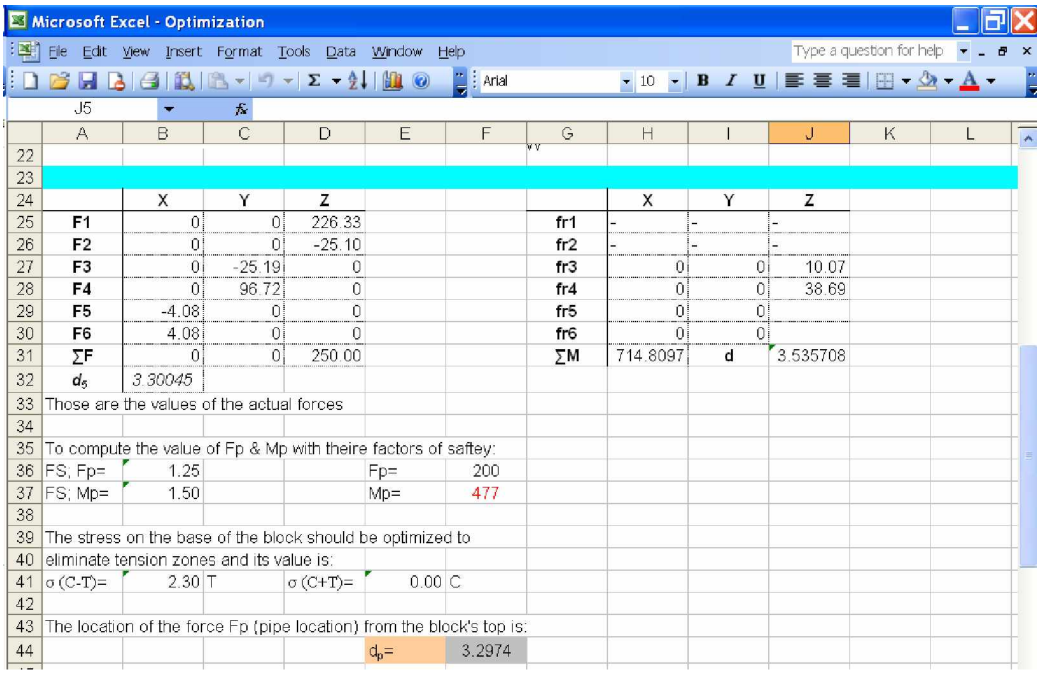


Figure 8: Input and Output Parameters

In order to start the analysis of the block we should connect these parameters via the formulas that we derived previously. The formulas governs the forces and moments were inserted in the Excel Sheet besides the constrains formulation, Figure 9.



	A	B	C	D	E	F	G	H	I	J
24		X	Y	Z				X	Y	Z
25	F1	0	0	226.33			fr1	-	-	-
26	F2	0	0	-25.10			fr2	-	-	-
27	F3	0	-25.19	0			fr3	0	0	10.07
28	F4	0	96.72	0			fr4	0	0	38.69
29	F5	-4.08	0	0			fr5	0	0	
30	F6	4.08	0	0			fr6	0	0	
31	ΣF	0	0	250.00			ΣM	714.8097	d	3.535708
32	d_5	3.30045								
33	Those are the values of the actual forces									
35	To compute the value of Fp & Mp with theire factors of saftey:									
36	FS, Fp=	1.25			Fp=	200				
37	FS; Mp=	1.50			Mp=	477				
39	The stress on the base of the block should be optimized to									
40	eliminate tension zones and its value is:									
41	σ (C-T)=	2.30	T	σ (C+T)=	0.00	C				
43	The location of the force Fp (pipe location) from the block's top is:									
44					d_p=	3.2974				

Figure 9: Moments and forces acts on the block

The sheet now is ready to use for analysis. It means that for a given input data (specified previously) we can analyze the block completely and get the forces that act on the block with their configuration as shown in Figure 9. The stresses under beneath the block are also obtained.

Now we are in a good position to start the optimization issue.

5. Optimization

To optimize the block we need to identify the parameters that are included in the optimization process. The parameter that we aim to optimize is the volume of the block. In order to do so the Solver Command is used. Let us start:

1. Go to Tools menu in the Microsoft Excel
2. Click Solver, Figure 10

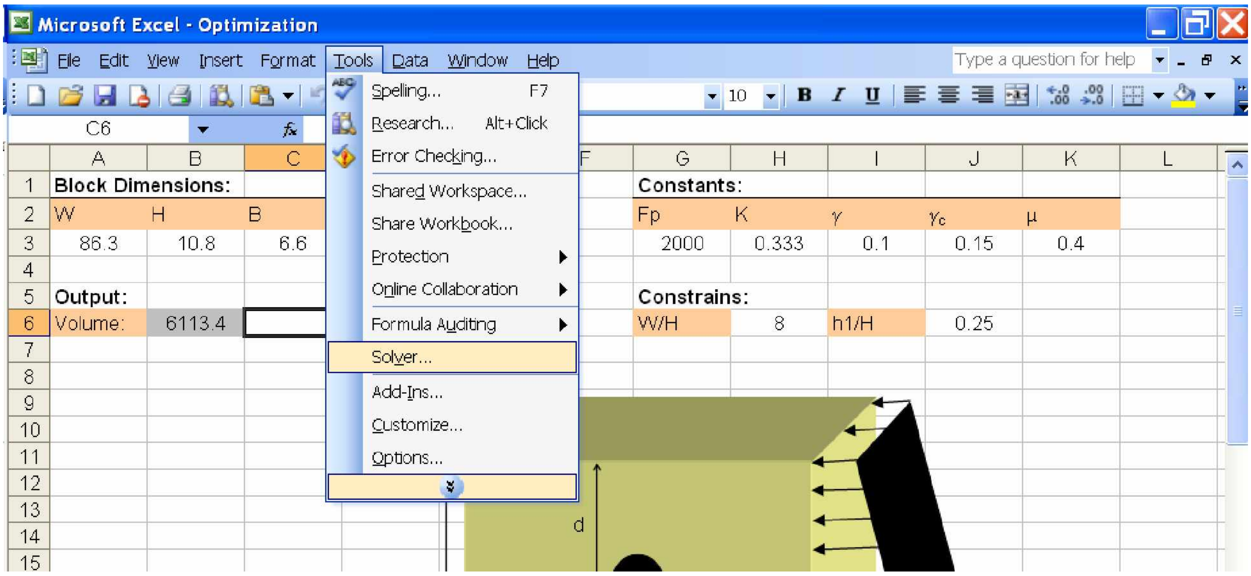


Figure 10: Where is Solver

3. Solver window is now activated, Figure 11
4. In this window set the target cell the you want to optimize (Volume in our case)

5. Choose Min since we aim to minimize the volume

6. Set the cells that you want to vary in the optimization process (*W*,*H*,*B* and *d*)

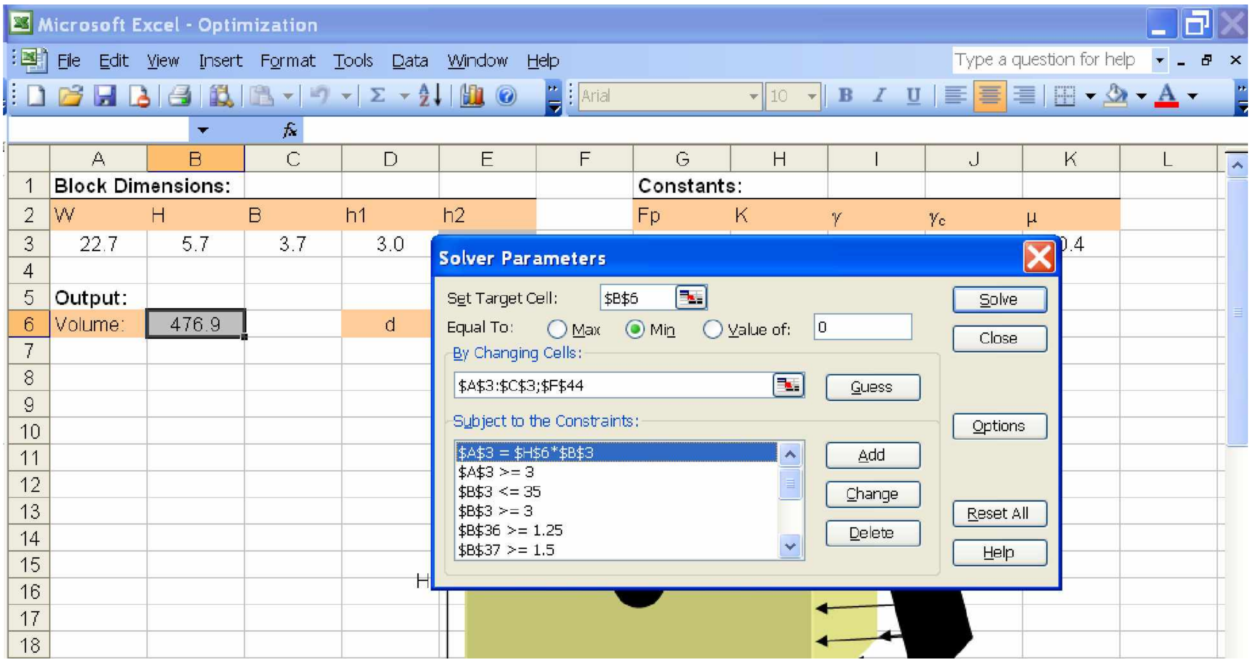


Figure 11: Solver window

7. Add the constrains, Figure 11

Constrains are mainly inserted in accordance with the concept stated in the Design Constrains section.

Since the problem is now modeled in Excel we need to test our model. In testing the model we got non-practical solutions as:

W	H	B
4312196	0.000355	0.000385

This is logical in order to obtain the minimum volume we should increase the width *W* of the block as much as possible, but this absolutely not practical.

In order to overcome this problem we added another constrain to our model that related *W* to *B* and by this we obtained more logical and acceptable solutions. Another constrain is we maintained our *H* and *B* values to be greater than 3 ft and less than 35 ft. In fact this constrain can be neglected since Solver aims to minimize the *H* and *B* and maximize *W*.

We have two different Excel Sheets for minimization. In the first sheet the distance *h1* is assumed to be variable that is specified by the user, which is normally taken as 3 ft in practice and we have the ability to change the ratio between *W* and *H*. Furthermore, in the other sheet, the user/engineer can specify also the ratio between *h1* and *H* in order to get the optimum/minimum volume for his desired block.

After describing the way by which we have constructed our model, let us verify the model's results.

6. Design Verification

To be certain about our Excel model results and the satisfaction of our constrains, we should check the values obtained from the model if they are consistent with that calculated by hand. A more advanced step is to compare our results with the current techniques used in finding the volume of the block and show the difference in volume between the two techniques.

6.1 Excel Model Verification

In order to do the check, a random force of 1400 lb was inserted, in the first optimization sheet (*W*=4*H* and *h1* = 3 *ft*), as an input besides the soil properties that are described in the Properties section. The results were as follows:

F	W	H	B	d
1000	42.5	10.6	6.6	6.4

Let us start our checking:

1. Computing the acting forces:

$$\vec{F}_2 = -\frac{k_a}{2}\gamma_s W\,H(2h_1 + H) = -\frac{0.333}{2}\times 0.1\times 42.5\times 10.6(2\times 3 + 10.6) = 125\,lb$$

$$\vec{F}_1 = \frac{\gamma_s}{2k_a} W\,H(2h_1 + H) = \frac{0.1}{2(0.333)} 42.5\times 10.6(2\times 3 + 10.6) = 1128\,lb$$

$$\vec{F}_3 = \gamma_s WBh_1 = 0.1\times 42.5\times 6.6\times 3 = 84\,lb$$

$$\vec{F}_4 = -(\gamma_s WBh_1 + HWB\gamma_c) = -(0.1\times 42.5\times 6.6\times 3 + 10.6\times 42.5\times 6.6\times 0.15) = 532\,lb$$

$$\vec{F}_5 = \frac{H}{2} Bk_a\gamma_s(2h_1 + H) = \frac{10.6}{2} 6.6\times 0.333\times 0.1\times (2\times 3 + 10.6) = 19\,lb$$

$$\vec{F}_6 = -\frac{H}{2} Bk_a\gamma_s(2h_1 + H) = -19\,lb$$

Important note: in the Excel model we have inverted the sign of the force in order to be consistent with Solver which is more convenient for us. This change does not affect the results at all.

2. Computing the most effective frictional forces:

$$\vec{fr}_3 = \gamma_s\,\mu\,WBh_1\gamma = 84\times 0.4 = 33.6\,lb$$

$$\vec{fr}_4 = \mu(\gamma_s\;WBh_1 + HWB\gamma_c) = 213\,lb$$

3. Check net soil resistance:

$\sum$resistance forces = $\sum(F_1 - F_2 + fr_3 + fr_4) = 1250\,lb > 1000\,lb \qquad (OK)$

4. Check factor of safety against sliding:

$\frac{R}{F_p} \geq 1.25 \qquad \Rightarrow \qquad \frac{1250}{1000} \geq 1.25 \qquad (OK)$

5. Check factor of safety against overturning moment (tipping):

$M_p = F_p (H - d_p) = 1000\ (10.6 - 6.4) = 4211\ lb.ft$

$M_R = F_4 (W/2) + F_2 (H - d_5) - F_1 (H - d_5) - fr_3 H - fr_5 (H - d_5) - fr_6 (H - d_5)$

$= 532(42.5/2) + 125(10.6 - 6.4) - 1128\ (10.6 - 6.4) - 33.6(10.6) = 6316\,lb.\,ft$

$\frac{M_R}{M_p} \geq 1.5 \qquad \Rightarrow \qquad \frac{6316}{4211} \geq 1.5 \qquad (OK)$

6. Bearing capacity check:

$$0 \leq \sigma \leq 3$$

$$\sigma = \sigma_{\max 1} + \sigma_{\max 2} = \pm \frac{6(Rd - M_p)}{WB^2} - (\gamma_s h_1 + \gamma_c H) = \pm \frac{6(1250 \times 6.99 - 4211)}{42.5 \times 6.6^2} - (0.1 \times 3 + 0.15 \times 10.6)$$

$\therefore \qquad \sigma_1 = 3\,ksf \qquad , \qquad \sigma_2 = 0.79 \qquad (OK)$

The requirement $0 \leq \sigma \leq 3$ is satisfied.

By now we checked the validity of the model and the next step is to compare those results with the current technique used in computing the blocks volumes in order to appraise our job (the project).

6.2 Current Technique Used in Dimensions Investigation

Volume of an anchor block required is done at present essentially by trial and error techniques which requires time and several trials to satisfy the constrains besides it does not give the optimum dimensions of the block.

In this section we are going to show the procedure at which the volume of the block is found nowadays.

In this example, we would like to find the dimensions for a block that can resist a force of 1502 *lb*, let us start:

a) Try block with dimensions $W \times H \times B = 25 \times 14 \times 9 \;\; ft^3$

1. Computing the acting forces:

$$\vec{F}_2 = -\frac{k_a}{2}\gamma_s W\,H(2h_1 + H) = -1051 lb$$

$$\vec{F}_1 = \frac{\gamma_s}{2k_a} W\,H(2h_1 + H) = 117 lb$$

$$\vec{F}_3 = \gamma_s WBh_1 = 67.5 lb$$

$$\vec{F}_4 = -(\gamma_s WBh_1 + HWB\gamma_c) = -540 lb$$

$$\vec{F}_5 = \frac{H}{2} Bk_a\gamma_s(2h_1 + H) == 41.96 lb$$

$$\vec{F}_6 = -\frac{H}{2} Bk_a\gamma_s(2h_1 + H) = -41.96 lb$$

2. Computing the most effective frictional forces:

$$\vec{fr}_3 = \gamma_s\,\mu WBh_1\gamma = 100.8 lb$$

$$\vec{fr}_4 = \mu(\gamma_s\; WBh_1 + HWB\gamma_c) = 252\,lb$$

3. Check net soil resistance:

$\sum$resistance forces = $\sum(F_1 - F_2 + fr_3 + fr_4) = 1177.5 < 1502 lb$ $\qquad (NOT\ OK)$

4. STOP

b) Try block with dimensions $W \times H \times B = 30 \times 16 \times 16 \ ft^3$

1. Computing the acting forces:

$$\vec{F}_2 = -\frac{k_a}{2}\gamma_s W\,H(2h_1 + H) = -175.8\,lb$$

$$\vec{F}_1 = \frac{\gamma_s}{2k_a} W\,H(2h_1 + H) = 1586\,lb$$

$$\vec{F}_3 = \gamma_s WBh_1 = 144\,lb$$

$$\vec{F}_4 = -(\gamma_s WBh_1 + HWB\gamma_c) = -1296\,lb$$

$$\vec{F}_5 = \frac{H}{2} Bk_a\gamma_s(2h_1 + H) = 93.77\,lb$$

$$\vec{F}_6 = -\frac{H}{2} Bk_a\gamma_s(2h_1 + H) = -93.77\,lb$$

2. Computing the most effective frictional forces:

$$\vec{fr}_3 = \gamma_s\,\mu WBh_1\gamma = 57.6\,lb$$

$$\vec{fr}_4 = \mu(\gamma_s\ WBh_1 + HWB\gamma_c) = 518.4\,lb$$

3. Check net soil resistance:

$\sum$resistance forces = $\sum(F_1 - F_2 + fr_3 + fr_4) = 1985.76 > 1502\,lb$ (OK)

4. Check factor of safety against sliding:

$\frac{R}{F_p} \geq 1.25 \quad \Rightarrow \quad (\frac{1985.76}{1502} = 1.32) \geq 1.25 \quad (OK)$

5. Check factor of safety against overturning moment (tipping):

Assume the pipe is located at the middle of the block's height.

$M_p = F_p(H/2) = 1502(16/2) = 12016\ lb.ft$

The location of the resultant forces of soil acting on the block is:

$$d = 4.77\ ft$$

So, the resisting moment M_R = 19840.3 *lb.ft*

$$\frac{M_R}{M_p} \geq 1.5 \qquad \Rightarrow \qquad (\frac{19840.3}{12016} = 1.65) \geq 1.5 \qquad (OK)$$

6. Bearing capacity check: $0 \leq \sigma \leq 3$

$$\sigma = \sigma_{\max 1} + \sigma_{\max 2} = \pm \frac{6(Rd - M_p)}{WB^2} - (\gamma_s h_1 + \gamma_c H)$$

$$\therefore \qquad \sigma_1 = 4.68\, ksf \qquad , \qquad \sigma_2 = 0.716 \qquad (NOT\ OK)$$

Since σ_1= 4.68 *ksf* > 3 *ksf*, we have to revise our model. One way to overcome this problem is by lowering the pipe position. Choose a distance of 9 *ft* from the top of the block to locate the pipe.

Following the same analysis we got σ_1= 3.57 *ksf* which is still more than our limit (σ_1= 3 *ksf*).

Change the location of the pipe to 10 *ft* instead of 9 *ft* and reanalyze, you will get σ_1= 3.05 *ksf* ≈ 3 *ksf*. In this case, lower the pipe to 9.5 *ft* and reanalyze. The result is σ_1= 2.78 *ksf* > 3 *ksf*. Now we are satisfied and confident with the constrains.

Our designed block has the following parameters:

$$W \times H \times B = 30 \times 16 \times 16\ \ ft^3, \qquad d = 9.5\ ft$$

and the volume of the block in this case is $W \times H \times B = 7680\ ft^3$.

6.3 Excel Model Importance

The previous example in which the volume of a block subjected to 1502 *lb* was found to be7680 ft^3. By inserting the same force in our Excel model (sheet 1; *h1* = 3 *ft* and perform the analysis with the condition that $W = 4\,H$ we got the following:

W	H	B	d
49.5	12.4	7.8	7.5

The volume of this block is $49.5 \times 12.4 \times 7.8 = 4796\ ft^3$, our saving in material (concrete) in this case is ~ 40 %. $\left(\frac{7680}{4796} = 0.624\right)$

By changing the condition to $W=8H$ the result is:

W	H	B	d
76.4	9.5	5.9	5.8

The volume of this block is $76.4 \times 9.5 \times 5.9 = 4288\ ft^3$, our saving in material (concrete) in this case is ~ 50 %. $\left(\frac{7680}{4288} = 0.558\right)$

This interesting result is shown in Figure 12.

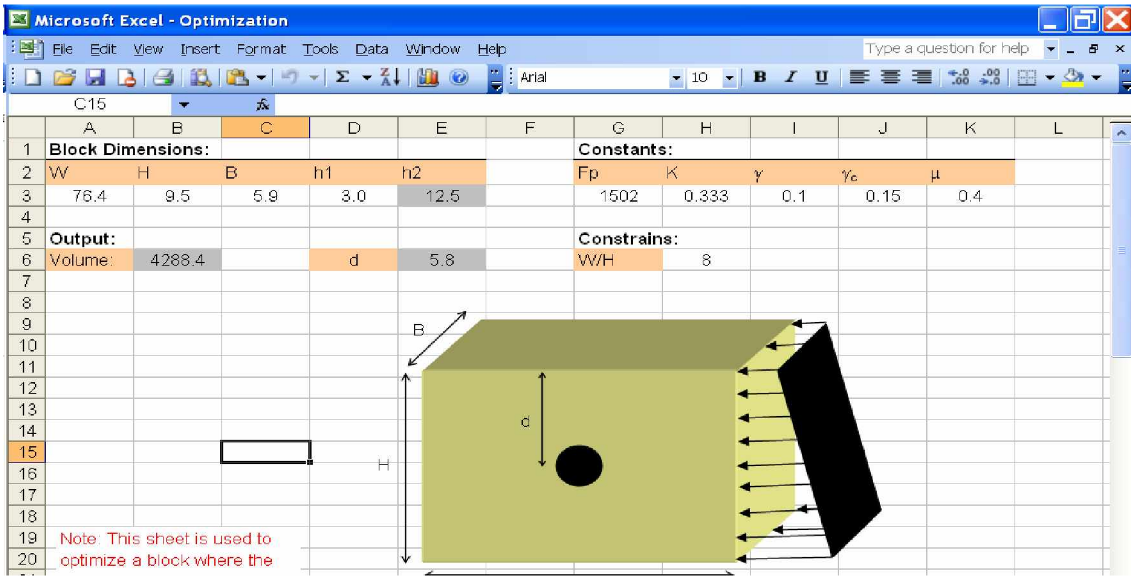


Figure 12: Output for Fp=1502 *lb*

This model can also be used to analyze and investigate the stresses that a block with its given dimensions will undergo.

We are now in a stage to construct our tables and study the behavior of the results.

6.4 Available Excel Choices

From the Excel constructed model, as explained in the Excel Model Construction section, we have two different sheets to optimize our block. The first is with a specified parameter *h1* and a variable *W/H* while the second enables the user to specify both ratios *W/H* and *h1/H*.

From those two different sheets several results (dimensions) are obtained for forces with a constant increment.

In the Appendix part Table 1 shows the dimensions required for blocks subjected to forces range between 200 *lb* to 2000 *lb* for *h1*=3 *ft* and different ratios of *W/H*.

The other table Table 2 displays the dimensions required for blocks subjected to forces range between 500 *lb* to 2000 *lb* with different *W/H* ratios and *h1/H*.

7. Output Analysis

For the case where *h1 = 3* ft and *W/H* = 8, the data obtained from the optimization for the forces applied and their corresponding dimensions are plotted in Chart1 -see the Appendix part-. All the dimensions but *W* seems to be linearly related to the force. If we re-plot our data in a semi-logarithmic scale, a better behavior is shown. By requesting for best fitting, Excel shows a fit that passes through all the points of the plot and shows also an exponential relationship (equations) between all the dimensions and their corresponding forces. From those figures we can say that for a given force the volume can be found by multiplying the exponential equations for each parameter that is involved in the volume computation and using the force as our abscissa value (x-coordinate).

For example for the case shown in Chart 3 we have:

$W = 3.2154\ x^{0.3736}$, $H = 0.8039\ x^{0.3736}$, and $B = 0.3968\ x^{0.4072}$

Then the volume of a block for a given force x can be expressed as: $V = W\ B\ H = 1.0257\ x^{1.1544}$

For example; if we are seeking for the dimensions of a block that resists a force of 1000

Then $W = 3.2154\ 1000^{0.3736} = 42.47 ft$, $H = 0.8039\ 1000^{0.3736} = 10.617 ft$ and $B = 0.3968\ 1000^{0.4072} = 6.6095 ft$

And the volume of the block needed is $V = 1.0257\ 1000^{1.1544} = 2980\ \text{ft}^3$ or in another way the weight of the concrete block is $\gamma_c V = 0.15\ (2980)$ kip

Another observation about the dimensions of the block that is for both the W and H dimensions, the variable x is raised to the same power. Also it is mentioned that $W=4H$ and for it is clear that the coefficient of W is four times more than that of H.

Let us investigate the situation with $W=8H$;

For the case shown in Chart 2 we have

$W = 4.3888\ x^{0.3905}$, $H = 0.5486\ x^{0.3905}$, and $B = 0.4239\ x^{0.3593}$

Then the volume of a block for a given force x can be expressed as: $V = W\ B\ H = 1.0206\ x^{1.1544}$

Another interesting observation is that the variable x for the volume is raised to a power of 1.1544 in the first case and to 1.1544 in the second case which may suggest that the volume has a variable x raised to a constant power in all situations.

Our expected parameters may be written on the form $W = i\ x^{\alpha}$, $H = j\ x^{\alpha}$ and $B = k\ x^{\beta}$ where $i=n\ j$ gives a volume of $V=i\ j\ k\ x^{2\alpha+\beta}$.

As a result from this, for two special cases that is when $W=4H$ and $W=8H$ we can express the dimensions of the block as a function of the force x as

For $W=4H$: $W = 3.2154\ x^{0.3736}$, $H= 0.8039\ x^{0.3736}$,$B=0.3968\ x^{0.4072}$

and $d=0.4661x^{0.3794}$

For $W=8H$: $W = 4.3888\ x^{0.3905}$, $H= 0.5486\ x^{0.3905}$, $B= 0.4239\ x^{0.3593}$

and $d=0.2743x^{0.4162}$

Where the x is expressed in pounds and the output dimensions are in feet.

Conclusion

In our project we gave a detailed analysis for a cubic anchor block and the target, the design part, was to obtain the optimum dimensions of the block which minimizes the cost. The block was optimized and the results were verified and compared with the current techniques used in calculating the dimensions of such blocks. It was found that about 50% of the material can be saved with this model.

The Excel model enables the user to optimize the block under several conditions and also give him the opportunity to analyze the forces act on a block with known dimensions. Several methods that meet the requirements of practical conditions were provided as well in order to find the proper dimensions of a block for a given pipe force. To find the proper dimensions, the user may use the tables, chart, or the provided formulas.

Appendix